

The Trigonometric Adaptive Worksheet Performance in Optimizing Trigonometric Thinking of Prospective Mathematics Teacher: Single Subject Research

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Abstract. *The ability to think in trigonometry supports the understanding of trigonometry concepts, which still pose challenges for first-year university students, including prospective teachers. The Trigonometric Adaptive Worksheet is one of the student worksheets to optimize trigonometric thinking abilities. This research aimed to assess the performance of the Trigonometric Adaptive Worksheet on the trigonometric thinking abilities of prospective mathematics teachers. Quantitative descriptive research with a Single Subject Research method and a basic design of A (baseline) - B (intervention) was employed as the research approach. Three prospective mathematics teachers from three categories of high, moderate, and low mathematical abilities were selected as research subjects. The data related to changes in individual behavior regarding progress in trigonometric thinking abilities were analyzed using within-condition analysis and between-condition analysis procedures. Based on the single-subject data analysis of the three research subjects, it is evident that moderate and low subjects tend to benefit more from implementing the Trigonometric Adaptive Worksheet. These findings have important implications, suggesting adaptive worksheets can enhance individual learning outcomes by tailoring content to individuals' needs. The worksheet adaptability and personalization provide an opportunity to improve trigonometry instruction's effectiveness.*

Keywords: *single subject research, trigonometric adaptive worksheet, trigonometric thinking skills*

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Introduction

The topic of Trigonometry is one of the learning outcomes targeted in the curriculum for prospective mathematics teachers, encompassed in the Calculus course. Its purpose is not only to deepen the understanding of trigonometry concepts and problem-solving but also to explore the material and strategies for presenting trigonometric ideas to students when they become mathematics teachers in the future. In its implementation, understanding Trigonometry needs to be supported by trigonometric thinking skills, which involve many relationships and angle comparisons within a triangle (Hamzah, Maat, & Ikhsan, 2021; Lu, Maknun, Rosjanuardi, & Jupri, 2020). From an algebraic perspective, trigonometry is seen as a comparison of unknown angles expressed through the ratios of sine, cosine, tangent, cotangent, and secant angles (Albahboh, Gingold, & Quaintance, 2019; Zhang & Zhang, 2023). According to Çekmez

(2020), trigonometric thinking skills refer to an individual's mental capacity to process information to recognize patterns of relationships, identify properties of facts, and relate them to the characteristics of trigonometry, namely, the lengths and angles of triangles. Such thinking processes are intended as recommendations for solving mathematical problems and providing solutions in real-world situations, such as the motion and velocity of objects, the length and wavelength of waves, or phenomena in various engineering fields (Körei, Szilágyi, & Török, 2021). Additionally, Spangenberg (2021) suggests that trigonometric thinking skills have the potential to stimulate individuals to think perceptively when analyzing problems and adaptively managing relevant information. Therefore, it is reasonable to consider trigonometric thinking skills as one of the priority competencies to be developed in individuals, especially in prospective mathematics teachers.

Several studies have revealed that Trigonometric thinking skills among students, especially first-year university students, are relatively low (Spangenberg, 2021; Urrutia, Urrutia, Loyola, & Marín, 2019). A closer examination of the difficulties faced by students in developing Trigonometric thinking skills includes (1) students struggling with determining Trigonometric functions for angles less than zero degrees and angles greater than ninety degrees, (2) students encountering challenges in translating Trigonometric functions from the perspective of the sides of a right triangle within the unit circle or from the numerical values of the functions, and (3) students having difficulty determining angle values using the unit circle through the deduction of symbols within Trigonometric functions (Oktaviyanti & Sholahudin, 2023; Mumcu & Aktürk, 2019). If these constraints are not addressed, they can impede students' learning performance in Trigonometry and hinder the development of students' problem-solving abilities (Hamzah et al., 2021; Maknun, Rosjanuardi, & Jupri, 2022). To anticipate potential obstacles or challenges students may face when learning trigonometry, it is essential to seek a solution so that students' trigonometric thinking abilities can be optimized.

