Thermo-Hydric Modeling of The Water Retention Curve Based on The Hydric Model of Van Genuchten

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Abstract – In this paper, we propose an extension for a model of unsaturated soils developed by Van Genuchten (1980) and obtain a thermos-hydric model to study the influence of temperature on the water retention curve. A brief presentation of the Model is described using the independent parameter modeling method. The proposed hydrometric Model makes it possible to predict, from the experimental measurements carried out on drainage-humidification paths for an ambient temperature, the WRC (water retention curve) at high temperatures while knowing the initial state of the soil studied (Compacted or in the form of a paste). We show in this Model the existence of the hysteresis phenomenon between the drainage and humidification path and the shift of the downward retention curves showing a slight decrease in the water content as the temperature increases. To validate this Model, three experimental results from the literature are simulated. The results obtained by simulating the experimental curves show the ability of the proposed Model to predict WRC at high temperatures. These results considerably reduce the number of experimental trials in geotechnical and geothermal unsaturated soils.

Keywords: Water, retention curve, Drainage, Humidification, Suction, Temperature

Introduction

Many authors have been interested in modeling the water retention curve. Since the relationship developed by Gardner (1958), many authors have endeavored to improve the mathematical adjustment function of the water retention curve. The different successive empirical approaches are based on power law and exponential functions. Among the main results are: Brooks and Corey (1964), Van Genuchten (1980), Williams et al. (1983), and others. In general, these models are based on the assumption that the shape of the characteristic curve depends on the distribution of the size of the pores at ground level. Fredlund et al. (1994); show that if the particle size distribution is uniform, deducing its characteristic curve is possible. However, these techniques appear to be limited in the case of gravelly soil. The models cited above give the water retention curve at ambient temperature, but higher temperatures in various applications may stress soils. We can cite the main ones: - The storage of nuclear waste, whether in deep geological formations or surface deposits. -High voltage cables buried in the ground at shallow depths.

The Van Genuchten (1980) model does not consider the effect of temperature on the water retention curve, although it is the Model most used in research work. This article aims to simulate the effects of temperature with the Salager et al. (2010) model by taking as a reference curve (at T=0°C) the Simulation made by the Van Genuchten model.
Modeling Development
Modeling of Thermo-Hydric comportment

The influence of temperature on the water behavior in porous media is due, in particular, to the thermal expansion of the phases as well as to variations in the surface tension of the liquid and the wetting angle (Bachman et al. 2002). The Model used results from the development of the suction differential ($\partial s$) with respect to the three independent variables: volumetric water content ($\theta$), temperature ($T$), and the void ratio ($e$). This development has already been developed by Salager et al., 2010, and the only difference is that the retention curve at ambient temperature will be modeled by the water model of Van Genuchten (1980). Only a water-loading path in drainage from a state saturated with pure water is considered here. This excludes any situation involving the phenomenon of hysteresis. The suction differential is expressed in the form:

$$ds = \left(\frac{\partial s}{\partial \theta}\right)_{T,e} d\theta + \left(\frac{\partial s}{\partial T}\right)_{\theta,e} dT + \left(\frac{\partial s}{\partial e}\right)_{T,\theta} de$$

(1)

This relation is developed by expressing the various terms of partial derivatives starting from physical relations. The volume water content ($\theta$) can be expressed as a function of the specific densities of the solid and liquid phases, the specific water content ($w$), and the void ratio ($e$):

$$\theta = \frac{\rho_s^* w}{\rho_e^*(1+e)}$$

(2)

The infinitesimal variation ($dq$) as a function of the three variables, water content, temperature, and void ratio, deduced from Eq.2, is expressed in the form:

$$d\theta = \frac{\rho_s^*}{\rho_e^*(1+e)} dw - \frac{\rho_s^* w}{\rho_e^*(1+e)^2} de + \frac{w}{\rho_e^*(1+e)} d\rho_s^* - \frac{\rho_s^* w}{\rho_e^*(1+e)^2} d\rho_e^*$$

(3)

By introducing the thermal expansion coefficients:

$$\beta_e = -\frac{1}{\rho_e} \frac{d\rho_e^*}{dT}$$ (Coefficient of thermal expansion for water phase)

$$\beta_s = -\frac{1}{\rho_s} \frac{d\rho_s^*}{dT}$$ (Coefficient of thermal expansion for solid phase)

We can express the infinitesimal variation ($d\theta$) in the form:

$$d\theta = \frac{\rho_s^*}{\rho_e^*(1+e)} dw + \frac{\rho_s^* w}{\rho_e^*(1+e)} (\beta_e - \beta_s) dT - \frac{\rho_s^* w}{\rho_e^*(1+e)^2} de$$

