

An Analysis of Errors on Mathematical Symbol as a Metaphor in Linear Programming

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Abstract. Symbol sense is crucial in the understanding of mathematical problems comprising various symbols. The misuses of symbols happen due to misinterpretation, which is considered the constraint to learn algebra more comprehensively, including in linear programming. Metaphor is defined as a means to carry over symbol sense, and is used to improve mathematical understanding. This present research was aimed at analyzing errors on mathematical symbol as a metaphor in linear programming. This research was conducted by means of descriptive qualitative design, with test and interview as the instruments. The test was administered to five eleventh graders selected according to highest rates of errors committed. This research has shown that the students committed a number of errors, such as representing symbols as variables, representing numbers, and interpreting symbols as relational operators. Errors which the students committed in constructing mathematical models covered defining the final value, representing numbers, applying inequality system, and interpreting symbols as operation counts. This present research has provided some ways for symbol sense, and thus the errors on mathematical symbol as a metaphor could be lessened. This research can be further followed up by reviewing the effectiveness of remedial instruction according to the committed errors on mathematical symbols.

Keywords: error, representation, mathematical symbol, metaphor

Introduction

The concept of algebra has vastly applied to any contexts of life (Fu'adiah, 2018; Malihatuddarjah & Prahmana, 2019). The varying algebra applications have become the most essential things to learn. It is alleged that algebra constitutes an abstract concept as it depicts a lot of Greek symbols (Yunarni, 2015). As a consequence, algebra is categorized as the hardest course amidst senior high school students (Rahmawati & Permata, 2018). Interpreting symbols, therefore, becomes crucial in the understanding of mathematical problems formulated through the use of various symbols (Rini, Hussen, Hidayati, & Muttaqien, 2021). The misuse of symbols constitutes a serious hindrance to learn algebra more deeply due to several factors, i.e., limited understanding on interpreting mathematical symbols, lack of creativity in connecting the basic concept of algebra with other mathematical concepts, inability to select and understand the most appropriate mathematical formulation, and tendency to memorize (Fridgo, Yenti, & Heriyanti, 2016; Malihatuddarjah & Prahmana, 2019). Many students commit errors in understanding problems in linear programming (Rahmawati & Permata, 2018). Errors in interpreting, understanding, and using the algebraic symbols, especially in linear programming context, are

compelling to be researched. An error, in addition, is defined as a distortion that occurs when solving mathematical problems (Herutomo & Saputro, 2014). Another definition refers the term 'errors' to a deviation that counterfeits something deemed right or existent (Wijaya, 2013; Wahbi & Bey, 2015). An error is also meant as a structured and constant deviation that violates the truth (Setiawan, 2018). In other words, an error signifies a distortion that intrudes upon the truth or the right procedures formerly arranged (Hidayati, 2019; Setiawan, Hapizah, & Hiltrimartin, 2018). Regarding the aforementioned notions, in this research, an error is indicated as a representation of distortion that violates laws or algorithms in solving mathematical problems of which truth has been formerly stipulated.

Algebra is one of many topics in mathematics that demonstrates how to set variables and sizes by means of symbols, alphabets, and other representative codes (Kaput, Carraher, & Blanton, 2017). According to Kaposi, Kovács, and Altenkirch (2019), algebra refers to a model of sets with various types of equation. Algebra in this research is referred to a branch of mathematics that has many things to do with numbers, mathematical operations, mathematical premises, variables, constants, and coefficients. Accordingly, all of those will be considerably helpful to solve specific problems encountered in daily life.

Mathematical symbols manifest the representatives of mathematical ideas formulated through codes or symbols (Goldin, 2020). Symbols that exist comprise numbers, operation counts, relational operators, and algebraic symbols (Miftah & Orlando, 2016; Sulastri, Marwan, & Duskri, 2017). In addition, symbols represent the external dimensions of thoughts concerning on mathematical ideas (Aristiyo, Rochmad, & Kartono, 2014; Feny, Budayasa, & Lukito, 2017). It is asserted that mathematical symbols constitute one of the most effective mathematical communication means to carry over, encoding upon mathematical ideas (Lutfianannisak & Sholihah, 2018; Zulfah & Rianti, 2018). In other words, mathematical symbols depict mathematical representatives as the means of interpreting, communicating, and converting mathematical ideas to the forms of numbers, operation counts, relational operators, and algebraic symbols.