The under-optimization of Trigonometric thinking skills among students today is partly due to the mismatch between the learning media used (Bedada & Machaba, 2022; Maki & Adams, 2019). Thinking skills, including Trigonometric thinking, need to be stimulated and explored through adaptive, guided, and patterned learning media, helping to facilitate the development of concepts from the simplest to the most complex (Ngu & Phan, 2023; Oktaviyanti & Sholahudin, 2023). One learning medium that can accommodate this is Trigonometric Adaptive Worksheets (Stovner & Klette, 2022; Chin, Choy, & Leong, 2021). The term "adaptive worksheet," referred to in this research, is a learning support tool designed to facilitate the mastery of subject matter concepts. Its content can be customized to meet the learning needs of students, considering both the difficulty level and the pace of material

absorption. The design of the adaptive worksheet emphasizes providing a personalized learning experience for students and adjusting the learning process based on their performance and abilities. In a study by Oktaviyanti and Sholahudin (2023), it was mentioned that using Trigonometric Adaptive Worksheets can stimulate students to think systematically and deductively. Furthermore, Nasution and Yerizon (2019) stated that Trigonometric Adaptive Worksheets allow students to see comparisons or equivalences of angle values through various representations. The structure designed within the Trigonometric Adaptive Worksheets can provide reasoning assistance, enabling students to construct patterns and make generalizations of concepts (Oktaviyanti & Sholahudin, 2023). Therefore, the implementation of Trigonometric Adaptive Worksheets should be pursued to optimize the Trigonometric thinking skills of students, especially prospective mathematics teachers.

Research related to worksheet development, especially in the context of trigonometric concepts, has been conducted extensively over the last five years, starting in 2019. Nasution and Yerizon (2019) focused on developing trigonometry concept worksheets, collaborating with a discovery learning approach to enhance problem-solving abilities, particularly for junior high school students. Meanwhile, Kadarisma, Sari, and Senjayawati (2020) and Nurdianto, Hartono, and Indaryanti (2020) emphasized the creation of worksheets for trigonometric materials based on inquiry and generative learning to optimize higher-order thinking skills. Hayuningrat and Rosnawati (2022) adopted a realistic mathematics approach in designing trigonometry worksheets to guide students in achieving generalization abilities. Furthermore, Bhoke and Bara (2021) investigated the feasibility of guided discovery-based worksheets aided by GeoGebra for high school students. Oktaviyanti and Sholahudin (2023) examined the format and content of the worksheet to determine how well students could adjust their trigonometric thinking. Most of the research that has been done focuses on the development of worksheets, and it generally indicates that these worksheets satisfy the requirements for validity and reliability of the instrument so that they are applicable to strengthen students' mathematical skills. Despite recent research on the fundamental concept of creating worksheets for trigonometric concepts, there is still a limited amount of information regarding the outcomes of worksheet implementation, particularly concerning trigonometric thinking abilities.

Therefore, based on the comparison between the ideal conditions and the real-world situation concerning the relatively unexplored trigonometric thinking abilities through the utilization of worksheets, the research aims to examine the performance of the trigonometric adaptive worksheet as an effort to optimize the trigonometric thinking abilities of prospective mathematics teachers.

Method

This study aims to assess how the intervention with trigonometric adaptive worksheets affects the progress of trigonometric thinking abilities of research subjects from three different levels of ability through quantitatively analyzed behavior changes. Typically, intervention research is associated with large groups, but we only need three subjects to represent varying abilities in this case. We consider Single Subject Research (SSR) as the suitable research approach for our study.

SSR research, referred to as single-case experimental research, aims to track behavioral changes in a single person or a small group of people in response to a treatment or intervention that is repeated over a set amount of time, such as once per day, once per week, or once per month (Catagnus, Garcia, & Zhang, 2023; Miller, Smith, & Pugatch, 2020). This approach enables researchers to investigate the impact of experimental interventions when obtaining a standard-sized group of subjects is challenging. Single-subject research has the potential to reduce common errors found in group comparison studies, such as inter-subject variability, as each individual in single-subject research serves as their control. Similar to experimental research in general, SSR depends on comparisons between the control and experimental groups but employs the same research subjects in various settings. These comparisons are then documented over time to show whether the interventions made a difference in the behavior of the research subjects (Collini, 2019; Lanovaz & Primiani, 2023). The use of trigonometric adaptive worksheets is the independent variable in this SSR study, while prospective mathematics teachers' trigonometric thinking ability is the dependent variable.

