

Implementing Problem-Based Learning to Enhance Students' Mathematical Proficiency in Primary School

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Abstract. Primary school students' mathematical proficiency is crucial to their academic progress. However, many students still struggle to understand math, especially fractions. This study examines how Problem-Based Learning (PBL) improves primary school students' mathematical proficiency. The study employed a quasi-experimental approach to investigate the effects of PBL academic achievement. The experimental group received PBL, while the control group received expository teaching. This study selected 54 students from two primary schools using purposive sampling. The experimental group consisted of 28 students, and the control group had 26. Students' mathematical proficiency was tested pre-and post-intervention. The findings reveal a significant improvement in mathematical proficiency in the experimental class exposed to PBL, as indicated by the independent samples t-test. The two-way ANOVA underscores the interaction influence of PBL and Early Mathematics Skills (EMS) factor on improving mathematical proficiency, supported by a high R-squared value (0.941). It suggests that PBL explains a substantial portion of the observed diversity in mathematical proficiency. The quasi-experimental approach has limitations, and the study shows that PBL improves mathematical proficiency. The study's quasi-experimental characteristics and potential lack of generalizability are acknowledged. While recognizing students' starting EMS, educators and policymakers should incorporate PBL into mathematics instruction, especially for complex concepts like fractions.

Keywords: fraction material, mathematical proficiency, primary student, problem-based learning

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Introduction

Mathematical proficiency comprehensively represents mathematical competence (Ala-Mutka, 2011). Strategic and adaptive reasoning comprises mathematical proficiency (Kilpatrick et al., 2001). Solving non-routine problems requires strategic competence and adaptive reasoning (Nahdi & Jatisunda, 2020). Strategic competence is the ability to formulate, represent, and solve problems (Kilpatrick et al., 2001). Adaptive reasoning is the ability to reason, reflect, explain, and justify (Kilpatrick et al., 2001). Strategic competence and adaptive reasoning are logical connections that reflect mathematical proficiency (Syukriani et al., 2016).

Strategic competence helps students apply the right strategy to solve problems (Zubainur, 2020). Students need to know the different ways and techniques appropriate for problem-solving. Once students can formulate difficulties, the next step is to represent them mathematically, numerically, symbolically, verbally, or graphically (Kilpatrick et al., 2001). Adaptive reasoning is the ability to think logically, estimate answers, explain the answer concepts used, and judge mathematical truth. Based on this description, strategic competence and adaptive reasoning are important ability bonds that students possess (Haryanto, 2017).

The five components of mathematical proficiency are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, which are interconnected, intertwined, and interrelated (Kilpatrick & Swafford, 2017). Conceptual knowledge, procedural fluency, and a productive disposition are the remaining pillars of mathematical proficiency (Samuelsson, 2010). Procedural and conceptual fluency are conceptualized as forms of mathematical knowledge, while strategic competence and adaptive reasoning are seen as mathematical skills. Mathematical proficiency is progress in mathematical skills and knowledge (Bartell et al., 2013). Mathematical proficiency encompasses engaging with mathematics by understanding its concepts, performing calculations, applying mathematical principles in real-world situations, reasoning logically, and demonstrating expertise, competence, and facility in mathematical problem-solving.

There is a significant correlation between conceptual understanding, procedural fluency, strategic competency, adaptive reasoning, and productive disposition in mathematics achievement (Awofala, 2017). However, mathematical proficiency is still low, resulting in an inability to understand non-routine problems and difficulty using their reasoning skills, impacting mathematics achievement (Zubainur, 2020). The tasks set by teachers do not meet the required standard for learners in higher order and critical thinking (Luneta, 2015). Developing mathematical proficiency requires complex pedagogical skills from teachers (Vale et al., 2019). Building ability ties begins with choosing material that challenges students' thinking. The concept of fractions was chosen because they are "important and difficult" (Margolinas, 2015). Competence with fractions is associated with mathematical achievement (Siegler et al., 2012a). It is a fact that fractions are considered to be one of the most complex and challenging primary school mathematical concepts (Lamon, 2020). Most students struggle with visual, verbal, and symbolic representations of fractions (Nicolaou & Pitta-Pantazi, 2016).

