

Exploring Students' Mathematical Understanding according to Skemp's Theory in Solving Statistical Problems

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Abstract. Understanding a concept is not only for mastering the next concept. It is the foundational skill used to solve mathematical problems. This study aims to explore students' conceptual understanding. This research is qualitative research with phenomenology. The participants in this study consisted of 18 eighth-grade students from one of Bandung's private secondary schools. After the students were given the test, six students were selected as research subjects. This aims to ensure that data can be obtained thoroughly and comprehensively. Data collection techniques included tests, interviews, and observations, with research instruments comprising a test and an interview guideline. Data saturation has been achieved. The results of this study are as follows: 1) students with relational understanding are fewer in number than students with instrumental understanding. 2) students find it most difficult to use the interconnections of various mathematical concepts in solving problems. 3) students' difficulties are due to their lack of ability in the concept of basic operations and their lack of understanding of the problems. Therefore, teachers need to strengthen students' understanding of basic operations and the concepts of mean, mode, and median to effectively connect these concepts with their problems.

Keywords: mathematical understanding, skimp's theory, statistical problems

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Introduction

Mathematics is one of the sciences closely related to real life. In learning mathematics, students are required to have the ability to solve problems well (Hsiao, Lin, Chen, & Peng, 2018; Özreçberoğlu & Çağanağa, 2018; Phonapichat, Wongwanich, & Sujiva, 2014; Siniguan, 2017; Spooner, Saunders, Root, & Brosh, 2017). The ability of students to solve problems can be seen from their success in associating the mathematical concepts they have mastered with the context of the problems they face (Helsa, Turmudi, & Juandi, 2023). One of the learning objectives in mathematics is to understand mathematical concepts (Permendikbud, 2014). One of the main objectives of mathematics education is to understand the process of knowledge (Rittle-Johnson, Siegler, & Alibali, 2001). Based on these learning objectives, it can be seen that students must have mathematical, conceptual understanding ability after learning mathematics. NCTM (2000) also said understanding material concepts is crucial for problem-solving. Therefore, conceptual understanding is fundamental for students to master.

Idris (2009) said that understanding is not just remembering concepts or following procedures. In other words, understanding mathematics requires more than just remembering facts. Hiebert and Carpenter (1992) stated that a mathematical idea or fact is considered understood when it becomes a natural part of an internal network. Khalid, Imawati, Swastika, Maharani, and Pradana (2021) said that mathematical, conceptual understanding ability means that the material taught to students is not just memorization but more than that, namely understanding so that students can better understand the concept in more depth within the subject matter itself. Conceptual understanding concerns answering questions correctly and recognizing correct answers at a metacognitive level (Kaltakci-Gurel, 2023). Through mathematical conceptual understanding ability, students are expected to have the ability to understand meaning in a scientific manner, both in theory and in everyday life (Dewi, Muslim, & Samsudin, 2019). Mathematical, conceptual understanding will help students and become their initial capital for solving mathematical problems (Minarti & Wahyudin, 2019). So, mathematical conceptual understanding is a person's ability to understand mathematical ideas and absorb mathematical material learned better so that students can better understand the concepts, not just memorize them.

The importance of conceptual understanding does not align with the quality of students' conceptual understanding abilities. The reality shows that the mathematics achievement of Indonesian students is still relatively low. TIMSS (Trends in International Mathematics and Science Study), an international study in the field of mathematics and science conducted to determine and obtain information about the achievement of mathematics and science achievements in participating countries, reported in 2015 that the average score of Indonesian students is 397, while the average international score is 500 (TIMSS, 2015). The 2018 PISA results showed that Indonesian students scored lower than the international average (OECD, 2019). Nationally, about 71% of them did not reach the minimum competency level in mathematics (Kemendikbud, 2019). The results of TIMSS and PISA studies show the low ability of students in Indonesia to master concept knowledge and solve non-routine problems. Research conducted by Widya and Sukoriyanto (2023) found that junior high school students' ability to solve PISA mathematics problems in statistics is still low. This also aligns with research conducted by Rohendi (2022) in Bandung, which states that many students still fall into the low category when solving statistics problems.

In understanding mathematical concepts, students must understand definitions, rules, theorems, how to solve problems, and how to correctly operate mathematics, factual, conceptual, procedural, and metacognitive (NCTM, 2000). The concepts in mathematics are organized mathematically, logically, and hierarchically, from the simplest to the most complex or from the

concrete to the abstract (Coles & Sinclair, 2019). In other words, understanding and mastering a material or concept is a prerequisite for mastering the next material or concept and making students more adaptable to new tasks (Skemp, 1976). Therefore, it can be understood that mathematical conceptual understanding is fundamental to learning mathematics, requiring students to master it in order to make their learning more meaningful.