Three prospective mathematics teachers who had high (S1), moderate (S2), and low (S3) mathematical abilities were selected as the research participants for this SSR study. They were chosen by non-probability sampling to determine the effectiveness of the trigonometric adaptive worksheet in maximizing abilities. The three research subjects were first-year students in the Department of Mathematics Education of Universitas Serang Raya who took Calculus I during the academic year 2022–2023, which is an odd semester. Based on the results of prospective mathematics teacher assessments conducted during the current half semester, categories of high, moderate, and low student skills are determined. The categorical subject indicators include the results of the basic mathematics entrance exam for university, the average report card grades, and initial observations during the first half of the semester.

The data collected pertains to the individual behavioral changes related to the progress of students' trigonometric thinking abilities in two phases: the baseline phase and the intervention phase. In the baseline phase, research participants used textbooks typically for learning Calculus to acquire trigonometry ideas and solve problems involving trigonometric functions. Before

receiving an intervention, the baseline phase aims to gather preliminary information on students' trigonometric thinking abilities. Additionally, a trigonometric adaptive worksheet was used during the intervention phase of research participants' learning and problem-solving to examine students' trigonometric thinking skills following the intervention. Furthermore, the development of the trigonometric adaptive worksheet is based on the work of Oktaviyanti and Sholahudin (2023). Each baseline and intervention phase was conducted in five sessions, each lasting 60 minutes for each study participant.

Behavioral changes in the research subjects were measured based on the acquisition of correct responses in solving trigonometric problems per session, with a maximum score of 100. To determine whether the independent variable can affect the dependent variable, the data is then analyzed using visual plotting analysis or visual inspection techniques to trace level, estimate trends, stability, immediacy effect, consistency of data patterns, and non-overlapping data in each evaluated phase (Byers, Leighton, Leggett, Krzensk, Adamson, Pollock, & MacMahon, 2022; Kemp & Fisher, 2021; Manolov & Onghena, 2022). The split-half technique, which splits the analysis into two conditions, analysis within conditions (within-phase data patterns) and analysis between conditions (between-phase data patterns), is another procedural reference for the visual analysis method (Sen, 2021). Analysis within conditions concentrates on baseline and intervention situations through procedures of (1) making AB notation, (2) calculating the number of sessions for each condition, (3) determining level of stability and range of data for each condition, (4) measuring changes in level and difference between the first and last values in each condition (relative and absolute level change), (5) determine trend using the split-middle trend estimation method, (6) calculating the percentage of data points within the stability limit for each condition (trend stability), and (7) using the free hand method (replacing the dotted line into a regular line) to evaluate data paths (Lane & Gast, 2014; Virues-Ortega, Moeyaert, Sivaraman, Rodríguez, & Castilla, 2023). While analysis between conditions is carried out to pay attention to whether there are differences between the baseline and intervention phases with the stages of (1) determining the number of variables that change between conditions (changed in trend direction), (2) identifying the direction of the trend in all adjacent conditions, (3) compare the decision of the percent of data points within the stability limits of each condition with the direction of the trend in adjacent conditions (changed in trend stability), (4) evaluate level changes between conditions, and (5) calculate the percentage overlap of data between conditions (Kratochwill, Horner, Levin, Machalicek, Ferron, & Johnson, 2023; Lane & Gast, 2014).

Results and Discussion

Analysis Within Conditions

Evaluation of the data patterns displayed by study participants S1, S2, and S3 in baseline and intervention circumstances is the goal of the analysis within conditions (within-phase data patterns). For the data pattern under evaluation to meet condition consistency norms, its stability must be guaranteed. According to Chiang, Jhangiani, and Price (2015), any changes in the condition should be rather easy to detect if a condition is consistent or in a steady state.

For each of the five sessions throughout the baseline and intervention phases, research subject data were collected (Table 1).

