

Characteristics of Students' Metacognitive Ability in Solving Problems using Awareness, Regulation and Evaluation Components

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Abstract. *The process of solving absolute value problems is not only associated with the simplification of equations or inequalities. Students also need to pay close attention, ask the right questions, carry out the right strategies, and acquire adequate information. This step is essential to prevent students with good metacognitive ability from drawing wrong conclusions. The research discusses the metacognitive characteristics of mathematics education students in solving absolute value problems from the awareness, regulation and evaluation components. Participants consisted of 101 students from four state universities in the city of Malang. Data were obtained through written answers, transcripts of think aloud, and interviews. The data collected were analyzed to determine their metacognitive abilities in terms of awareness, regulation and evaluation components. The result showed that the metacognitive ability of low-skilled students only exists in the awareness component, which is thinking about what is being asked. Furthermore, those medium capable of the awareness component still lack adequate thinking ability. In the regulation and evaluation components, students do not realize that there are still inappropriate steps in solving problems and fail to check the correctness of their answers. However, high-ability students can solve problems in different ways and easily distinguish accurate information using effective strategies. Learn how the metacognitive characteristics of students in solving non-routine absolute value application questions, provides space for educators to be able to create appropriate learning models.*

Keywords: *Absolute value, awareness, evaluation, metacognitive, regulation, evaluation*

Introduction

Metacognitive abilities is one of the 21st-century skills increasingly used by students to solve absolute value problems (Zhao, Wardeska, McGuire, & Cook, 2014). According to Elia, Özel, Gagatsis, Panaoura, and Özel (2016) and Magiera and Zawojewski (2011), this ability tends to affect students' success in solving math related-problems. This skill set is important because it includes awareness of problem thinking and monitoring and regulation of cognitive processes (Aurah, Koloi-Keaikitse, Isaacs, & Finc, 2011; Kelemen, Frost, & Weaver, 2000; Young & Fry, 2008). Furthermore, it enables students to select appropriate strategic interventions, monitors the implementation of strategies, and evaluate their effectiveness in solving problems (Li & Nietfeld, 2007). Metacognition is a process in which a person thinks to build strategies to solve problems (O'Neil & Brown, 1998; Setiawan & Supiandi, 2019). Arends (2012) and Gregory Schraw, Gregory and Moshman (1995) defined this skill as peoples'

abilities to know and regulate their cognitive processes. Van de Walle (2007) stated that metacognition refers to conscious monitoring and regulation of one's thoughts. Good problem solvers always monitor their thinking abilities regularly and automatically. They recognize when they have been misled or can't fully understand something and decide to switch strategies, rethink the problem, look for related content, or start from the beginning.

Students with good metacognitive abilities are able to solve mathematical problems adequately, therefore, each needs to possess this skill (Güner & Erbay, 2021; Memnun & Akkaya, 2009). Students' metacognitive abilities show their activities in building relationships, choosing information, and analyzing questions. These activities require control for them to solve the problems at hand adequately.

Several previous studies have discussed metacognition (Boekaerts, 1999; Demirel, Aşkın, & Yağcı, 2015; Efklides, 2006; Oz, 2016; Panaoura, Gagatsis, & Demetriou, 2009; Zhao et al., 2014). For instance, Zhao et al. (2014) stated that students' metacognitive abilities contributed to the process of improving learning in any field, such as mathematics. Purnomo, Nusantara, Subanji, and Rahardjo (2017) researched the patterns and characteristics of students' mathematical metacognition processes in solving calculus problems. Similarly, Misu, Budayasa, Lukito, Hasnawati, and Rahim (2019) described the metacognition profile of mathematics education students based on gender in understanding the concept of integral calculus. According to Kuzle (2019), students' metacognitive actions in solving problems are revealed through action cards. Zhao et al. (2014) explored the metacognition process cognitively in accordance with the learning and problem-solving strategies. Kazemi, Reza, and Bayat (2010) examined metacognitive behavior in solving combinatoric math problems. However, these preliminary studies have not examined metacognition in problem-solving of absolute value. According to Almog and Ilany (2012), El-khateeb (2016), Elia et al. (2016), Yong (2015), Abdallah, Haddou, and Migot, (2018), and Sierpinska, Bobos, and Pruncut (2015) absolute value problem also needs to be studied more deeply because it is difficult for students to solve.