(4)

Eq.2 also leads to:

$$\theta = \frac{\rho_s^* w}{\rho_e^*(1+e)} \left(\frac{\partial s}{\partial \theta}\right)_{T,e} = \frac{\rho_s^* (1+e)}{\rho_s^*} \left(\frac{\partial e}{\partial w}\right)_{T,e}$$

(5)

Water retention is mainly due to capillary forces. It can therefore apply Jurin’s law which connects the suction to the surface tension of water ($\sigma_s$) to the mean radius of the pores ($r$) and the wetting angle ($\phi$):

$$s = \frac{2\sigma_s \cos \phi}{r}$$

(6)

The partial derivative of the suction with respect to the temperature, deduced from Jurin’s law, is:
\[
\theta = \frac{\rho_s \cdot w}{\rho_w (1 + e)} \left( \frac{\partial s}{\partial T} \right)_{\theta, e} = s \frac{d\sigma_s}{dT} - s \tan \phi \frac{d\phi}{dT}
\] (7)

The relation of Eq. 7 shows that the surface tension and the wetting angle depend only on the temperature. By noting \((F_w)\) is the inverse of the retention curve function at constant temperature and constant void ratio, and by introducing Eq. 3, Eq. 5, and Eq. 7 in Eq. 1, we obtain a new expression of the suction differential:

\[
ds = F_w dw + \left( F_w w (\beta_e - \beta_s) + s \frac{d\sigma_s}{dT} - s \tan \phi \frac{d\phi}{dT} \right) dT + \left( \frac{\partial s}{\partial e} \right)_{T, \theta} - \frac{F_w w}{(1 + e)} de
\] (8)

Equation (8) consists of three terms that reflect the respective contributions of variations in water content, temperature, and void ratio, to the infinitesimal variation in suction. The first two terms strongly link suction, water content, and temperature.

The third term becomes negligible in front of the two others in the case of a deformable solid. Its experimental determination throughout a thermo-hydro-mechanical process is not easy. Moreover, the infinitesimal variations of the void ratio are expressed here by external mechanical stresses null, which limits the deformations. In the first approach, this term will be neglected. The relevance of this choice will be discussed later. Likewise, the variation of the specific density of the solid phase as a function of the temperature will be neglected compared to that of water. Finally, the absence of experimental data does not allow the variation in the wetting angle to be considered a temperature function. All these considerations lead to a simplified expression of Equation (8) in the form:

\[
ds = F_w dw + \left( F_w w (\beta_e - \beta_s) + s \frac{d\sigma_s}{dT} - s \tan \phi \frac{d\phi}{dT} \right) dT + \left( \frac{\partial s}{\partial e} \right)_{T, \theta}
\] (9)

In order to obtain an equation for the effects of thermohydralic change in Eq. 8, we can consider, in the case of the hydric loading path, a specified suction \((s = cst, ds = 0)\). In the specific case of the thermohydric process, the void ratio varies with suction and temperature \((e = e (s, T))\). However, the temperature has a negligible effect on the void ratio concerning suction (Salager et al., 2010), so \(e = e (s)\). Under these assumptions, is specify a suction result for a specific void ratio \((ds = 0 and de = 0)\). Finally, Eq. 8 can be reduced to a ratio of water content and temperature:

\[
dw = \frac{F_T}{F_w} dT
\] (10)

\(F_T\) and \(F_w\) equations are defined as:

\[
F_T = F_w w (\beta_e - \beta_s) + s \frac{d\sigma_s}{dT} - s \tan \phi \frac{d\phi}{dT}
\] (11)

\[
F_w = \left( \frac{\partial s}{\partial w} \right)_{T, e}
\] (12)

With:

\[
\frac{\partial s}{\partial w} = \frac{1}{w_{sat} \left[ 1 + \left( \frac{a}{a} \right)^n \right]}
\] (13)

After determining \(F_T\) and \(F_w\), Eq. 10 allows us to determine a water retention curve for a specified temperature \(T_0\) and any arbitrary temperature \(T\). For a specified suction, the \(F_T\) and \(F_w\) are negative; as the temperature increases, the water content decreases.
Because of the complex mineralogical composition of soils, the dependence of temperature on the density of the solid phase (represented by the coefficient $\beta$) is difficult to determine. However, this dependence is known to be generally weaker compared with the dependence of the temperature on the liquid phase (represented by the coefficient $\sigma$); one can neglect the coefficient $\beta$ while remaining at a good approximation. In addition, few results are reported about the contact angle and its variations with temperature.