Table 1. Subject's score gaining in each session

Session	Subject Score					
	S1		S2		S3	
	A	B	A	B	A	B
1	70		60		55	
2	85		65		50	
3	75		65		65	
4	80		70		55	
5	85		70		65	
6		80		85		75
7		85		80		80
8		85		90		80
9		85		85		85
10		90		90		90

The data from Table 1 is represented in a visual display, as shown in Figure 1. Figure 1 illustrates the data acquisition for study subjects S1, S2, and S3 for each phase in AB notation and area boundary of data.

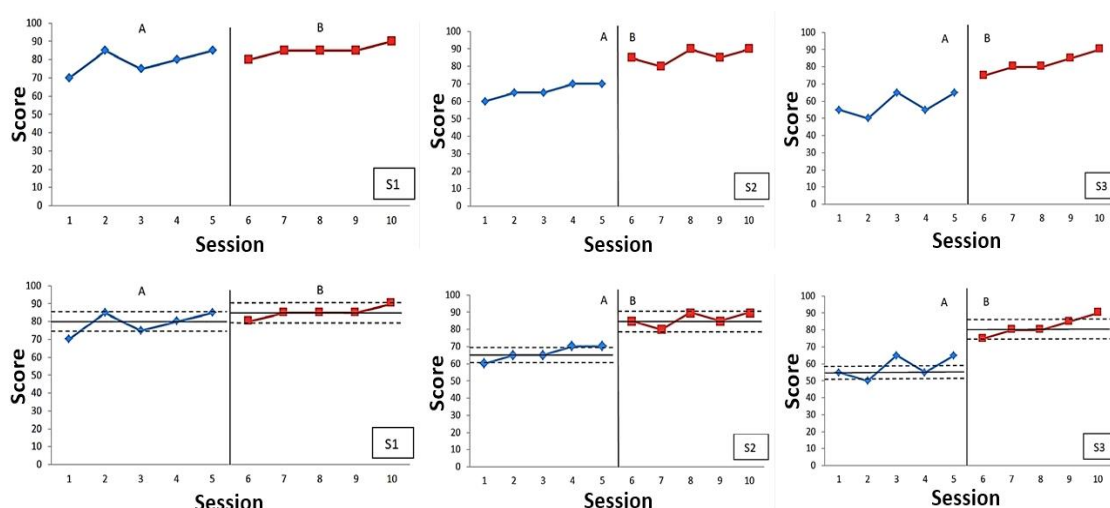


Figure 1. A – B notation and boundary of data stability for data S1, S2, and S3

Based on Figure 1. in the first row, the range and level of data stability for each condition can be determined through the average, the median, and the difference between the highest and lowest data values. Meanwhile, the level of data stability is calculated by the percentage of data

points at 15% of the median. The stability criterion is met if around 85% of the data in a phase is 15% of the median for all data points (Lobo, Moeyaert, Cunha, & Babik, 2017).

An illustration of the 15% area boundary of the mean data for all data points in the baseline and intervention phases for each study subject is shown in Figure 1 in the second row. For the S1 data, it is observable that many data points remain within the boundaries of the baseline phase (A), namely 4 data points. The percentage of stability in the baseline phase (A) was discovered by comparing the data points in the baseline phase (A) with all data points in the baseline phase (A) multiplied by 100% to obtain 80%. While the data points that remain within the limits of the intervention phase (B) are 5 data points and with the same procedure the percentage of stability in the intervention phase (B) is 100%. The calculation of the range and level of data stability in the baseline phase (A) and the intervention phase (B) for each research subject is detailed in Table 2.

Table 2. Recapitulation of data range and phase stability levels per study subject

subject	Phase							
	Baseline (A)				Intervention (B)			
	Mean	Median	Range	Stability	Mean	Median	Range	Stability
S1	79	80	15	80%	85	85	10	100%
S2	66	65	10	60%	86	85	5	100%
S3	58	55	15	60%	82	80	15	80%

Table 2 shows that each research subject has an average performance during the baseline phase (A), which is lower than the average performance during the intervention phase (B). This indicates a change in level from the baseline phase (A) to the intervention phase (B). To ensure that the level changes are consistent, pattern tracking is necessary.