Comparing fractions can be challenging for students and may result in errors, such as perceiving $\frac{1}{4}$ as greater than $\frac{1}{2}$ due to the misconception that four is larger than 2 (van Hoof et al., 2013). Comprehending fractions and creating effective teaching methods are significant educational subjects in contemporary times (Albin & Brown, 2019). According to Kilpatrick and

Swafford (2017), the capacity to depict mathematical ideas in various forms and comprehend the usefulness of diverse representations is a significant indicator of conceptual comprehension. Developing educational materials that incorporate the properties of fractions is a challenging task. It can enhance the correlation between strategic competence and adaptive reasoning abilities.

Scholars believe Problem-Based Learning (PBL) can enhance mathematical proficiency using fraction material as a student problem-solving tool. Originated in medical education during the mid-1950s (Savery & Duffy, 1995), PBL fosters collaborative problem-solving and knowledge acquisition among students, aligning with the goal of mathematics education to develop problem-solving skills (Hmelo-Silver, 2004; Norman & Schmidt, 2000; Schoenfeld, 2016; Senocak et al., 2007). Active participation is crucial in PBL (Jatisunda & Nahdi, 2020). The process involves presenting problems to students, who then use their knowledge to comprehend the underlying principles (Espey et al., 2007a; Zwaal, 2019). The teacher's role is to facilitate learning and enhance students' understanding (Barrows, 1996).

In PBL, students learn, analyze, and work together to solve unstructured problems (Tan et al., 2019; Trinter et al., 2015). Typically not immediately resolvable, open-ended, sometimes not resolvable (Bishara, 2016; Kim & Lim, 2019; Özcan, 2016), and the problem is complicated (Branch, 2015; King & Smith, 2020; MacLeod & van der Veen, 2020). Problem-solving requires prior knowledge of concepts and rules, understanding concepts and ideas in related subjects, and metacognitive knowledge (Belland, 2014; Gill et al., 2010; Son & Lee, 2021). However, Task complexity will disadvantage students (Van Merriënboer et al. 2003). Working memory capacity can only hold so much information at any time, so the complexity of tasks in PBL is severely limited (Sweller et al., 2019). PBL will be used successfully if the students have a solid conceptual understanding, but it can be detrimental to students with a shallow level of knowledge (Holmes & Hwang, 2016). Therefore, it is imperative to find a way to maximize the learning process, where teachers must pay attention to the student's intellectual potential (Merrienboer et al., 2006). Additionally, students who lack a basic understanding of mathematics cannot solve problems (Fitzmaurice et al., 2019).

According to prior research conducted by Dhlamini and Luneta (2016), it has been shown that students frequently exhibit insufficient mastery of fundamental mathematical concepts. Hence, the significance of teachers employing effective learning models cannot be overstated (Sudiarta & Widana, 2019). If learning remains ineffective (Evans, 2007). PBL is designed to establish a conducive and efficient educational setting (Tarmizi et al., 2010). Previous research studies have demonstrated the effectiveness of PBL in enhancing students' mathematical proficiency (Abdullah et al., 2010; Masitoh, 2019). The primary objective of this study is to examine the impact of PBL implementation on mathematical proficiency, specifically in the

domain of fractions. Another aim of this study is to investigate the correlation between PBL and early mathematical skills (EMS) factor in enhancing mathematical proficiency.

Method

This study is quasi-experimental. It examines the impact of PBL on mathematical proficiency. The success of PBL can be assessed by examining students' mathematical proficiency in both the experimental and control groups. This study employs a quasi-experimental design due to practical, ethical, and methodological considerations regarding sample selection (Campbell & Stanley, 2015). The studies utilized a pre-test and post-test control group design. The procedure is described below.

<i>O</i>	<i>X</i>	<i>O</i>
<i>O</i>		<i>O</i>
<i>Pre-test</i>	<i>Treatment</i>	<i>Post-test</i>

Figure 1. Matching-only pre-test-post-test control group design (Fraenkel et al., 2011)

Both groups were initially administered a pre-test to ensure their mathematical proficiency was equivalent. The experimental and control classes received different treatments, as indicated by the X and no symbols in the figure above. The experimental group underwent problem-based learning, while the control group was subjected to expository instruction. The study was conducted over two months, from March 12th to April 20th, 2023, in two public primary schools in Majalangka, West Java, Indonesia. The study commenced with instrument preparation, mathematical proficiency assessment, sample selection, learning implementation, and research data analysis. The study included 54 students who were allocated into two classes. The experimental class comprised 28 students, while the control class had 26 students. The sample was non-probabilistically selected based on the subjects, investigation of mathematical ability, and interviews with school teachers. Their analytical proficiency in the subject matter determined the students' early mathematics skills (EMS). The research samples were drawn from classes with varying levels of prior knowledge. The research process was repeated seven times, with the initial and final meetings as pre-test and post-test sessions, respectively. Table 1 outlines the many stages of the PBL process in the experimental class.