Skemp (1976) classified conceptual understanding into two types: instrumental and relational. Instrumental understanding is the ability to apply mathematical procedures or rules without knowing the reasons (Skemp, 1976). In other words, students know “how” but do not know “why.” Relational understanding is the ability to apply mathematical rules and know the reasons for using them (Skemp, 1976), which form “schemas” (Skemp, 1982). Kinach (2002) said that Skemp's instrumental understanding is equivalent to content-level understanding, while relational understanding is equivalent to concept understanding, problem-solving, and epistemic understanding. This indicates that students who have relational understanding have a stronger foundation in their understanding. If students forget the formula, they still have the opportunity to solve the problem by using other related concepts. Skemp (1976) identified four key advantages of relational understanding in the context of learning mathematics. Firstly, it facilitates the solving of complex mathematical problems. Secondly, it enhances the retention and comprehension of mathematical concepts. Thirdly, relational understanding supports the attainment of learning objectives. Lastly, it fosters the generation of original ideas. Students should not only grasp mathematics at the instrumental level but also strive to achieve relational understanding (Idris, 2009; Skemp, 1976). Researchers are motivated to study Skemp's theory because it promotes the development of a deeper and lasting comprehension of mathematics through relational understanding.

One of the materials that students must master is statistics. Statistics are very useful in everyday life (Li, 2016). Statistics is even used in all scientific fields, such as economics, sociology, and health. However, students still have difficulty solving statistical problems (Batanero, Godino, Vallecillos, Green, & Holmes, 1994; Koparan, 2015; Budayasa & Juniati, 2018). This study focuses on exploring the students' mathematical conceptual understanding of solving statistical problems. In this case, researchers describe students' instrumental and relational understanding and difficulties. Consequently, teachers can later plan a solution that can be used to improve students' mathematical conceptual understanding, especially in statistical materials. The novelty of this research is that it uses Skemp's mathematical conceptual understanding indicators. The approach used in this study is different from previous research because it utilizes a phenomenological approach. Another novelty of this research is its exploration of the intersection between mathematical conceptual understanding and problem-solving skills in

statistics. This research is important because it contributes to the theory of student understanding, such as types of instrumental understanding and relational understanding in statistics. Therefore, according to Skemp's theory, researchers are interested in exploring students' mathematical and conceptual understanding of solving statistical problems.

Method

This is qualitative research with phenomenology. Phenomenology research aims to describe the state of a phenomenon. Phenomenology is a suitable methodology for gaining insight into the essence or structure of life experience (Walker, 2007). Qualitative research aims to understand the phenomena experienced by research subjects holistically by describing them through words and language in a scientific context and utilizing various scientific methods (Ulfa, 2020). Qualitative research can be defined as an approach to exploring and understanding the meaning ascribed to a social or humanitarian problem by an individual or group (Creswell, 2014).

This research aims to explore and describe the students' mathematical conceptual understanding abilities, especially in statistics. Researchers describe the instrumental and relational understanding of students along with their difficulties. This research was conducted in April 2023. The participants in this study were eighth-grade students in one of Bandung's private secondary schools, for 18 participants. In this study, the researchers have obtained participant consent for data collection. The 18 participants completed the given test. After the students took the test, six were chosen as research subjects for an in-depth interview. They are chosen based on the characteristics of their answers while taking the test. This aims to ensure that data can be obtained thoroughly and comprehensively as needed and assist researchers in drawing quality conclusions. The data collection techniques used were a test, interviews, and observation. The main instrument in qualitative research is the researchers themselves. The researchers, as instruments, will be collecting, processing, and interpreting data (Creswell, 2014). The supporting instruments are a test of statistical problems and an interview guideline. The questions were given in the form of long-answer questions so that researchers could examine the flow of students' thinking through the answers that had been given. Before being given to students, the questions were validated by mathematics education experts on content, construct, and face validity. Content validation ensures that a measurement instrument covers all relevant aspects of the concept being measured (Lawshe, 1975). Construct validation concerns the extent to which the instrument measures the concepts adopted theoretically and empirically (MacKenzie, Podsakoff, & Podsakoff, 2011). Face validity considers how suitable the test content appears on the surface (Raphael, 2022). The outcomes of descriptive validation meet the valid criteria at a rate of 82%. The interview aims to obtain more in-depth information. The interview used in this study was

semi-structured. Semi-structured interviews are in-depth interviews that also allow for more in-depth discoveries (Magaldi & Berler, 2020). Interviews were conducted until no more new information could be obtained from participants or until they felt they had provided sufficient information.

The stages in this study were introduction, compiling a test of mathematical conceptual understanding ability, collecting data, analyzing data, and drawing difficulties. Activities in data analysis included three things, namely, data reduction, data presentation, and conclusion or verification. Data reduction involves simplifying and transforming raw data to extract essential information, while data presentation entails visually representing the data through charts, graphs, and tables to communicate insights effectively. Finally, the conclusion or verification phase involves making informed inferences and drawing conclusions. Miles, Huberman and Saldana (1994) stated that validity in qualitative research, namely credibility, transferability, dependability, and confirmation. Researchers used four questions to measure students' concept understanding: two questions with instrumental understanding indicators and two with relational understanding indicators, as in Table 1.