Baseline (A) and intervention (B) phase, both relative and absolute. Relative level changes are carried out using the split-middle method, which involves the phase into two repeating parts to obtain quarters containing the same data points. The median point of each phase in each quarter is detectable and used to estimate the trend in the next stage. Meanwhile, absolute level changes are found by calculating the difference between the first and last values in each phase. Table 3 below provides information on the relative and absolute level changes in each phase for each research subject.

Information in Table 3 explained that, in general, there were quite consistent level changes in each study subject in the baseline (A) and intervention phase (B), with changes ranging from +5 to +15. Next, determine the trend investigated from the distribution of data points, whether it is monotonically rising (the trend is accelerating) or monotonically decreasing (the trend is decelerating). Using the split-middle technique, an estimate of the trend of each

phase and the area of stability for each research subject can be obtained, as illustrated in Figure 2.

Table 3. Changes in relative and absolute levels for each condition for S1, S2, and S3

Level Change Aspect	Research subject					
	S1		S2		S3	
	Phase A	B phase	Phase A	B phase	Phase A	B phase
Relative Change						
Median first half (1 st quartile)	75	85	65	85	55	80
Median second half (3 rd quartile)	85	85	70	90	65	85
Relative Level Change	+10 Increase	0 Stable	+5 Increase	+5 Increase	+10 Increase	+5 Increase
Absolute Change						
First Value	70	80	60	85	55	75
Last Value	85	90	70	90	65	90
Absolute Level Change	+15 Increase	+10 Increase	+10 Increase	+5 Increase	+10 Increase	+15 Increase

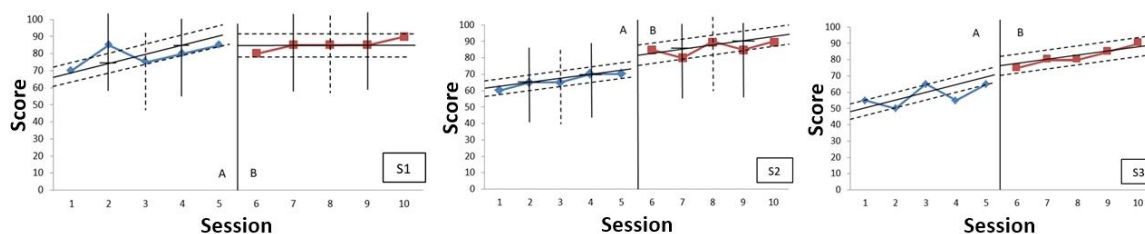


Figure 2. Estimation and areas of trend stability in each phase for data S1, S2, and S3

From Figure 2, it is possible to calculate the number of data points that remain within the boundaries of the baseline (A) and intervention (B) phases. Using the same procedure in Figure 2, the percentage trend for trend stability in the baseline (A) and intervention (B) phases can be determined as shown in Table 4 below.

Table 4. Percentage of *trend stability* in each phase for S1, S2, and S3

Research Subject	Percentage of Stability at and in Phase	
	Phase A	B phase
S1	$(4/5) * 100\% = 80\%$	$(5/5) * 100\% = 100\%$
S2	$(5/5) * 100\% = 100\%$	$(5/5) * 100\% = 100\%$
S3	$(4/5) * 100\% = 80\%$	$(5/5) * 100\% = 100\%$

Table 4. shows that the percentage of trend stability is in the range of 80% - 100%, which indicates stability criteria are met. Then, from Figure 2 and Table 4, trends can be traced from each baseline (A) and intervention (B) phase for each research subject, which includes the direction of the trend, trend stability, and path in the trend. The overview is illustrated in Table 5.

Table 5 shows that the S2 and S3 research subjects tend toward an accelerated trend with stable trend stability both in the baseline phase (A) and the intervention phase (B).