The study gathered research data by conducting mathematical proficiency tests on students belonging to the experimental and control groups. The study employed mathematical proficiency tests as research instruments. Strategic competence and adaptive reasoning emerged from (Kilpatrick et al., 2001). The assessment of instruments' validity and reliability necessitates expert evaluation and trials in diverse settings. Upon completion of these steps, the researcher will ascertain whether the instruments developed meet the validity and reliability standards based on

the established criteria. The validity coefficient for each test item is approximately 0.75 (high), and the reliability coefficient is 0.81 (high). The score of each item measures mathematical proficiency ability based on the item's difficulty level. The test of mathematical proficiency consists of five questions. The optimal maximum score is determined to be 50, as questions 1 to 5 encompass a scoring range spanning from 0 to 10. The mathematical proficiency test instrument on fraction content is presented in Figure 2.

Table 1. Problem-based learning treatment in this research

Activities Description of PBL	Teacher Activities	Student Activities
Introduction	The teacher imparts apperception, motivation, fundamental competencies, and educational goals to students.	Students attend to the teacher's apperception, motivation, fundamental competencies, and learning objectives.
Core activities	The teacher disseminates non-routine problem worksheets to students.	Students generate problem ideas from issues outlined in a provided worksheet.
	The teacher facilitates the establishment of the problem theme by the students.	- Students establish the learning environment. - Students in small groups allocate tasks to solve problems on the worksheet. - Students engage in practical activities based on the learning context. - Access relevant learning resources and gather necessary tools and materials as documented in their worksheets. - Students in small groups illustrate the fraction-related problem to be solved.
	The teacher supervises each student group's production of a problem activity report.	- Students execute activities - Students seek solutions to problems in small groups based on pertinent learning resources. - Each group prepares a report and presents it to the class.
Closing activities	The teacher rewards students' success.	- Students complete exams given by the teacher - Students verify their comprehension with the instructor's explanation

Answer the following questions correctly:

1. Pa Ibnu has a garden of 10 hectares. He has planted 12% maize, 0.18% manioc and 2.1% rice. How many hectares are left unplanted?
2. A trader bought cooking oil in two containers, one containing $3\frac{4}{5}$ litres and the other containing $1\frac{2}{5}$ litres. To sell the oil, he puts it in bottles. How many bottles can he fill?
3. Ghazia has a sideline making syrup. She has $6\frac{1}{4}$ kg of sugar and buys another $4\frac{1}{2}$ kg. She then sells it in bottles, each requiring $\frac{1}{4}$ kg of sugar. So the number of bottles needed is?
4. Susi has just made tomato juice and poured it into a 12-litre thermos but only filled $\frac{3}{4}$ of it. She put $\frac{1}{3}$ of it in the fridge. The rest was put into plastic bags. Each bag could hold $\frac{1}{2}$ litre of tomato juice. So the number of plastic bags that Susi has to provide is...
5. Doctor Rizmi gave his patient 40 tablets of medicine, $1\frac{1}{4}$ tablets daily. In how many days will the medicine run out?

Figure 2. The mathematical proficiency test instrument

The data analysis begins with conducting normality and homogeneity tests on the variance data, followed by post-test and N-gain analysis. The Kolmogorov-Smirnov test was used to check the two data sets for normality, and the F test was used for homogeneity of variance. The data were analyzed using the t-test on independent samples and a two-way ANOVA test with Univariate General Linear Model (GLM) analysis. The t-test on independent samples was conducted to determine differences in the ability to solve mathematical problems between the two classes. On the other hand, ANOVA was conducted to find the difference in the increase in mathematical problem-solving skills between the two classes based on the Early Mathematics Skills (EMS) category level. Hypothesis testing was carried out using a significance level of 0.05. The categorization of n-gain based on Meltzer (2002) is presented in Table 2.