Symbols in mathematics are included into a metaphor, which takes some forms, such as object collection, small-ranked assessment without counts, and set matching in one-to-one correspondence (Wagner, 2013). Additionally, metaphor is also the core of mathematical instruction and the vein of our thoughts about mathematical ideas. It is used to develop abstract mathematical concepts and is a challenging phase to be represented through concrete analogies (Presmeg, 2013). Further, metaphor is defined as a means of making meanings over symbols (Veraksa, 2013). In other words, metaphor offers an ease to learn mathematics more broadly (Malviya, 2019). According to a previous research about symbols as a metaphor, it showed that

students were unable to represent symbols as a metaphor (Zukhrufurrohmah & Putri, 2019). Another research also revealed that the seventh graders relatively acquired low level of competence in using symbols and formulas (Primayanti, Suwu, & Appulembang, 2018). Further, a research conducted to analyze the results of eight items on PISA test demonstrated that most of students were capable of representing symbols, particularly in using formula to define the area of a square, in spite of their inability to formulate problems into the expected mathematical expressions (Zulfah & Rianti, 2018). The other research on nine students showed that the moderate to low achievers could not apply mathematical symbols properly (Arifin, Trapsilasiwi, & Fatahillah, 2016). Accordingly, this present research aimed to analyze errors on mathematical symbol as a metaphor in linear programming. The linear programming, in general, includes a number of symbols, and is in need of reasoning in solving any given problems (Fannie & Rohati, 2014). More importantly, this current research was focused on the students' errors in interpreting mathematical symbols existing in linear programming. Prior to this, Zukhrufurrohmah and Putri (2019) had researched students' recognition in representing derivative partial symbols as metonymy and metaphor. To be particular, the statements of the problem of this present research are formulated as follows: [1] what are the errors on mathematical symbols that appear as the metaphor in linear programming? and [2] how are the errors on mathematical symbols that appear as the metaphor committed in linear programming?

Method

This current research employed a descriptive qualitative design, specifically a case study. There were five senior high school students recruited as the research subjects; all of whom were from one of senior high schools in Gresik Regency, East Java, Indonesia. The subjects were selected based on the rates of errors committed, namely the highest (S1), the moderate (S2 and S3), and the lowest (S4 and S5). To collect the data, a test and interview were conducted, with the test items and interview guideline validated by the experts. The instruments underwent necessary revisions according to the feedback and suggestions from the experts. The series of activities were recurrent until the instruments were stipulated as valid. The test items (Question 1 and 2) are shown in Figure 1.

In the beginning, the test was administered to the ten senior high school students selected. The test was essay-formatted and related to linear programming. The test, furthermore, was analyzed to locate errors on mathematical symbol as the metaphor. Then, five students with the highest rates of committed errors were reselected in order to obtain deeper information about the symbol errors, to comprehensively investigate the students' understanding on interpreting the symbols, and to examine the causes of errors committed by the students.

1. A company produces two types of goods, A and B. To produce the goods, two machines are needed. Goods A is produced using Machine I for two hours and Machine II for two hours. Meanwhile, Goods B is produced using Machine I in an hour and Machine II in three hours. Only 8 hours remain for Machine I and 12 hours for Machine II. If the company earns a profit of about IDR 5,000 from Goods A and IDR 7,000 from Goods B, how much profit could the company earn?
2. A mother buys 12 grams of butter and 24 grams of flour as she is about to make Rainbow and Brownies cakes. It is estimated that each Rainbow cake will need 2 grams of butter and 8 grams of flour; while the Brownies cake needs approximately 3 grams of butter and 3 grams of flours. If the mother wants to make at least one baking sheet for each cake, how is its mathematical model? Then, define the distribution of the area.