Table 5. Overview of direction, stability, and trend lines

Aspects of the Review	Research Subject					
	S1		S2		S3	
	Phase A	B phase	Phase A	B phase	Phase A	B phase
Trend direction (deceleration/acceleration)	Acceleration	Stable	Acceleration	Acceleration	Acceleration	Acceleration
Trend Stability (stable/unstable)	Stable	Stable	Stable	Stable	Stable	Stable
Path in trend (multiple/single)	Single	Single	Single	Single	Single	Single

Analysis Between Conditions

The purpose of carrying out analysis between conditions (between-phase data patterns) is to observe differences in the behavior of research subjects in the baseline phase (A) and the intervention phase (B). Differences in the behavior of the research subjects were traced from the trigonometric thinking skills, which are expected to change from the baseline phase (A) to the intervention phase (B). Therefore, the trigonometric thinking ability is the only dependent variable measured by changes between phases in this study.

Tracing differences in the trigonometric thinking ability of the research subjects was evaluated from changes in levels between phases, changes in direction and trend stability, as well as the percentage of overlap. The change in level between phases from baseline (A) to intervention (B) is calculated from several values, including the average value, the median value, the difference between the highest and lowest data values, and the median value of the quarter of each phase. Changes in levels between phases contribute to changes in trend direction and trend stability. In detail, the level changes between phases are shown in Table 6.

Table 6. Changes in level between baseline (A) to intervention (B) phases

Level Change Aspect	A → B Level Change		
	S1	S2	S3
Relative Change			
The median of the first half of condition B	85	85	80
The median of the second half of conditions A	85	70	65
Relative Level Change	0	+15	+15
	Stable	Increase	Increase
Absolute Change			
First Value Condition B	80	85	75
Last Value Condition A	85	70	65
Absolute Level Change	-5	+5	+10
	Decrease	Increase	Increase
Changes in Median Value			
Median Value of Condition B	85	85	80
Median Value Condition A	80	65	55
Median Level Change	+5	+20	+25
	Increase	Increase	Increase
Change in Mean Value			
Condition B Mean Value	85	86	82
The Mean Value of Condition A	79	66	58
Mean Level Change	+6	+20	+24
	Increase	Increase	Increase

Based on Table 6 on the four aspects of level changes, it can be seen that S2 and S3 have a consistent trend of increasing. The tendency for changes in levels to consistently increase implies an increasing trend direction and a tendency for stable stability between phases from baseline (A) to intervention (B) for S2 and S3. This means that the actions taken during the intervention phase (B) affect changes in behavior observed in research subjects.

Furthermore, the effect size of an intervention can be calculated using the percentage of non-overlapping data or PND (Percentage of Non-overlapping Data). An intervention is effective if the performance shown during the intervention phase (B) does not overlap with the performance observed during the baseline phase (A). The percentage of data that does not overlap is calculated by comparing the data values above the highest from the baseline phase (A) with the number of sessions in the intervention phase (B) multiplied by 100%. PND illustrations for each research subject are presented in Figure 3.

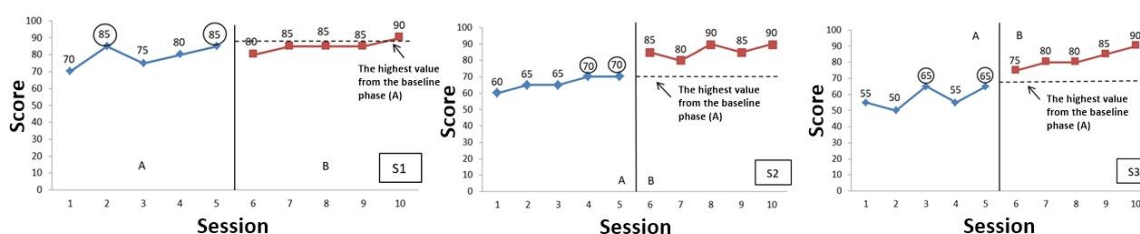


Figure 3. Illustration of PND determination

Figure 3 shows that the percentage of non-overlapping data for the S1 subject is 20%, while S2 and S3 have the same gain of 100%. The PND value in the S1 subject is included in the unreliable treatment category, meaning that the use of action in the intervention phase (B) does not impact changes in the dependent variable. At the same time, the PND for S2 and S3 subjects are in the criteria for highly effective treatment, which means that implementing actions in the intervention phase (B) impacts changes in the dependent variable.