Table 2. N-gain category

Category	Score
High	$g > 0.7$
Medium	$0.3 < g \leq 0.7$
Low	$g \leq 0.3$

Results and Discussion

Data from the pre-test and post-test findings regarding mathematical proficiency were obtained from the study. The students' EMS was tested by administering questions examining their comprehension of the fundamental ideas for learning fractions. The outcomes obtained from the EMS were comparable to the first test results, confirming the suitability of the two student groups for comparison. The post-test and N-gain data were used to elucidate the disparity in enhancing mathematical proficiency between the experiment and control class. The subsequent explanation provides a rationale for the hypothesis that will be utilized to assess enhancements in mathematical proficiency:

1. The students' mathematical proficiency in the experimental class was better than the control class.
2. There is an interaction between the learning model factors given and the EMS category factors toward increasing students' mathematical proficiency.

The experimental class demonstrates dominance over the control class, as evidenced by the disparity in the average mathematics proficiency of students, as presented in Table 3. The experimental class had a higher mean post-test score than the control class. Therefore, these findings suggest that the experiment potentially had a beneficial effect on students' mathematical proficiency. However, it is essential to highlight the consistency and significance of the observed differences. An independent samples t-test is used to test the first hypothesis. Table 4 shows the test results.

Table 3. Descriptive statistical data on students' scores on post-test of students' mathematical proficiency

	Class	N	Mean	Std. Deviation	Std. Error Mean
Post-test	experiment class	32	72.12	9.15	2.25
	control class	32	59.75	13.27	1.92

Table 4. The results of the t-test for average score of the post-test about students' mathematical proficiency between experimental and control group

Levene's Test for Equality of Variances	F	Sig.	t	df	Sig. (2-tailed)
Equal variances assumed	11.38	.002	5.86	62	.000

Asymp, the Sig (2-tailed) value for mathematical proficiency between the experimental and control groups is 0.00. It means H_0 is rejected. The experimental class's mathematical proficiency was superior to the control class. Furthermore, a two-way analysis of variance (ANOVA) with GLM-Univariat was used to test the second hypothesis. Table 2 displays the findings of the study. We first measured the n-gain score in the experimental and control classes to analyze it. The results are presented in Table 5.

Table 5. N-gain score

	Class	Score	Category
N-gain	experiment class	0.38	Medium
	control class	0.17	Low

The findings from the N-gain analysis revealed that the students in the experimental group exhibited a medium improvement in their mathematics proficiency, as indicated by an N-gain score of 0.38. In contrast, the students in the control group had a rise of 0.17, falling within the lower range. The N-gain value, falling within the medium category, suggests that implementing PBL may benefit students' mathematical proficiency in fractions.

Table 6. Analysis of variance of data on improving mathematical proficiency based on the mathematics learning model and EMS

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	4223.071 ^a	5	839.62	190.16	.000
Intercept	218578.62	1	211687.62	46541.44	.000
Model	1743.10	1	1755.20	360.45	.000
EMS	2431.53	2	1266.24	264.21	.000
Class * EMS	122.28	2	61.64	13.28	.000
Error	275.92	58	4.71		
Total	216910.00	64			
Corrected Total	4511.99	63			

a. R Squared = .941 (Adjusted R Squared = .941)

The two-way analysis of variance (ANOVA) findings are presented in Table 4, which was conducted to assess the impact of the mathematics learning model and EMS components on improving mathematical proficiency. The findings indicate that the mathematics learning model had a notable impact on enhancing mathematical proficiency, as evidenced by a statistically significant F value of 190.16 ($p < 0.05$). This finding suggests that the mathematics learning

model employed in the research had a beneficial impact on enhancing mathematical proficiency. Another notable element that had a significant role was the EMS factor, which substantially impacted the enhancement of students' mathematical proficiency ($F = 264.21$, $p < 0.05$). It implies that variations in students' initial mathematical proficiency levels significantly influenced the research outcomes. Furthermore, it is worth noting that there was a notable impact of the interaction between class and EMS ($F = 13.28$, $p < 0.05$), which confirms that the influence of PBL can differ depending on the degree of EMS. The R-squared value of 0.941 suggests that the mathematics learning model can account for around 94.1% of the observed variability in enhancing mathematical proficiency. Hence, this discovery provides evidence.