Some researchers explain that the metacognitive component is divided into two according to Tarricone, namely cognitive knowledge and regulation (Flavell, 1979; Oz, 2016; Zhang & Seepho, 2013). Cognitive knowledge is the general understanding and awareness regarding cognition (Oz, 2016). It is divided into three types (Schraw & Dennison, 1994; Young & Fry, 2008), namely (1) declarative, (2) procedural, and (3) conditional. Declarative knowledge is having an adequate idea about someone, including strategies and effective information needed to fulfill the task. Procedural knowledge is the awareness that presents sequences and steps in assembling and carrying out a job. Conditional knowledge is a combination of declarative and procedural information about someone. Metacognitive regulation is peoples' ability to control

their cognitive skills. It consists of three components: awareness, regulation, and evaluation, which are related to students' activities to solve mathematical problems (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004). The awareness activities are rethinking the following, 1) what was asked, 2) the information on the problem, and 3) the next steps that need to be taken to solve the problem. Regulation activities are rethinking the following, 1) ways to solve problems, 2) different ways to solve problems, and 3) what to do next. Evaluation activities are rethinking the following 1) every step that has been taken, 2) whether other considerations are needed, and 3) whether the answer to the problem is correct (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004).

In previous studies, no one had studied the characteristics of students' metacognitive abilities in solving problems. Therefore, based on the facts above, this study aims to describe the metacognitive characteristics of students in the awareness, regulation and evaluation components when solving non-routine absolute value application questions. This study is considered necessary due to its potential ability to help educators determine the metacognitive characteristics of students in solving non-routine absolute value application questions and appropriate learning models. Therefore, the research question is how the characteristics of students' metacognitive ability in solving problems using awareness, regulation and evaluation components?

Method

This is a qualitative research to describe students' metacognitive abilities in solving the absolute value problem. The subjects of this study were 101 semesters VI students of the Mathematics Education study program at four universities in the city of Malang, Indonesia. This semester was used because it provides absolute value problems in calculus and real analysis courses. The selection of several universities was taken to enrich and deepen data about the characteristics of students' metacognitive abilities. Characteristics of metacognitive abilities are categorized into three groups, namely low, medium, and high metacognitive abilities. Snowball sampling was used as the subject taking technique in this study. The snowball sampling technique was carried out by taking data sources that were originally from two universities. However, due to the small number of data sources and unable to provide in-depth data, the data sources were added to four universities.

The instrument developed in this research is a test consisting of one item of absolute value that is used to determine awareness, regulation, and evaluation. The problem instrument is used to determine the set of solutions for absolute value inequalities. To identify the metacognitive characteristics of students in solving absolute value questions, it is necessary to

have absolute value questions that are solved by students by thinking hard. The researcher observes and observes how the characteristics of students' metacognitive awareness appear when students solve absolute value questions.

In addition to the think-aloud test, interviews were also used to validate test result data. The absolute value problem test and interview guide were previously validated by two mathematicians and two educational experts before being distributed to students (Purnomo et al., 2017). The results of the validation showed that the tests and interview guidelines were feasible to be used to explore student characteristics in solving absolute value problems. The developed test instrument is shown in Figure 1.

Determine all values x that satisfy $ x - 1 + x^2 - 3x \geq -\frac{13}{4}$

Figure 1. Absolute value problems distributed to students

Each of the 110 students was given 60 minutes to solve problems and think hard with written answers for clarification. Students are asked to record their own student activities when solving problems in think aloud. Based on the recordings, researchers can analyze students' metacognitive processes repeatedly so that more accurate data is obtained. Data analysis was carried out in six stages, namely 1) transcribing recorded student activities, such as written answers and interviews, 2) performing abstraction to summarize data and reduce those that are needed, 3) performing data coding, 4) grouping the recorded activity results, written answers and interview into 3 parts, namely those with low, medium and high metacognitive abilities, 5) taking one student from each group to explore the characteristics of metacognitive abilities, and 6) making conclusions based on data analysis and discussion previously conducted (Creswell, 2009).

Results and Discussion

The components of metacognitive abilities are measured by administering tests, students think hard when working on questions, and interviews related to problem solving. The metacognitive ability component consists of three components: awareness, regulation, and evaluation, which are related to students' activities to solve mathematical problems (Magiera & Zawojewski, 2011; Wilson & Clarke, 2004). Based on the test results and thinking load, it can be seen that students who have low metacognitive abilities are 35 students or 35%, students who have moderate metacognitive abilities are 56 students or 55%, and students who have high metacognitive abilities are 10 students. or 9%. Subjects in the low metacognitive ability group are S1. Subjects in the medium metacognitive ability group are S2. The subject of the high metacognitive ability group is S3. The following is an explanation of the written answers from S1, S2, and S3.