Figure 1. The test items

The data analysis was conducted in three phases, comprising data reduction, data display, and conclusion drawing. Data reduction was done by selecting necessary data according to the results of test and interview. Data display was presented in a form of narrative text containing the misuses of mathematical symbols as the metaphor in linear programming completed with the possible causes. Further, the errors in representing the symbols were focused on numbers, operation counts, relational operators, and variables applied either in the mathematical models or in the procedures of solving linear programming problems. The causes of the errors were described based on the symbols that appeared as the metaphor, which was referred to interpreting the mathematical symbols (Malviya, 2019). At last, the results of the test and interview with the subjects were concluded on the basis of errors committed in interpreting the symbols centered to numbers, operation counts, relational operators, and variables.

Results and Discussion

Errors in Representing Symbols in The Mathematical Model

Alluding to the analysis on the results of the test and interview with the subjects who committed errors in designing mathematical models, the first error highlighted the point of “*the mother is about to make at least one baking sheet for each cake*”. According to the excerpt, a mathematical model in the form of symbolic representation was possible to design, $x \leq 1$ and $y \leq 1$. However, in this case, S1, S3, and S4 could not make any representations. The subjects, contrariwise, showed the tendency of not writing nor adding any representative mathematical models.

This sort of error could be caused by some factors. First, the subject was unable to interpret the question related to the design of verbal representation requiring conversion to symbolic representation. Second, the subject did not acquire the basic concept of mathematics

and linier programming. Such an incapability made the subject unable to formulate the verbal form into the symbolic representation. A sample error committed by S4 is shown in Figure 2.

Diketahui =

	Roti A	Roti B	Persediaan
Mentega	2	3	12
tepung	8	3	24

① $2x + 3y \leq 12$
 ② $8x + 3y \leq 24$

Ditanya :
 Daerah penyelesaian ?

Figure 2. The error committed by S4 (incomplete representation of the problem into the model of $x \leq 1$ and $y \leq 1$).

In addition, the following demonstrates the excerpt of interview with S4 regarding the committed error:

- P : Could you please show me the mathematical models that represent Question 1 and 2?
- S : For Question 1, it's $2x + y \leq 8$ and $2x + 3y \leq 12$; while for Question 2, it's supposed to be $2x + 3y \leq 12$ and $8x + 3y \leq 24$.
- P : What is the most appropriate model to represent 'to make at least one baking sheet of each'?
- S : Perhaps, I need to add $x \geq 0$ and $y \geq 0$ because the phrase 'at least' probably means that 12 and 24 appear to be the minimum limit. So, it can exceed the values, I guess.

The second error was committed by S1 and S4 in interpreting symbols as variables. Such an error happened since the subjects had yet to understand the variables. During the interview on this typical error, some of the subjects interpreted the notion of variable as: 'an object used to locate operational numbers, which is regularly symbolized through non-capital alphabets'; and 'a value that is possible to change within a set of given operations and is commonly stated using alphabets, both capital and non-capital'. According to the interview result, it can be summed up that variables are interpreted as symbols that substitute numbers with their values that remain unknown. The subjects, in addition, also assumed that variables constituted any objects represented by means of capital and non-capital alphabets. Another error was detected when the subjects were still incapable of interpreting the variables correctly. The subjects represented Variable x to substitute *the butter* and Variable y for *the flour*. Ideally, Variable x could be referred to the number of Brownies cakes; while Variable y is for the Rainbow cakes, and vice versa. The subjects believed that converting the problems to the other forms of variables could

help them solve the problems so that double representations on the variables existed. The students were accustomed to representing the number of Brownies cakes as Cake A, and Cake B for the number of Rainbow cakes, and *vice versa*. Afterwards, they began to represent them into the other forms of variables, x and y , which they thought it would help them effectively. The series of errors committed by S1 and S4 are showed in Figure 3.