PBL is currently a popular mathematics learning model in Indonesia, as evidenced by several studies, such as research by Anjelina et al. (2021). PBL has successfully improved students' mathematical proficiency. The study's findings indicate that students who engage in PBL exhibit superior mathematical creative thinking and self-concept abilities compared to those who receive conventional learning (Purwasih & Sariningsih, 2017). The analysis suggests that PBL is more effective than conventional learning in enhancing students' mathematical representation skills (Fitri et al., 2017). The study's results indicate that implementing PBL in schools is a viable option for enhancing students' higher-order thinking skills, particularly in schools with middle or high-ability students (Napitupulu et al., 2016). In general terms, PBL has successfully improved students' mathematical skills in Indonesia. It is inseparable from the characteristics of PBL, which is based on the theory of constructivism. Students are required to think critically, find solutions to real-world problems (Jensen et al., 2019), and be able to make evidence-based arguments (Belland, 2014). The provided description aligns with the first finding in this study, which revealed that the mathematical proficiency of students in the experimental group class was better than that of the control class.

The findings of this study are further supported by the outcomes of PBL investigations conducted in primary school settings. PBL effectively improves literacy and numeracy in primary schools (Masliah et al., 2023). At the primary school level, PBL is advised as a learning approach because it can help grow higher-order thinking skills (Zainal, 2022). Students' critical thinking skills in mathematics learning are greatly influenced by problem-based learning (Ariyani & Prasetyo, 2021; Haryanti & Febriyanto, 2017; Ruli & Indarini, 2022). PBL positively impacts students' problem-solving abilities (Widyastuti & Airlanda, 2021). Problem-based learning models can improve student learning outcomes (Fauzia, 2018). Results also show that the Prezi-based PBL model significantly impacts science literacy (Kristiantari et al., 2022). Based on the description above, it is clear that PBL is an effective way to improve mathematical proficiency in primary school.

PBL is a learning model for introducing a problem and fostering comprehension of its overarching concepts through theory (Espey et al., 2007b). The researcher aims to use PBL to respond to today's dynamic society, where social, political, and technological conditions are constantly changing. This situation requires students to think critically, justify hypotheses, collect and interpret data, and communicate processes and results to others (Yager, 2000). PBL involves real problems and can lead to changes in knowledge, skills, or attitudes needed to make decisions (Rangachari & Crankshaw, 1996). The PBL environment positions students as professionals and confronts them with problems that require them to define a problem that is not structured, develop hypotheses, access, analyze, and use data from multiple sources, revise initial hypotheses as the data collected evolves, and justify solutions based on evidence and reasoning (Sungur et al., 2006). Naturally, PBL can improve mathematical proficiency and become a strong reason for accepting the second hypothesis: "There is an interaction between the learning model factors given and the EMS category factors toward increasing students' mathematical proficiency."

Mathematics teaching in primary schools often incorporates real-world contexts to enhance problem-solving (de Kock & Harskamp, 2014; Merriënboer, 2013). In this study, the researcher provided non-routine problems (core activities) with settings relevant to students' lives to address issues that cannot be solved conventionally or require more advanced thinking and problem-solving. Schoenfeld (1983) defines problem-solving as not knowing how to execute a task using regular processes. The key idea of PBL is that learning is "constructive, self-directed, collaborative, and contextual" (Schwartz et al., 2001). Students actively engage with significant problems, solve them in a collaborative environment, construct mental learning models, and develop self-directed learning habits through practice and reflection (Dolmans et al., 2002) with the help of prior knowledge as active information who organize new relevant experiences into personal mental representations or schemata.

Most educational theories assume that engaging students' prior knowledge affects teaching and learning (Simonsmeier et al., 2022). Research shows that prior knowledge affects learning (Bodner et al., 2014). Low-prior knowledge students were more influenced by their environment and others, which impeded their concentration (Yeo et al., 2022). If only stored in long-term memory, prior information cannot affect learning (Simonsmeier et al., 2022). This research uses PBL to help students leverage prior knowledge. Prior knowledge influences PBL, which mediates learning outcomes. Prior knowledge improves problem-solving tactics (Schneider et al., 2011) and evaluates sources and new information (Lombardi et al., 2016).

The description above strengthens the results of the second hypothesis that PBL interacts with EMS on mathematical proficiency; this shows that the effect of learning on improving mathematical proficiency differs for individuals with different early mathematics skills. By

knowing this interaction, we can design more appropriate learning strategies to improve mathematical proficiency to varying levels of EMS. It also gives us the information to predict students' mathematical ability after a particular learning program based on their early mathematics skills. It can help in planning effective and efficient learning programs.