S1 Data Exposure

Figure 2 shows that S1 was unable to determine the information contained in the question. Therefore, the problem was solved by ignoring the absolute value sign then simplifying the inequality. The author stated that S1 does not understand the use of absolute value signs in solving equations or inequalities. S1 simplifies the form of the inequality by combining like terms without any clear strategy.

$$\begin{aligned}
 &1) |x-1| + x^2 - 3x \geq -\frac{13}{4} \\
 &\text{Jawab:} \\
 &x-1 + x^2 - 3x \geq -\frac{13}{4} \\
 &-2x - 1 + x^2 \geq -\frac{13}{4} \\
 &x^2 - 2x - 1 \geq -\frac{13}{4} \\
 &x^2 - 2x - 1 + \frac{13}{4} \geq 0 \\
 &4x^2 - 8x - 4 + 13 \geq 0 \\
 &4x^2 - 8x + 9 \geq 0
 \end{aligned}$$

Figure 2. S1 written answer

During problem-solving, S1 did a think-aloud, and the process was transcribed as shown in Figure 3.

In the matter $|x-1| + x^2 - 3x \geq -\frac{13}{4}$. Means searched x . How is this... answer... what to do. Then I didn't find the answer. How about it then. I do not know. Confused. Confused. $x^2 - 2x - 1 - 13/4$ also, cannot be factored. $-13/4$ transferred. Match the denominators. If you look for the factors, what do you use, you can't... there are no factors. Then I don't know anymore.

Figure 3. Think-aloud S1 transcript in solving problems

The underlined sentence indicates that S1 reread the question and restated the problem based on the think-aloud transcript. Furthermore, S1 tried to determine the value of x in the question with difficulty and failed to determine the information before solving the problem. S1 repeatedly stated the word "confused," which showed an inability to understand the process. The sentences "Continue to do" and "Then what are you doing" indicate that S1 did not know the next steps to take. The sentence "Then look for x times yes" indicates that S1 is not sure of the right strategy.

Next, S1 was interviewed to obtain the confirmation of the answers based on the problem-solving process.

- P* : Do you understand what is being asked?
S1 : Got it.
P : Please state what is asked in the question!
S1 : What is asked does x satisfy the equation.
P : How did you know what was asked?
S1 : Based on what is asked in the question.

- P : What information do you know regarding the question?*
S1 : It consists of absolute value.
P : How did you know about this information?
S1 : From the question.
P : What is your thought on the sign of absolute value?
S1 : The absolute value is always positive.

The interviews related to problem-solving indicate that S1 knows and understands the question by looking for the value of x that satisfies the equations and inequalities. According to S1, the information contained in the problem indicates that there was an absolute value and fraction sign. This shows that S1 cites information only from the stated question and cannot obtain new ones from the existing absolute marks. Furthermore, S1 also experienced a wrong concept in understanding absolute value material. The absolute value of a real number x is always positive, and S1 assumes that in simplifying an absolute of x , the sign needs to be omitted because it does not affect.

S2 Data Exposure

S2's written answer in Figure 4 shows that it obtains absolute value information based on two possibilities from the elaboration $|x - 1|$. Furthermore, S2 uses a strategy by simplifying the quadratic inequality on each possibility. According to the authors, it has trouble in determining the roots of the quadratic inequality. Therefore, S2 uses the *abc* formula to conclude that the solution to the inequality is in the form of an imaginary number without comparing it with absolute value conditions. Hence, S2 failed to determine the missing step before concluding.

∴ Karena semua nilai x yang memenuhi memuat bilangan imajiner, maka persamaan tersebut merupakan bagian dari bilangan kompleks

Figure 4. Conclusion written answer S2

Since all values of x that satisfy the equations are imaginary numbers, the inequality is part of the complex number.

Figure 5. Translation version

S2 conducted the think-aloud process during problem-solving, which have been transcribed, as shown in Figure 6.

After I noticed an absolute value sign in the problem, I divided it into two parts, namely $x - 1 \geq 0$ and $x - 1 < 0$. Furthermore, I simplified and tried to determine the factor without success. Finally, I tried using the *abc* formula, and it turned out that all values x satisfied the inequality and contained imaginary numbers. This showed that the roots are unreal. Therefore, this inequality is the result of a complex number because they are imaginary.