Diketahui =

3. Mentega x
Tepung terigu y

Rainbow cake $> 2x + 8y$
Kue brownis $> 3x + 3y$

	Roti A	Roti B	Persediaan
Mentega	2	3	12
tepung	8	3	24

① $2x + 3y \leq 12$
② $8x + 3y \leq 24$

Ditanya :

Daerah penyelesaian ?

Figure 3. The errors committed by S1 and S4 in interpreting variables

The third error occurred in representing numbers. The question is, 'each Rainbow cake needs 2 grams of butter and 8 grams of flour; while each Brownies cake needs 3 grams of butter and 3 grams of flour.' The verbal representations that follow could be the best mathematical symbols for the problem, $2x + 3y \leq 12$ and $8x + 3y \leq 24$. However, in this case, they committed errors in representing symbols as numbers for they wrote the mathematical model as $2x + 8y \leq 12$ and $3x + 3y \leq 24$. In the same questions, but different statements, it is stated that *the mother is about to make at least 'one baking sheet' for each cake* (Rainbow and Brownies). The verbal representations of such a statement were possible to be notated as $x \leq 1$ and $y \leq 1$. Nonetheless, the subjects interpreted the mathematical model as $x \geq 0$ and $y \geq 0$. When the students were interviewed and showed that they made errors in using the procedures, they tried to reply, "Well, perhaps the good answer is supposed to be $x < 0$ and $y < 0$." The uttered statements indicated mathematical symbol errors the students committed in representing numbers. This was because the subjects were not able to understand the problem so as to make them think that $x \geq 0$ and $y \geq 0$ are permanently set as the primary law to answer questions in linear programming. This could happen since the subjects did not completely acquire the concept of basic mathematics satisfactorily, particularly on numbers. This weakness caused them to be incapable of representing numbers. One of the erroneous answers was made by S5 as shown in Figure 4 below.

Nama Kue	Bahan	Bahan
Rainbow cake	mentega 2 gr	tepung terigu 8gr
Kue Brownis	mentega 3gr	tepung terigu 3gr
tentukan daerah penyelesaiannya!		
$2x + 8y \leq 12$		
$3x + 3y \leq 2y$		
$x \geq 0$		
$y \geq 0$		

Figure 4. The error committed by S5 in representing numbers

There were some errors committed by S2 and S5 in interpreting relational operators. In the question, it is stated that *the mother would make at least one baking sheet of each cake* (Rainbow and Brownies cakes). The phrase *at least one baking sheet of each cake* meant 'lower than' or 'equal to' ($\leq$) 1. However, in this case, the errors were found in the interpretation of the terms '*at least one baking sheet of each cake.*' S5 interpreted it as 'more than' or 'equal to' ($\geq$) 0; while S2 wrote $x < 0$ and $y < 0$. Based on the committed error, S5 changed the meaning into '*the mother did not make any or she made more,*' with 0 value included. The student also interpreted the terms '*at least*' as the minimum limit, which was referred to the lowest value which might overvalue the others. Meanwhile, the error committed by S2 was evident in the altered concept into '*the mother did not make any cake or less than 0.*' To depict the errors, the following is the excerpt of interview with S2 and S5 in accordance with their misinterpretation upon symbols.

- P : How will you notate the most appropriate mathematical model to represent the phrase '*making at least one baking sheet of each cake*'?
- S2 : It's supposed to be $x \geq 0$ and $y \geq 0$
- S5 : Uhm, I think it will be $x < 0$ and $y < 0$."

Their answers strongly indicated the errors they committed in the application of mathematical symbols regarding relational operators. The errors, furthermore, were caused by some factors, namely that the students could not understand the problem, and that they misinterpreted the phrase '*at least*' as 'greater than its minimum value.' It happened due to the fact that the students did not really master the basic concept of mathematics, especially related to relational operators. As a consequence, the students could not represent the symbols as relational operators. Figure 5 shows the error committed by S2 in answering one of the questions.