Table 1. Test of conceptual understanding ability in statistical problems

Mathematical conceptual understanding	Aspects/Indicator	Question Number
Instrumental	Applying the mean formula to find the mean value of a data set.	1
Instrumental	Apply the ideas and concepts of statistics when solving simple math problems.	2
Relational	Use the interconnections of statistical concepts and algebraic concepts to solve problems correctly.	3
Relational	Perform mathematical operations consciously with the use of various mathematical rules or principles.	4

Adaptations from Nurkaeti, Pratiwi, Aryanto, and Gumala (2019)

Results and Discussion

Based on the analysis of the test results, it was found that the subject's mathematical conceptual understanding ability was still relatively low. The students' low mathematical conceptual understanding ability obtained by researchers is shown by the results of the students' answers. Namely, many students still have not been able to correctly answer the questions given. Researchers investigated students' mathematical conceptual understanding ability to see the profile of students' instrumental and relational understanding. Of the four questions on the test given to students, the following is the percentage of students who answered the questions correctly for each question (see Table 2).

Table 2. Percentage of students answering correctly

Mathematical conceptual understanding	Aspects/indicators	Percentage answering correctly	Mean Percentage
Instrumental	Applying the mean formula to find the mean value of a data set.	72.2%	69.4%
	Apply the ideas and concepts of statistics when solving simple math problems.	66.6%	
Relational	Use the interconnections of statistical concepts and algebraic concepts to solve problems correctly.	27.7%	33.2%
	Perform mathematical operations consciously with the use of various mathematical rules or principles.	38.8%	

Adaptations from Nurkaeti et al. (2019)

Table 2 shows that the mean percentage of students who answered correctly on instrumental understanding questions is 69.4%. While the mean percentage of students who could correctly answer the relational understanding questions was only about 33.2%. This shows that students with relational understanding are fewer than students with instrumental understanding. So, it can be concluded that the students' mathematical conceptual understanding ability is still low. This is in line with a study conducted by Anggraini, Yohanie, and Nurfahrudianto (2022), which shows low students' mathematical conceptual understanding and only a few students with relational understanding.

From Table 2, it can be seen the percentage of students answering correctly for each indicator. It can also be seen from the four indicators that students are easiest to work with on instrumental understanding questions with indicators of applying mathematical formulas or rules in a mathematical calculation or operation, which is 72.7%. Conversely, students have the most difficulty working on relational understanding questions with indicators of using the interrelationship of various mathematical ideas and concepts in solving problems, which is only 27.7%.

Analysis of Student Answers Based on the Conceptual Understanding Test

Question Number 1

Question number 1 is a question to measure students' mathematical conceptual understanding abilities with indicators of applying mathematical formulas or rules to a mathematical calculation or operation.

Question number 1 is:

Given the data: 5, 6, 8, 2, 7, 4, 8, 10, 7, 8, 6, 1. Determine the mean of the data.

Based on Table 2, it can be seen that 72.2% of students were able to apply formulas in a calculation. 27.8% of students could not apply the formula in a calculation correctly. The results of students' mathematical conceptual understanding in applying formulas can be seen in Figure 1.