Figure 6. S2 think-aloud transcript in solving problems

Based on the think-aloud transcript shown in Figure 6, the underlined sentence indicates that S2 carefully asked the questions. S2 thinks about the sign of absolute value in the problem and makes it into two parts. Furthermore, S2 relate the information to obtain a good strategy. However, S2 did not rethink whether the strategic steps taken were appropriate or not, therefore it was impossible to determine the missing data. Next, S2 was interviewed to obtain clarification on the answers provided. The following is an excerpt of an interview with S2 on problem-solving.

- P : What information do you know about the question?
 S2 : Absolute value equation/inequality.
 P : How did you know about this information?
 S2 : Based on the form of the questions presented.
 P : How do you connect the information you obtained to solve the problem?
 S2 : By applying the rule to absolute values.
 P : How did you solve the problem?
 S2 : To determine all x values, first, I divided the equation into two parts, namely for $x \geq 1$ and $x < 1$
 P : What problem-solving strategy did you follow?
 S2 : Already ma'am
 P : Are the steps used to answer the problem, correct?
 S2 : Maybe something went wrong ma'am.
 P : Why did you choose this strategy?
 S2 : I follow as usual, ma'am.

The interviews showed that S2 knows and understands the problem-solving strategy by determining the value of x that meets the equations and inequalities. Furthermore, S2 made a strategy by dividing the absolute value sign into two parts, namely $x \geq 1$ and $x < 1$, and failed to determine the right step left before reaching a conclusion.

S3 Data Exposure

The written answer S3, as in Figure 7, is used to solve the problem of absolute value. S3 makes two cases based on the definition, with each consisting of simpler inequality, such as $x \in R$. Furthermore, S3 analyzed the shape of the inequalities where $x \in R$ apply to inequalities.

☐	$(x-1)^2 \geq -\frac{5}{4}$	$(x-2)^2 \geq -\frac{1}{4}$
☐	$\Downarrow$	$\Downarrow$
☐	untuk sebarang $x \in R$	untuk sebarang $x \in R$
☐	maka $(x-1)^2$ bernilai tidak negatif	maka $(x-2)^2$ tidak negatif
☐	dan jelas akan lebih dari	positif dan jelas akan
☐	Sama dengan $-\frac{5}{4}$	lebih dari sama dengan
☐		$-\frac{1}{4}$
☐	sehingga dapat disimpulkan bahwa semua nilai x yang	
☐	memenuhi adalah $x \in R$ atau dengan kata lain Hp: $\{x x \in R\}$	
☐	(Kita ambil gabungannya)	

Figure 7. Written answer S3 (strategy 1)

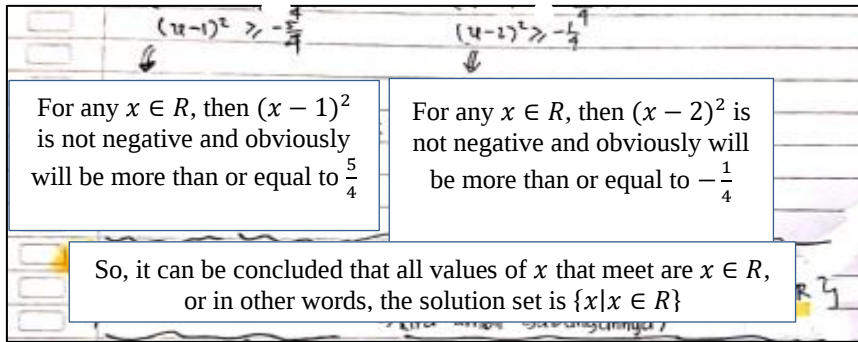


Figure 8. Translation version

While working on the assignment, S3 used the think-aloud process, which was further transcribed. The think-aloud S3 transcript is shown in Figure 9.

For the first one, we were given a question asked to determine all values that meet x is $|x - 1| + x^2 - 3x \geq -\frac{13}{4}$. So here we will use the definition of absolute value. So for $|x - 1|$ it will be $x - 1$ worth for $x \geq 1$ and will be $-(x - 1)$ worth when $x < 1$, with a note here $x \in R$. So here for the first case, we will describe it so that after I simplify it, I write $(x - 1)^2 \geq -\frac{5}{4}$. So, from here, we can analyze that for any value $x \in R$ then $(x - 1)^2 \geq -\frac{5}{4}$, it is not negative and obviously will be. Well for the second case is also similar after simplifying, I found that $(x - 2)^2 \geq -\frac{1}{4}$. $(x - 2)^2 \geq -\frac{1}{4}$ and obviously will $\geq -\frac{1}{4}$.