Ditanya : model matematika ?
$= 2x + 3y \leq 12$
$8x + 3y \leq 2y$
$x \geq 0, y \geq 0$

Figure 5. The error committed by S2 in interpreting relational operators

The errors regarding the notation of mathematical models of specific problems by means of mathematical symbols could happen in some cases. Firstly, the students were unable to encode verbal into symbolic representations. It means that the students were not capable of converting the problems into the appropriate mathematical models. Secondly, the errors occurred due to inconsistency of the students in representing variables. The students also mistakenly interpreted relational operators, such as the use of the symbol ' $\leq$ ', which was due to their incapability of interpreting symbols, especially related to relational operators and variables (Irfan, 2017). The finding of the errors on symbol as the metaphor was also in line with that of the previous study. It was found that the subjects committed errors in designing mathematical models as they could not perfectly convert verbal into symbolic representations, and they could not interpret variables properly as they did not understand the concept (Rahmania & Rahmawati, 2016). Further, the errors committed by the research subjects were similar to those in another previous study. It was shown that the subjects could not interpret symbols holistically and precisely (Zukhrufurrohmah & Putri, 2019). In addition, the errors in designing mathematical models through mathematical symbols were also identical with a previous research that found the students' difficulties in designing mathematical model that fitted the given problems (Hidayah, 2016). Technical errors on variables constituted one of numerous types of errors according to Kastolan's error theory that had something to do with the results of this present research, with the subjects committing errors in interpreting variables (Raharti & Yunianta, 2020).

Errors in Representing Symbols on the Procedure of Solving Linear Programming Problems

The procedure that the students performed was by understanding the given problems, followed by interpreting and converting the problems into appropriate mathematical models. After the models were prepared, the subjects could solve the problems through several phases; some of which were to define general equation of objective function, to complete inequality system, to determine the outer value to depict the target area precisely, and to state the functional value of each of the outer values. In addition, there were some ways to design mathematical models, depending on what was required in the questions. In fact, the subjects were not able to complete the mathematical model correctly. The varied errors were committed by the subjects. Such errors caused inaccuracy in finding out the expected final results. Figure 6 shows the errors in solving the problem.