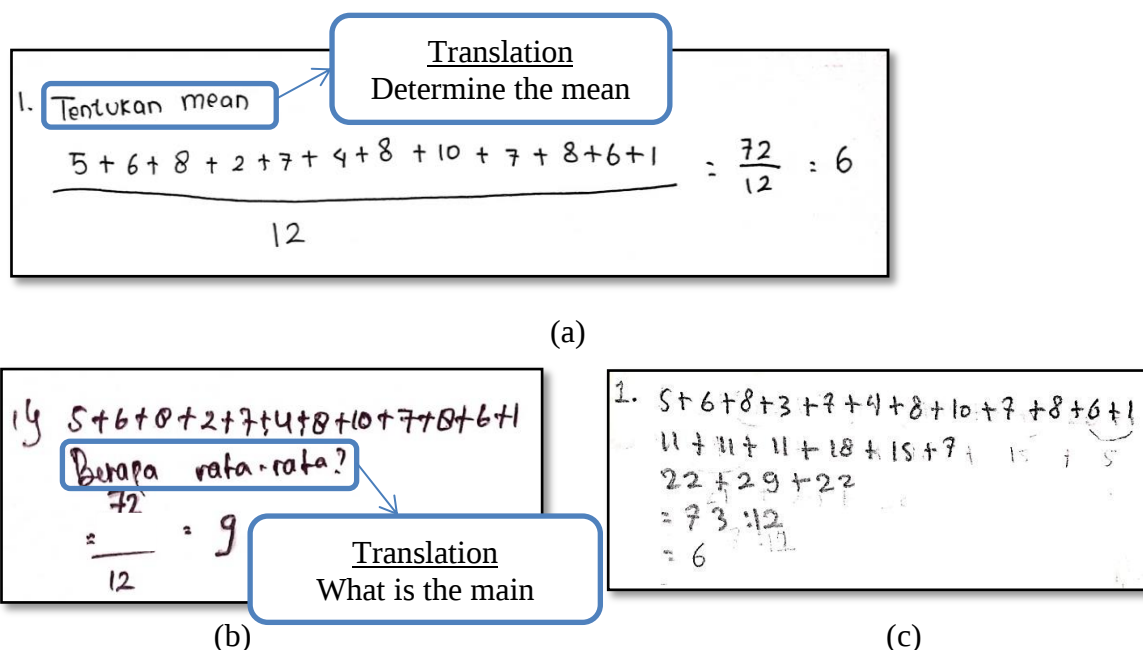


Figure 1. The results of answer number 1

Instrumental understanding of the concept of averaging consists of simply knowing the computational rules for calculating the simple average of a set of numbers (Idris, 2009). For question number 1, the majority of students already knew the mean formula. They knew that the mean of a data set is found by adding all the numbers in the data set and then dividing by the total number of data points in the set. Answer 1(a) is an example of a student answer that can correctly use a formula in mathematical operations. Figures 1(b) and 1(c) are examples of student answers that are still incorrect in answering the question. Figures 1(b) and 1(c) show that these students have not been able to use the formula in mathematical operations correctly. In Figure 1(b), the student divided incorrectly. The student wrote $72/12 = 9$, whereas the correct answer is $72/12 = 6$. In Figure 1(c), the student made a mistake in addition. The student wrote that the sum of the data is 73, whereas the actual sum is 72. During classroom observation, researchers saw similar things happening to some students. Some of them still made mistakes with basic operations, especially the concept of division. This is in accordance with Yusuf, Titat, and Yuliawati (2017), who stated that students cannot solve statistical problems properly due to their weaknesses in performing basic operations and making incorrect calculations. Siregar, Siagian, and Wijaya (2023) stated that one requires a comprehensive understanding of the basic concepts in order to manipulate mathematical symbols. The level of pupils' mathematical understanding of basic concepts significantly impacts their ability to comprehend more advanced mathematics (Diarni, Ikhsan, Zaura, & Johar, 2023). Errors in basic math concepts will cause children difficulty learning further (Jarmita, 2015).

Question Number 2

Question number 2 is a question to measure students' mathematical conceptual understanding ability with indicators of applying ideas and concepts in solving simple math problems.

Question number 2 is:

The math test scores of 15 students are as follows: 7, 5, 4, 6, 5, 7, 8, 6, 4, 4, 5, 9, 5, 6, 4. Determine the number of students whose scores are above average.

Based on Table 2, it can be seen that 66.6% of students were able to apply concepts when solving simple math problems. 33.4% of students could not apply concepts when solving simple math problems. The results of students' understanding of mathematical concepts applied to solving simple math problems can be seen in Figure 2.