So after that, I also analyzed that $|x - 1| + x^2 - 3x \geq -\frac{13}{4}$, nowhere for $x^2 \geq 3x$, $x = 0$, $x \geq 3$, $x \in R$ and $x \in R^-$. This $0 < x < 3x \in R$ also applies to inequalities. So from here or from some of the analyzed cases, for the solution, we take the combination. So hereafter I checked too, but I didn't include it, then all x values that meet the absolute value inequality in number 1 are $x \in R$

Figure 9. S3 think-aloud transcript in solving problems

Based on the written answer and think aloud, S3 stated that all values were used to satisfy the inequality. Furthermore, S3 reported that the absolute value definition was used to divide two cases of inequality. Therefore, S3 simplified each inequality to determine a solution to the problem. S3 also seems to know the steps needed to make each strategy in accordance with the absolute value rule. Next, a different strategy was determined by analyzing the problems that exist in the questions. Based on this, a different way of solving the problem was analyzed using the Written Answer before concluding the solution to the inequality.

Furthermore, interviews were conducted to obtain the confirmation of their written answers and think aloud. The following is S3 excerpt based on the problem-solving process.

P : What information do you know regarding the question?

S3 : Information I got from $|x - 1| + x^2 - 3x \geq -\frac{13}{4}$ is that for any x value in $|x - 1|$, x^2 is always greater than or equal to $-\frac{13}{4}$. For $-3x$ we must recheck the value to get the right answer. This is because there is an absolute sign in the question, which is defined in two cases. Therefore, in problem number 1, we use the concept of absolute value to find all x values that satisfy the inequality.

P : How did you connect the information you obtained to solve the problem?

- S3 : *From the information, I figured out the resolution strategy to use.*
 P : *How did you solve the problem?*
 S3 : *Using absolute value definition, I divided the case of inequality into two, which allowed me to find a solution.*
 P : *What problem-solving strategy did you follow?*
 S3 : *The appropriate problem-solving strategy.*
 P : *Are the steps you made, correct?*
 S3 : *I'm sure it's correct because I rechecked them.*
 P : *Why did you choose this strategy?*
 S3 : *Because I think it is in accordance with the concept.*
 P : *Have you tried other strategies before?*
 S3 : *Yes, I also used the analysis of absolute and squared values.*
 P : *Are you sure of your answer?*
 S3 : *Yes, because I double-checked and substituted my answers into the given questions.*

Based on interviews with S3 related to problem-solving, the concept of absolute value was determined. Furthermore, the right information was obtained from the problem using two events. The first uses the definition it, while the second analyzes the shape of the inequality to find a solution to the inequality.

Based on the written answers, think-aloud transcripts, and interview results showed that S1(low metacognitive abilities) only thinks of the question without concluding the awareness component. The components of the S1 regulation are not rethinking whether the strategy made is right or wrong to solve the problem. Furthermore, the S1 evaluation component does not recheck the solutions to determine the right strategies. Based on the above, the metacognitive ability of undergraduate students is low because they do not rethink the information contained in the questions. Santoso, Napitupulu, and Amry (2019) states that students with low problem-solving abilities are at the level of using metacognitive tacit. According to Wilson and Clarke (2004), a person's awareness while carrying out a task helps determine the information. Adinda, Purwanto, Parta, and Chandra (2021) stated that one of the failures of students' metacognitive awareness in solving absolute value problems is failing to obtain information. Swartz and Perkins (2017) stated that the lowest level of individual metacognitive abilities is that individuals solve problems and make decisions without really thinking. This shows that S1 in the awareness component only think of the questions without fulfilling the characteristics of the regulation and evaluation components.

Based on the written answers, think-aloud transcripts, and the results of interviews with S2 (medium metacognitive abilities), the characteristics in the awareness component are related to the information obtained to solve problems. The regulatory component is used to determine the strategy used to solve the next problem differently. In the evaluation component, S2 failed to rethink the characteristics of the steps, which are still wrong. Purnomo et al. (2017) stated that a person's awareness component includes relating questions to similar problems obtained and

solved previously. According to Purnomo et al. (2017), students do not realize that the strategic steps in solving the absolute value problem are still wrong or still missing. Amram, Dagan, Satianov, and Yoshpe (2019) reported that one of the difficulties students have been solving absolute value problems using many technical steps.