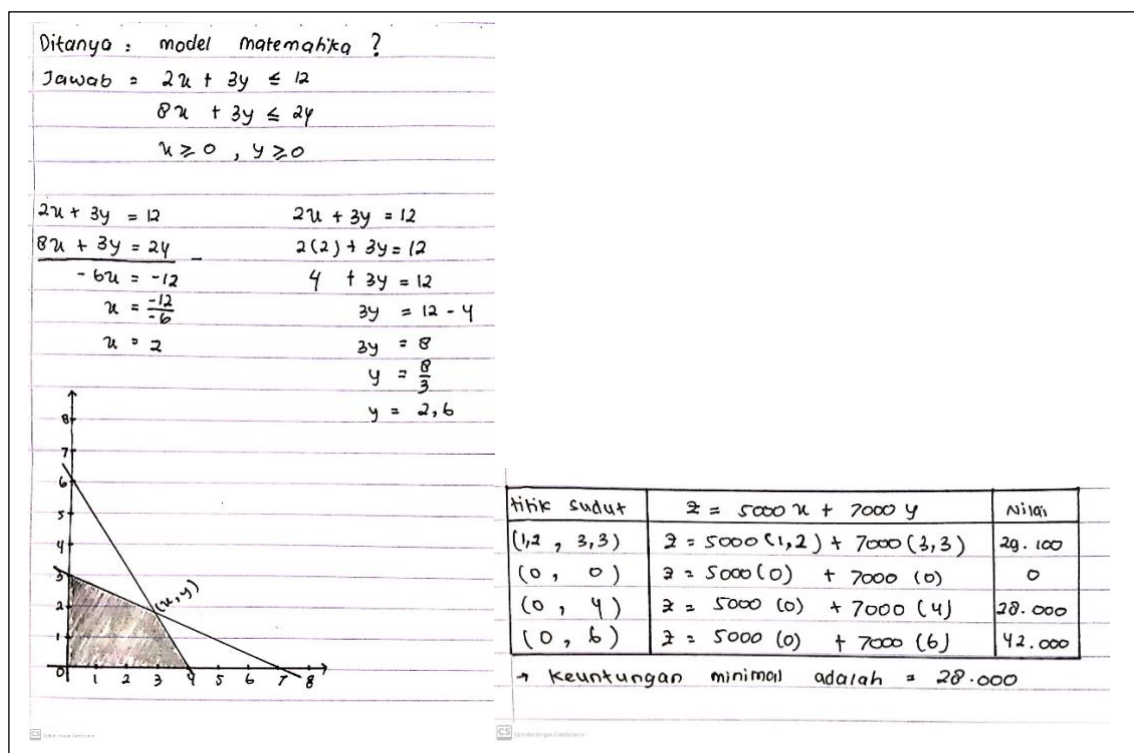


Figure 6. The error committed by S2 in finding out the final result

Inaccuracy in defining the final results occurred when the subjects tried to answer the questions by means of mathematical symbols. Consequently, the errors were present; one of which was committed by S4 in answering the question regarding linear inequality system of two variables. The answer was supposed to be preceded by using substitution procedure with $x = 2$ inserted into the equation of $2x + 3y \leq 12$. It was supposed to be $2(2) + 3y \leq 12 \leftrightarrow 4 + 3y \leq 12 \leftrightarrow 4 - 4 + 3y \leq 12 - 4 \leftrightarrow 3y \leq 8$, and so on (until the value of y was unveiled). Instead, the student committed an error by notating $2x + 3y \leq 12 \leftrightarrow 2(2) + 3y \leq 12 - 4$. It was clear that the student simply wrote the value of the operational result of $2(2)$, which was 4 in the same line with that of in $2(2) + 3y \leq 12 - 4$. In fact, the stage still belonged to the operational procedure in which the result remained mystified. This indicated misrepresentation over mathematical symbols especially on numbers. When getting interviewed to investigate the reasons why such an error happened, the student admitted, *'That's the fastest way.'* Based on the error, the main cause was that the student did not fully understand how to count using the substitution procedure, and believed that the way he took was the fastest step to solve the problem. Such an inability, as also demonstrated by S4, led to the error in representing numbers. To be specific, one of the answers that indicated the error is shown in Figure 7.

$$\begin{array}{l} 2x + 3y \leq 12 \\ 2(2) + 3y \leq 12 - 4 \\ 3y \leq 8 \\ y \leq 2,6 \end{array}$$

Figure 7. The error committed by S4 in representing number

The typical error occurred when the students attempted to work on inequality system by means of the substitution method. Prior to using the method, the elimination method resulted in $y=2$, so that for the substitution in search of x in $2x + y = 8$, it was supposed to be $2x + (2) = 8 \leftrightarrow 2x + 2 - 2 = 8 - 2 \leftrightarrow 2x = 6 \leftrightarrow \frac{2x}{2} = \frac{6}{2}$. Then, $x = 3$ was found. Nonetheless, in such a case, the student committed the error in the application of substitution method. The student mistakenly notated $2x + y \leq 8$, and found that $y = 4$. In fact, the y value remained identified through the elimination method. This sort of error occurred due to several factors. According to the interview session, the student admitted, *"To be honest, I still can't understand how to apply substitution method so I bet I can't do it quite well."* Therefore, it was obvious that the student could not understand the basic concept of substitution method, and thus he could not apply the proper method to solve the problem. S5 indicated the committed error through one of his answers as shown in Figure 8.

$$\begin{array}{l} 2x + 3y = 12 \\ 2x + y = 8 \\ \hline 2y = 4 \\ y = \frac{4}{2} = 2 \end{array} \quad \left\{ \begin{array}{l} \text{Substitusi nilai } y \text{ ke pers } 2x + y \leq 8 \\ \text{diperoleh } y = 4. \\ \text{dengan demikian } c(4, 2) \end{array} \right.$$