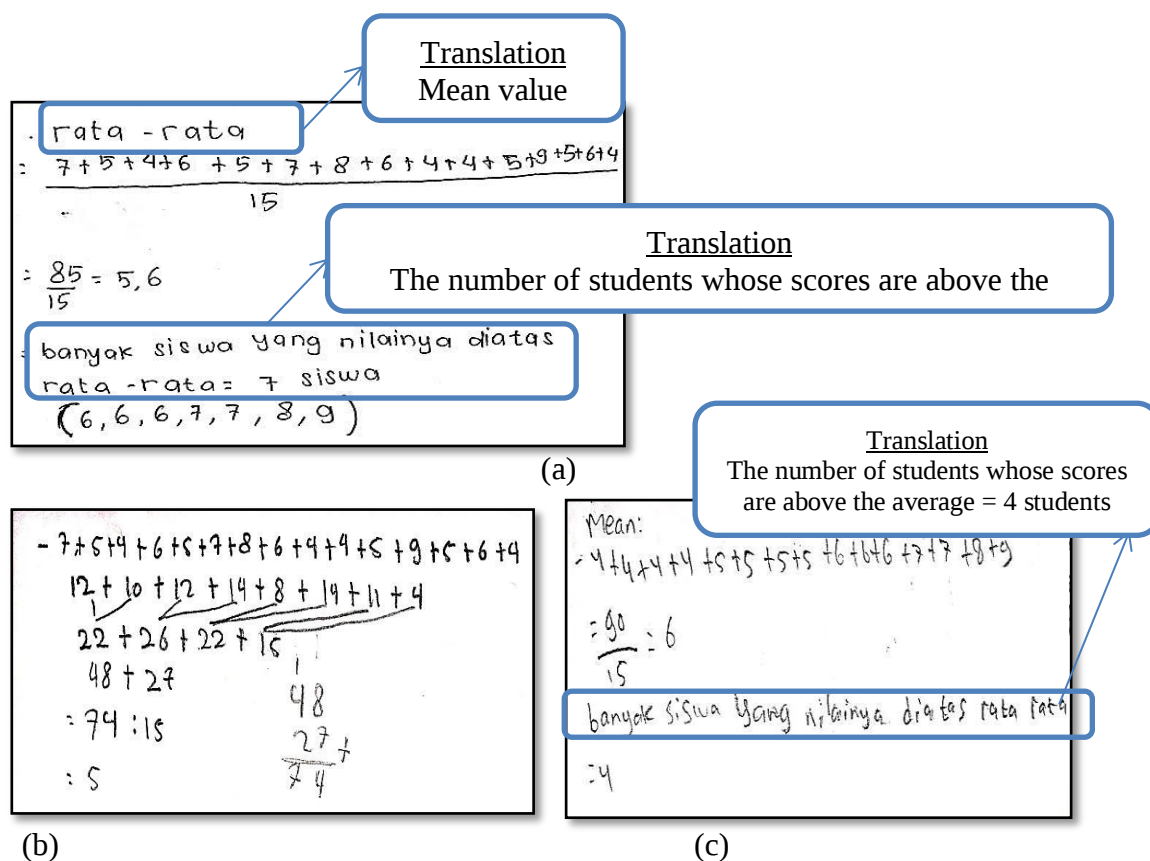


Figure 2. The results of answer number 2

Figure 2(a) is an example of an answer from a student who can apply concepts to solving simple math problems. From Figure 2(a), it can be seen that the student is correct in performing basic operations. The student can determine the mean of the data correctly. In addition, this student can also correctly determine the number of students whose scores are above average. Figures 2(b) and 2(c) are examples of incorrect representations of students' answers. In Figures 2(b) and 2(c), students made mistakes in basic operations. Both students were wrong about

addition and division operations. In figure 2(b), the student wrote $74/15 = 5$. After the interview, it was found that the student got the final result as a decimal number. However, the student was confused and felt the question was wrong and only wrote down the whole number part and did not write down the fractional part of the number. The following is an interview excerpt of the researcher and a student who provided an answer to Figure 2(b).

- R : How did you find the answer to question 2?*
S : By finding the mean, right?
R : That's correct. How do you calculate the mean?
S : You add up all the values and then divide the result by 15
R : Exactly. Your formula is correct, but you still made a mistake
S : Oh, really? What did I do wrong?
R : This one (researcher points out the student's mistake in addition and division)
S : Oh, I see. I made a mistake in adding them up
R : So, how did you get $74/15 = 5$?
S : I was confused. Miss. $74/15$ gives a decimal result, but I thought the answer couldn't be a decimal. So, I thought there was something wrong with the question. That's why I thought the closest value to $74/15$ is 5
R : Why did you think the result couldn't be a decimal?
S : Because as far as I knew, the result of an average has always been a whole number, Miss

In Figure 2(c), it can be seen that the students first arranged the numbers from least to greatest and then looked for the mean. To find the mean of the data, it is not necessary to arrange the data first. From the answers and interview results, it can be seen that the students did not understand the average concept well. The following is an interview excerpt of the researcher and the student who provided an answer to Figure 2(c).

- R : How did you find the answer to question 2?*
S : I was looking for the mean. I added all the values and then divided them by 15
R : That's correct. So, why did you list the data in order?
S : Isn't that how it should be done, miss?
R : When finding the mean, you don't need to list the data in order. However, if you're finding the median, that's when you should list the data in order.
S : Oh, I see
R : And you also made a mistake in addition. The sum is 85, not 90
S : Oh, yes, miss. I made a mistake in adding them up.

In line with that, during classroom observation, researchers went around to see how students worked on the problems given. When working on problem number 2, some students were too lazy to continue solving question number 2 because the mean of the data given was in decimal form. Some of them felt that what they did was wrong because the answer they obtained was not a whole number. The researchers saw that students were still trying hard to find the division result, and they seemed hesitant when the result obtained was a decimal number. Therefore, students need to understand the decimal number, which is the foundation for correct calculations (Pierce, Steinle, Stacey, & Widjaja, 2008). Students will face difficulties in learning if they make mistakes

with basic mathematical concepts (Jarmita, 2015). Some students were also confused by the question "determine the number of students who scored above average." They have not understood the problem properly or the questions given. So, some students did not answer this follow-up question. Many students only find the mean value without finding the number of students whose scores are above average.

Question Number 3

Question number 3 is a question to measure students' conceptual understanding ability with an indicator of using the interconnection between various mathematical ideas and concepts in solving problems. This question is a relational understanding question. Researchers investigate how capable students use the various concepts they know to solve problems. Researchers also investigate how students can understand questions and the connection between concepts. In question number 3, there are statistical and algebraic concepts. In addition, students must also understand the concept of basic operations in order to work on the problem correctly.