However, teachers often lack the pedagogical knowledge to teach problem-solving properly and to differentiate their teaching to heterogeneous groups of students (Jatisunda, 2013). Teachers' pedagogical attitudes determine the success or failure of the education system in developing students' problem-solving skills (Vicente et al., 2022). Of course, the student's perspective also needs to reinforce this attitude as it will affect their confidence in solving new problems (Yusof & Tall, 1998). Other literature also mentions that it is not enough for students to have prior knowledge, concepts, rules, and knowledge of concepts and ideas in a related domain but also metacognitive knowledge to combine these in solving problems (Kim et al., 2018). It can make PBL difficult for students, especially if they find it difficult to cope with the complexity of the problem. (Könings et al., 2019). Students' cognitive activities are fundamentally limited by their working memory, which can only process a certain amount of information at a time (Sweller, 1988a). Optimizing the learning process requires teachers to consider the student's cognitive capacity (Van Merriënboer, 2013). In line with the explanation above, in the context of this study, we found an indication that the guidance given in the learning process was not optimal, which was reinforced by the n-gain score, which only reached a medium level for the experimental class and a low level for the control class with fractional material.

The primary topic of the mathematics material taught to students in the research is the concept of fractions, as learning fractions is essential for students (Copur-Gencturk, 2021). A firm grasp of fractions is a reliable predictor of students' achievement in advanced mathematics (Pedersen & Bjerre, 2021; Siegler et al., 2012b), such as algebra and statistics (Bailey et al., 2012). Globally, students face challenges in mastering fractions (Torbeyns et al., 2015; Utley & Reeder, 2012), having misconceptions (Lamon, 2007), difficulty identifying equal fractions (Wong, 2010), and difficulty solving narrative problems, solving addition and subtraction problems with unlike denominators, and simplifying fractions (Aminah & Kurniawati, 2018; Imaroh, 2021; Made, 2018). Fractions are essential material, but students find it challenging to learn. In the Indonesian context, fractions are also complex for students to learn. Students' procedural ability to solve fraction problems is still low (Galuh et al., 2022). Difficulty in adding fractions with the same denominator (Ibrahim et al., 2022) and identifying didactic and epistemological obstacles (Rohmah, 2019). Based on this description, it is a challenge for researchers to solve, but the research results align with the report.

Fractions remains a complex subject for students to learn, although the first hypothesis is accepted by showing that PBL is better than expository if we look at the n-gain score. It is defined by Hake (1998) as used to measure changes in student performance. The n-gain score for students in the experimental group was 0.36, performance in the medium category, and then for students in the control class, the n-gain score was 0.17, performance in the low sort. It is due to students being confused in developing their learning due to minimal guidance, so the expected depth of material is not fully achieved (Miflin et al., 1999). The cognitive psychology view of minimal guidance to students in learning leads to learning goals not being achieved (Clark et al., 2012). In addition, the PBL process is not effective and efficient because the problems exceed the students' working memory capacity (Jalani & Sern, 2015). It is differentiated student success in problem-based learning (Sweller, 1988b). Presenting PBL to students will fail if teachers do not understand students' abilities. Research does not guarantee with a high degree of certainty that PBL improves students' knowledge more effectively than traditional learning (Dolmans et al., 2016; Wilder, 2015).

Conclusion

This study examined how PBL affects students' mathematical proficiency, focusing on fractions. PBL improved students' mathematical proficiency, as shown by the independent samples t-test analysis results, which indicated significant improvements in the experimental class compared to the control class. The two-way ANOVA showed that PBL and EMS components interact to improve student mathematical proficiency. The R-squared value (0.941) suggests that the PBL explained much of the observed diversity in mathematical proficiency. The significant R-squared value implies that PBL enhances students' mathematical proficiency in this research. The study indicates that PBL improves students' mathematical proficiency, especially in complicated subjects like fractions. The results emphasize the need for customized mathematical learning models and student EMS in math instruction. The study's quasi-experimental nature must be acknowledged, which may limit generalizability. The findings may not correctly represent different student populations due to their environment. Despite its limitations, the study shows that PBL improves mathematical proficiency, notably in fractions. Given real-world educational constraints, the quasi-experimental design was adopted for practical, ethical, and methodological reasons. Future studies should examine mathematical proficiency beyond fractions and various student demographics to improve external validity. Longitudinal research may reveal how PBL affects mathematical proficiency across time. Educators and policymakers should use PBL for complicated concepts like fractions in mathematics instruction. Adjusting education to students'

starting competence levels improves intervention efficacy. Teachers' professional development could focus on PBL in mathematics instruction.

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