Figure 8. The error committed by S5 in answering the question of inequality system

The students, S1, S2, and S5, were found to commit errors in defining the outer value. The errors happened due to their prior errors related to the procedure of defining the outer value. One of the errors referred to the misinterpretation over operational counts. It was notated $2x + y \leq 8$ to define the outer value. With $x = 0$, it was supposed to be $2x + y = 8 \leftrightarrow 2(0) + y = 8 \leftrightarrow y = 8$. Instead, the students mistakenly wrote $2x + y \leq 8$. Then, for $x = 0$, the value became $2x - y = 8 \leftrightarrow 2(0) - y = 8 \leftrightarrow -y = 8 \leftrightarrow \frac{-1y}{-1} = \frac{8}{-1} \leftrightarrow y = -8$. Such an error happened due to several reasons. First, the students interpreted $2x + y \leq 8$ as a constraint function. Second, the students interpreted that $2x + y \leq 8$ was different from $2x - y = 8$ assuming that the operational symbol of addition "+" needed replacement with that of reduction "-". These two factors were referred to what they expressed in the interview, *" $2x + y \leq 8$ is the constraint function; while $2x - y = 8$ is the target function. Therefore, I guess*

both are different that the symbol of addition should be replaced with reduction". This sort of interpretation caused the mathematical symbol errors in regards to operation counts. Further, they existed due to the fact that the students did not really acquire the whole concept of basic mathematics, especially operation counts. Such an inability made the students commit errors in notating the operation counts. Figure 9 shows how S2 committed the error.

$$\begin{array}{l}
 \bullet > 2x + y = 8 \\
 \text{dikur } x = 0 \\
 2x - y = 8 \\
 2(0) - y = 8 \\
 -y = 8 \\
 y = -8 \\
 (0, 6)
 \end{array}$$

Figure 9. The error committed by S2 in notating an operation count symbol

A number of errors were committed by the students in constructing mathematical models, which was parallel with the previous research indicating that the students committed errors when answering essay questions, taking substitution procedure, and applying operation count representations for they were incapable of understanding and interpreting the questions from verbal into symbolic representations in addition to their lack of understanding on the mathematical concepts (Andriyani, 2018; Rahmania & Rahmawati, 2016). Typical errors were also present in another previous research in which the students committed errors in defining the final and outer values (Ayuningsih, Setyowati, & Utami, 2020). The errors that occurred when defining the final value in this research were predictable by a theory of error highlighting mistaken procedures (Hutami, Trapsilasiwi, & Murtikusuma, 2020).

This present research recommends that the teachers be intensive to provide their students with the understanding on the basic concept of mathematics and to drill them with a series of exercises. As this research was relatively limited to the errors committed by the students in mathematical symbols for linear programming, further researches are expected to review the effectiveness of remedial instruction in response to the students' errors in mathematical symbols.

Conclusion

The students' errors on mathematical symbol as a metaphor in linear programming existed in wider extents. First, the students misrepresented mathematical models. It means that they had mistakenly notated verbal forms to symbolic representations. In addition, they committed the errors in defining the final value within the procedures of answering the questions related to linear programming. Next, the students had also committed the errors in

representing numbers, applying equation in inequality system, and interpreting symbols for operation counts. The errors occurred due to the fact that the students did not fully understand the questions, did not acquire the basic concept of mathematics and linear programming, and could not make the mathematical models. Likewise, the students flunked to understand the concept of substitution and how to apply it into a linear inequality system. Based on the findings, the researchers recommend that the students be provided with meaningful instructions that put much focus on mathematical symbols, either through teaching media or a new learning method. The reviewed errors in this present research potentially inspire further researchers to investigate the teaching of mathematical symbols more comprehensively. Accordingly, a proper method should be proposed to reduce students' errors when solving mathematical problems. Further researchers can also review symbol errors in other algebraic concepts.

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