Question number 3 is:

The basketball team consists of five students with an average weight of 45 kg. The difference between the heaviest and lightest weights is 15 kg. One person is the heaviest, and the others have the same weight. Determine the weight of the heaviest student.

Based on Table 2, it can be seen that 27.7% of students were able to use the interconnection between various mathematical ideas and concepts in solving problems. 72.3% of students could not use the interconnection between mathematical ideas and concepts to solve problems. The number of students who answered incorrectly was higher than those who answered correctly. This shows that students have difficulty solving problems with this indicator. The results of the student's work on problem number 3 can be seen in Figure 3.

Figure 3(a) is an example of a student who has used the interconnection of various mathematical ideas and concepts to solve problems correctly. In this case, the student used the concepts of statistics and algebra to work on the problem. The student understands that if five data values have an average of 45, then the sum of the data values is 225. But the interesting thing is that students use trial and error to find the heaviest weight.

The following is an interview excerpt of the researcher and the student who provided an answer to Figure 3(a).

- R : How did you find the answer to question 3?*
S : Well, it said that the average weight of 5 students is 45. So, I calculated their total weight as 45 multiplied by 5, which is 225
R : Then how did you determine that the heaviest student weighs 57?
S : Since the total is 225, and it's mentioned that "the difference between the heaviest and lightest weights is 15 kg," and "there is one person who is the heaviest, and the others have the same weight," I used a trial and error approach, Miss. I found the lightest weight to be 42 and the heaviest to be 57

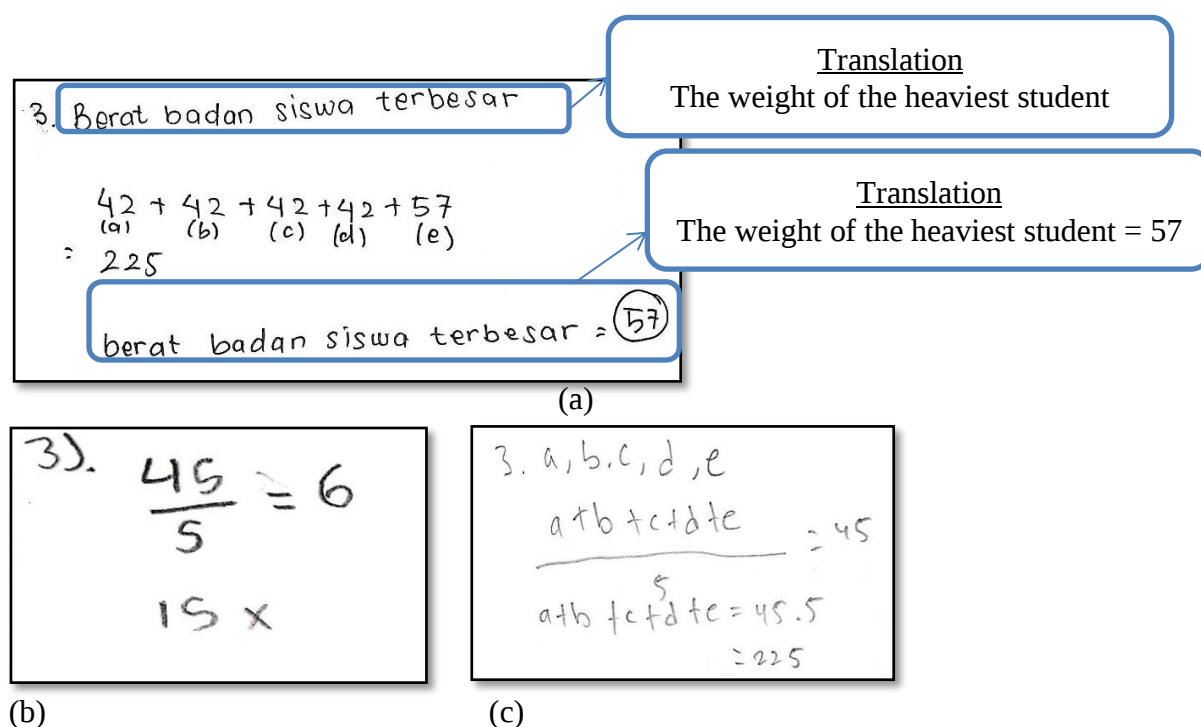


Figure 3. The results of answer number 3

Figures 3(b) and 3(c) represent students' incorrect answers. Figure 3(b) shows that the student does not understand the question's meaning. The student wrote an answer that was very different from the actual answer. The following is an interview excerpt of the researcher and a student who provided an answer to Figure 3(b).