For several sandy soils, Bachmann et al. (2002) showed that the contact angle was smaller than 5°. These authors measured the change in the contact angle as a function of temperature and found that the change was small. So it is normal to set this angle to zero. These considerations imply that the term $\frac{\tan \phi}{dT}$ can be neglected as a good approximation by comparing it with the term $(d\sigma_s/dT)$.

This leads to a simplified expression of the function $F_T$:

$$F_T = F_w w \beta_e + \frac{s}{\sigma_s} d\sigma_s/dT$$  \hspace{1cm} (14)

In the general case, $F_w$ depends on the suction, the temperature, and the void ratio. Nevertheless, experimentally, we note that the retention curves obtained at different temperatures (whether these curves are determined with constant void ratio or not) are almost parallel; their slopes are almost identical. This finding consolidates the hypothesis that the function $F_w$ is independent of temperature. The effect of temperature on water content can be decomposed into two sub-effects: that due to the expansion of the liquid phase and that due to a second term that considers the variation in surface tension. This decomposition is demonstrated by rewriting Eq.9 by replacing $F_T$ with the expression of Eq.14; we receive the following Equation, which models the thermo-hydric interaction in unsaturated soils:

$$dw = -\left(w \beta_e + \frac{s}{F_w \sigma_s} \frac{d\sigma_s}{dT}\right) dT$$ \hspace{1cm} (15)

The coefficient of thermal expansion and the surface pressure of water has been the subject of several studies, and certain expressions have been proposed for their variations with temperature (Salager et al., 2010). For the present Model, we will use the formulas cited by Salager et al. (2010):

$$\beta_e(T) = -4.20 \times 10^{-8} T^2 + 1.17 \times 10^{-5} T - 1.87 \times 10^{-5}$$ \hspace{1cm} (16)

$$\sigma_s(T) = -2.73 \times 10^{-7} T^2 - 1.40 \times 10^{-4} T + 7.56 \times 10^{-2}$$ \hspace{1cm} (17)

In these expressions, the unit of temperature is °C.

The derivative of surface pressure with respect to temperature is determined as follows:

$$\frac{d\sigma_s}{dT}(T) = -5.46 \times 10^{-7} T - 1.40 \times 10^{-4}$$ \hspace{1cm} (18)

From the data of the retention curve at temperature $T_0$, Eq.15 makes it possible to determine, for each suction value, the water content variation as a function of the variation in temperature. The retention curve is obtained at any temperature $T$ (Figure 1).
Simulations of Water Retention Curves

We will present qualitative simulations to compare the numerical results with the experimental results of three authors of the literature, namely: Ghembaza et al., 2007, Wan et al., 2014; and Belal et al., 2019.

Simulation for results of Ghembaza et al., 2007

Experimental tests on drainage and humidification paths at several temperatures: 23 °C, 40 °C, 60 °C, and 80 ° are carried out with a sandy clay prepared in the laboratory in the form of a paste at 1.5 wL.

The parameters of the water retention curve determined from the experimental points at 23 °C are given in Table 1. Given the dispersions of the measurements, only the tests at 23°C and 80°C are simulated.

The parameters are necessary for the Simulation of the water retention curve are taken from the curve at ambient temperature, which is why in Table 1, we find the temperature of 23 °C.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T (°C)</th>
<th>a (kPa)</th>
<th>w_hat (%)</th>
<th>n</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage</td>
<td>23</td>
<td>4110</td>
<td>0.39</td>
<td>3.355</td>
<td>0.702</td>
</tr>
<tr>
<td>Humid</td>
<td>23</td>
<td>750</td>
<td>0.39</td>
<td>2.702</td>
<td>0.630</td>
</tr>
<tr>
<td>Drainage</td>
<td>80</td>
<td>290</td>
<td>0.39</td>
<td>1.686</td>
<td>0.407</td>
</tr>
<tr>
<td>Humid</td>
<td>80</td>
<td>104</td>
<td>0.39</td>
<td>1.630</td>
<td>0.387</td>
</tr>
</tbody>
</table>

The parameters \( n \) and \( m \) are determined after the numerical slope of the experimental curves using formula (11b). The other parameters can be measured directly on the graph of the experimental results, namely:

\( \alpha \) is a parameter relating to the water content \( w_h \), corresponds to the inflection point of the water retention curve.
Figure 2. Definition of the quantities necessary for the determination of the parameters $a$ and $w_i$

The different tests are presented, where the material is subjected to water stress at the different imposed temperatures. It can be seen that the Simulation of the retention curve at temperature $T_0$ approaches the experimental curve; presented by Ghembaza et al. (2007) (Figure 3). The computational-experiment comparisons show a good agreement between the experimental measurements and the Model's predictions.