Based on written answers, think-aloud transcripts, and the results of interviews with S3, the awareness process is related to the information obtained to solve the problems. The S3 regulatory process has rethought the steps in solving problems, using different strategies. According to Akben (2020), people with good metacognitive abilities need to determine the right strategy. Swartz and Perkins (2017) also stated that they must organize their thinking to select these strategies. In the Evaluation process, S3 rethinks the suitability of the strategy to determine whether the solution is correct. The metacognitive characteristics of S3 are also in accordance with the indicators in the research carried out by Wilson and Clarke (2004). According to Jia, Li and Cao (2019), S3 can be classified as a student with good creative thinking skills. Besides that, good metacognitive abilities can also improve students' critical thinking and creative process (Syaiful, Huda, Mukminin, & Kamid, 2022).

High-ability students are able to find solutions in two ways, namely procedural and by paying attention to the shape of the inequality in the problem. Meanwhile, students are always capable of errors, therefore, they do not realize that by analyzing the form of the question without working procedurally, the problem can be solved. This is in accordance with the research carried out by As'ari, Mahmudi, and Nuerlaelah (2017), which stated that students are used to carrying out procedural activities, therefore, they are not unable to determine real problems.

Data analysis in this study documented 3 characteristics of students' metacognitive abilities, namely awareness, regulation, and evaluation components. An explanation of the characteristics of each type is shown in Table 1.

Table 1 showed the characteristics of low metacognitive abilities. Students in this group did not draw and link information with previous knowledge. Students had low regulation and evaluation components. This lowly group had no ability at all. This was in line with the lowest metacognitive level developed by Swartz and Perkins (2017). At this low level, students made decisions without really thinking about them. Students did not have a strategy for solving problems. Adinda et al., (2021) stated that students failed to solve absolute value problems because students were unable to draw complete information. Karaali (2015) also stated that a lack of motivation resulted in students not trying to find solutions to problems. Characteristics of medium metacognitive abilities already had an awareness component. Students draw information and related problems to prior knowledge. Students in this group only had a strategy

for the regulation component. However, the student did not rethink the steps and could not seek a different strategy. Students on the evaluation component had no ability. Suryaningtyas and Setyaningrum (2020) stated that only some students were able to use their metacognitive abilities in solving problems. Amram, et al. (2019) stated that one of the reasons students experienced difficulties was because the problem-solving process involved a lot of technicalities.

Table 1. Characteristics of students' metacognitive abilities in solving absolute value problems

No.	Type of Metacognitive Ability	Components of metacognitive ability	Characteristic Description
1	Low Ability	Awareness	- think about what to ask
		Regulation	-
		Evaluation	-
2	Medium Ability	Awareness	- think about what to ask
			- relate it to similar problems that have been obtained previously
			- rethinking of the information on the problem
		Regulation	- think of a way to solve the problem
3	High Ability	Awareness	-
			- think about what to ask
			- link existing problems with similar ones that have been obtained previously
			- have a rethink of the information on the problem
		Regulation	- think back on what to do
			- think of a way to solve the problem
			- rethink the steps
		Evaluation	- think of a different way or strategy to solve the problem.
			- rethink the different ways or what they use.
			- Rethink the suitability of the strategy or the way it is used to solve the problem.
			- Rethink the sequence of steps in solving problems.
			- Rethink the correctness of the solution obtained

Characteristics of high metacognitive abilities used all indicators for each metacognitive component. This is in line with the high level developed by Swartz and Perkins (2017) that students use all their metacognitive abilities, before, during, and after the completion process.

Conclusion

This research analyzed the various types of students' metacognitive abilities when solving absolute value problems using the awareness, regulation, and evaluation components. The results showed that students at low ability only solve problems without any special strategies by trial and error. Furthermore, the metacognitive ability of low-skilled students only exists in the awareness component, which only thinks about questions. Students that are capable of solving problems in a procedural way do not realize that there are still inappropriate steps. In the evaluation component, they do not check the correctness of the answers obtained. Students with

medium ability solve problems in a procedural way. In the regulation component, students do not realize that there are still inappropriate steps in solving problems. In the evaluation component, students do not check the correctness of the answers they have obtained. High-ability students are able to solve problems in different ways and are able to distinguish the process of obtaining accurate information and think of effective strategies to solve problems.

This research is limited to absolute value problems with limited student participants. This study also did not present the demographics in more detail to the participants. There needs to be a follow-up study with more subjects with more specific demographics, having different characteristics, as well as studying different mathematical problems so that research is more convincing and broad characteristics are obtained.

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