- R : How did you find the answer to question 3? Why did you use 45/5?
- S : It's written, "The basketball team consists of five students with an average weight of 45 kg." So, I thought it was about the average value; that's why I used 45/5 miss
- R : That's incorrect. The 45 there represents the average value, not the total weight of the five students. What is it that you wrote 15x?
- S : I'm sorry, miss. I don't understand the problem given well. It's too long and difficult for me to understand, miss

The student above no longer understands the concept of mean after being given a slightly different type of question, presented as a word problem. While Figure 3(c) shows that the student is close to the correct answer. It was just that the student did not continue the process because he was confused about the next step. Only a few students were able to answer this question. The following is an interview excerpt of a researcher and a student who provided an answer to Figure 3(c).

- R : How did you find the answer to question 3?
- S : Well, it said that the average weight of 5 students is 45. So, I calculated the total weight of those five students, which is 225, Miss. But when it comes to determining the heaviest student's weight, I'm not sure anymore, miss. I'm confused about how to find it. miss

The student above struggles to connect statistical concepts with algebraic concepts effectively. When faced with problems that require them to bridge various ideas, the student finds it difficult to provide well-structured answers. Most students did not correctly understand the meaning of the problem, or, in other words, many students failed to understand this problem. Many students failed to understand the relationship between sentences. Many students skipped and chose not to answer problem number 3 because they had already given up when reading this problem. Several studies found that difficulties in understanding problems resulted in students having difficulty solving problems (Mokhtar, Ayub, Said, & Mustakim, 2019; Phonapichat et al., 2014). Students find it difficult to understand the problem because they cannot associate the mathematical concepts they have mastered with the context of their problems (Helsa et al., 2023). Some students were able to solve this problem well. Students who worked on problem number 3 correctly found the answer through trial and error.

Question Number 4

Question number 4 is a question to measure students' conceptual understanding ability with an indicator of performing mathematical operations consciously using various mathematical rules or principles.

Question number 4 is:

Make 7 data values with $\bar{x} = Mo = Me$.

Based on Table 2, it can be seen that 38.8% of students were able to perform mathematical operations through the use of various mathematical rules or principles. 61.2% of students were not able to perform mathematical operations through the use of various mathematical rules or principles. The results of the student's work on question number 4 can be seen in Figure 4.

Question number 4 requires students to determine data where $\bar{x} = Mo = Me$. Students who were able to work on this problem are those who understand mean, mode, and median formulas. Figure 4(a) is an example of a student who can answer the question well. This student understands the concepts of mean, median, and mode. Figures 4(b) and 4(c) represent incorrect students' answers. The students are still wrong when performing addition and multiplication operations. In Figure 4(b), the student has understood the concepts of mean, median, and mode. However, there are still calculation errors in finding the mean of the data. The student wrote $1 + 2 + 3 + 4 + 4 + 4 + 3 = 13$ and $13/7 = 4,2$, and the answers are still wrong. From Figure 4(b), it can be seen that the student did not understand the question. The data provided by students is not data with $\bar{x} = Mo = Me$.