Figure 3. Simulation of drainage-humidification curves at $23^\circ C$ of sandy clay in paste form

In order to study the effect of temperature on the drainage and humidification path, we present the simulation results at elevated temperatures in Figure 4. We generally notice a good agreement between the experimental results and the Model's.
In order to demonstrate the influence of temperature on the water retention curves, the simulation results are superimposed with the experimental measurements at different temperatures, which are represented in Figure 5. There is a slight shift towards the low water retention curves when the temperature increases, reflecting the material's densification. The Model reproduces this phenomenon well. Furthermore, the simulations of the retention curves correctly present the phenomenon of hysteresis for a given temperature.
Simulation for results of Wan et al., 2014

Wan et al., 2014, are studied the drainage paths of bentonite compacted at a density of 1700 kg/m$^3$ and at four temperatures 20 °C, 40 °C, 60 °C, and 80 °C. Experimental results and simulation results of the retention curve for the four temperatures are shown in Figure 6. The parameters of the characteristic curve determined from the experimental points obtained on the drainage path at temperature $T_0$ are summarized in Table 2. These parameters are introduced into the Model to simulate the tests.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T (°C)</th>
<th>a (kPa)</th>
<th>w$_{sat}$ (%)</th>
<th>n</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage</td>
<td>20</td>
<td>700</td>
<td>0.25</td>
<td>1.286</td>
<td>0.222</td>
</tr>
<tr>
<td>Drainage</td>
<td>40</td>
<td>300</td>
<td>0.25</td>
<td>1.243</td>
<td>0.196</td>
</tr>
<tr>
<td>Drainage</td>
<td>60</td>
<td>100</td>
<td>0.25</td>
<td>1.199</td>
<td>0.166</td>
</tr>
<tr>
<td>Drainage</td>
<td>80</td>
<td>50</td>
<td>0.22</td>
<td>1.156</td>
<td>0.135</td>
</tr>
</tbody>
</table>

It can be seen that the simulations of the water retention curve at different temperatures agree well with the experimental results (Figure 6). We can say that the Model correctly reproduces the drainage path, and we can see the shift of the retention curves downwards when the temperature increases.

Simulation for results of Belal et al., 2019

Belal et al., 2019 performed an experimental study on a compacted silty soil treated with cement from the region of Sidi Bel Abbes (Algeria) at 20°C and 50°C (Figure 7). The parameters of the characteristic curve determined from the experimental points obtained on the drainage path at temperature $T_0$ are summarized in Table 3.
Table 3. Parameters deduced from experimental points (Belal et al., 2019)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T (°C)</th>
<th>a (kPa)</th>
<th>w_sat (%)</th>
<th>n</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage</td>
<td>20</td>
<td>0.24</td>
<td>115.6</td>
<td>1.601</td>
<td>0.375</td>
</tr>
<tr>
<td>Drainage</td>
<td>50</td>
<td>0.24</td>
<td>75.38</td>
<td>1.601</td>
<td>0.375</td>
</tr>
</tbody>
</table>

The water retention curves, at 50°C, are shifted downwards. It can also be seen in the retention curves that the high-temperature curves of 50°C have small water contents compared to the ambient temperature curves for the same suction values, which confirms the results of several researchers who observed this decrease in water content at high temperatures (Bachmann et al., 2002, Ghembaza et al., 2007, Fleureau et al., 2007; Jacinto et al., 2009, Salager et al., 2010). This decrease may be due to the evaporation of a quantity of water caused by the high temperature and the heat resulting from the chemical reaction between the soil and cement. Moreover, the Model used correctly reproduces the drainage path.

Figure 7. Simulation of the drainage curve of compacted silt at 20°C and 50°C

Conclusion

The Thermo-Hydric behavior modeling of unsaturated soils remains limited because of the complexity of experimental tests, which represent the basis of numerical modeling and constitutive laws. Indeed, experimental results in this area are not advanced enough to understand and control this behavior well. The water retention curve, which gives the variations in water content as a function of the suction applied, following drainage paths (increasing suction) or humidification (decreasing suction), is a fundamental characteristic of unsaturated soil. It makes it possible to quantify the extent of the attraction exerted by the soil on the water at each water content or degree of saturation. The extent of the suction, which also depends on the microstructure, conditions the movements of water in unsaturated soils and strongly influences the mechanical properties. The Model has proven its efficiency and can predict the water retention curve at high temperatures. In addition, the Model reproduces the hysteresis phenomenon well at different temperatures.
References