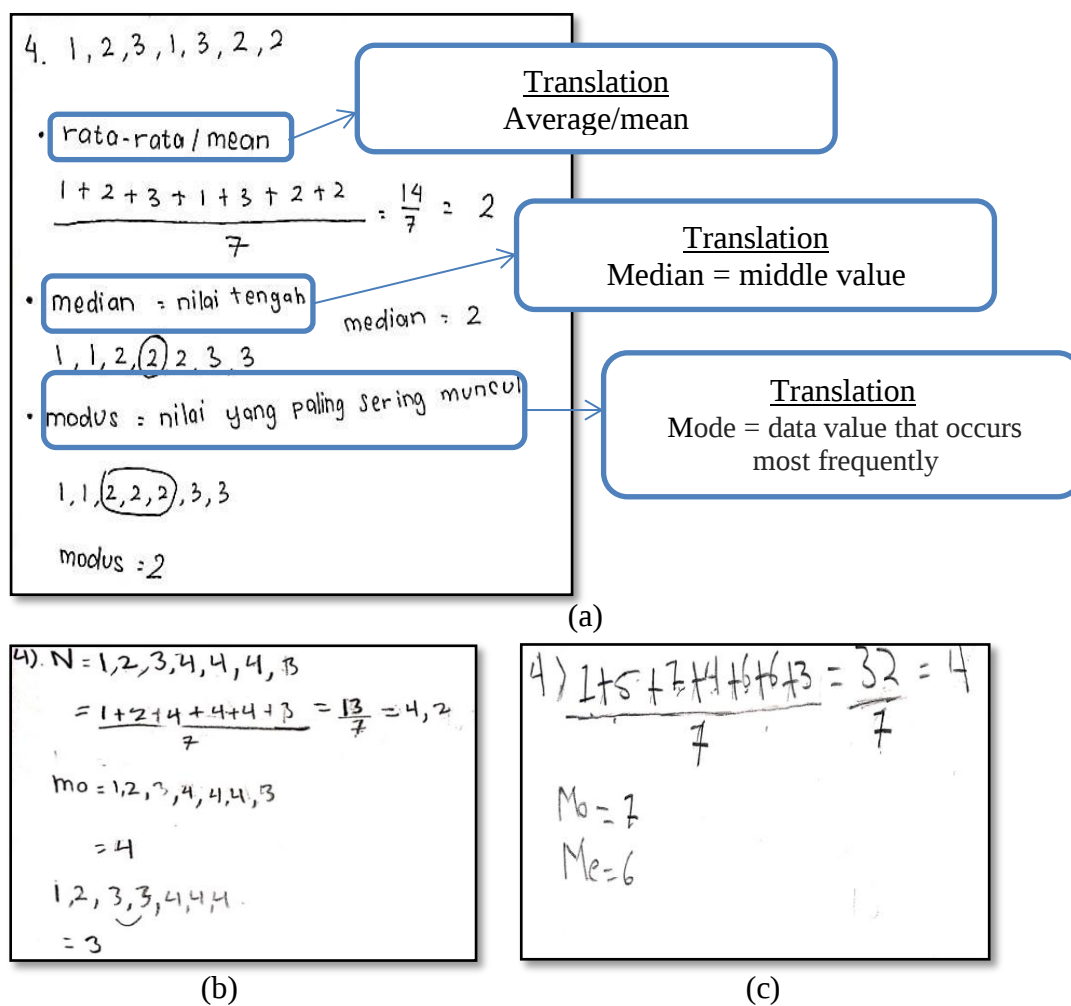


Figure 4. The results of answer number 4

The following is an interview excerpt of the researcher and a student who provided an answer to Figure 4(b).

- R : How did you find the answer to question 4?
- S : I randomly selected 7 data values, Miss. Then, I calculated the mean, median, and mode.
- R : Are you sure that's the way to do it? Please read the question again
- S : Isn't that how it's done, Miss? I'm not quite sure
- R : What you did was incorrect. What you should have done was find seven data values where the mean is equal to the median, which is also equal to the mode.

Figure 4(c) shows that the student understood the concept of mean but not median and mode. However, although the concept of the mean is correct, namely that the mean is found by adding all numbers in the data set and then dividing by the number of values in the set, in its application, students still make mistakes. Student errors are errors in basic operations. Students are not careful when doing addition and division. Students also do not understand the questions given. Students find it challenging to understand the problem because they cannot associate the mathematical concepts they have mastered with the context of their problems (Helsa et al., 2023).

Mathematics can be highly abstract, making it challenging for students to grasp. Skemp's theories on mathematical understanding suggest that students might struggle with mathematics when they view it as a set of disconnected symbols and rules rather than meaningful concepts. Ausubel's theory of meaningful learning emphasizes building on existing knowledge. If students do not have a solid foundation in the basics of mathematics, more complex concepts become more challenging to grasp.

Several implications can be carried out in the classroom to improve conceptual understanding, both instrumental understanding and relational understanding in statistical material, to strengthen students' basic arithmetic operations concepts: addition, subtraction, multiplication, or division. Teachers also need to strengthen basic operations with integers and fractional numbers. The basic operations influence students' mathematical abilities (Pierce et al., 2008). In addition, teachers need to strengthen the concepts of mean, mode, and median so that when students are faced with questions where they must be able to connect concepts, they can connect concepts with the problems they face (Helsa et al., 2023). Teachers also need to provide non-routine questions to students to familiarize them with problem-solving questions (Beghetto, 2017). So, it is essential to incorporate real-world examples and applications of mathematical concepts into teaching. This can make math more relevant and easier to understand. Provide structured support to help students build on their existing knowledge. As per Bruner's theory, gradually remove this support as students become more proficient. Engaging students in problem-solving activities, discussions, and hands-on experiences is also important to promote meaningful learning, aligning with Ausubel's theory.

Conclusion

Students still tend to have a low level of mathematical conceptual understanding. The mean percentage of students with instrumental understanding is 69.4%, while the mean percentage of students with relational understanding was only about 33.2%. This shows that students with relational understanding are fewer than students with instrumental understanding. Students are easiest to work with from the four indicators on instrumental understanding questions with indicators of applying mathematical formulas or rules to a mathematical calculation or operation. Conversely, students have the most difficulty working on relational understanding questions with indicators of using the interrelationship of various mathematical ideas and concepts in solving problems. Students' difficulty in understanding statistical concepts is due to their lack of ability in basic mathematical concepts, namely arithmetic operations. Difficulties in arithmetic operations will have a negative impact on students' ability to understand statistical concepts. In addition, another difficulty is the lack of understanding of the problem among students. Students

don't understand the problems in the questions because they don't understand statistical concepts well, so they find it challenging to connect statistical concepts with the problems they face. This study may not have thoroughly explored instructional strategies or interventions to improve students' mathematical understanding. Future research should investigate effective teaching methods and interventions that enhance mathematical understanding.

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