The analysis of subjects' performance in terms of level, trend direction, and trend stability in each study phase. From the conditions presented in Tables 2 and 3, it can be concluded that the performance levels of subjects S1, S2, and S3 in both the baseline (A) and intervention (B) phases did not experience a decline. This suggests that using the Trigonometric Adaptive Worksheet, the intervention offered the probability of enhancing trigonometric thinking abilities, particularly for those with different mathematical capabilities. Furthermore, the graph tendencies in Figure 2 indicated a consistent upward trend for S2 and S3, while S1 exhibited a more stable pattern. The degree of trend stability was determined through the calculation presented in Table 4, which showed consistent trend stability. Based on the direction and stability of the trends, it is evident that data points with upward trends and the potential for

better outcomes from the implementation of the Trigonometric Adaptive Worksheet are observed in subjects S2 and S3.

Transitioning to the results of the between-phase analysis provides information about the immediacy effect, consistency of data patterns, and non-overlapping data. The immediacy effect is observed from the median level changes between phases in Table 6, which show increases for each subject, S1, S2, and S3, respectively, of +5, +20, and +25. Furthermore, the immediacy effect is further sharpened by the level changes between the last three data points in the baseline (A) phase and the first three data points in the intervention (B) phase, as visualized in Figure 3. If the last three points in Phase A and the first three points in Phase B exhibit rapid changes, there is an indication that the intervention in Phase B has a direct effect (Natesan Batley, Minka, & Hedges, 2020). In Figure 3, rapid changes are detected in data points for S2 and S3. This means that the intervention, in the form of implementing the Trigonometric Adaptive Worksheet, directly affects the trigonometric thinking ability of subjects S2 and S3.

Furthermore, the consistency of data patterns is traced through a comparison of changes in trend direction obtained from absolute-level changes in within-phase and between-phase analyses, as shown in Tables 3 and 6. Absolute level changes in subjects S2 and S3 show consistency in both within-phase and between-phase analyses. However, this is not the case for subject S1, which experiences a decrease in absolute level changes in the between-phase analysis. Thus, it can be said that subjects meeting the data pattern consistency standard are S2 and S3. Then, to calculate the effect size of the study as a procedure for knowing how effectively an intervention works to provide improvement, the PND (Percentage of Non-overlapping Data) method is used, as illustrated in Figure 3. Based on the PND calculation, it was known that the interventions carried out on S1 subjects were unreliable, while S2 and S3 subjects were effective. This means that using the Trigonometry Adaptive Worksheet in the intervention phase has a greater chance of impacting the trigonometric thinking skills of S2 and S3 subjects.

By considering the results of the analysis within and between conditions, it was discovered that there was a change in individual behavior related to the development of students' trigonometric thinking skills with the assistance of the Trigonometric Adaptive Worksheet. S2 and S3, who had low and moderate mathematics ability, showed consistent changes in behavior. This result suggests that, particularly for students in the moderate and low mathematics ability categories, the Trigonometric Adaptive Worksheet presents progressive potential in optimizing trigonometric thinking skills. It is possible because the Trigonometric Adaptive Worksheet displays an integrative activity structure from various representations of trigonometry concepts rather than just taking deductive workflows into account as a guide for

students thinking in detecting rule consistency (Oktaviyanthi & Sholahudin, 2023). Accordingly, Edelman and Wang (2020) emphasized that students can be given structured instructions for exploring the idea of comparing the lengths and angles of triangles with unit circles and numerical function values. These schematic worksheets correspond to the characteristics of trigonometry processing. Additionally, Schlatter, Molenaar, and Lazonder (2022) claim that worksheets with collaboratively designed guided problem instructions offer a better chance to support successful student learning.

This research not only strengthens research analysis procedures using the Single Subject Research approach, particularly in the field of mathematics education (Manikmaya, Charitas, & Prahmana, 2021; Nuari, Prahmana, & Fatmawati, 2019; Widodo, Cahyani, & Istiqomah, 2020). It also measures the effectiveness of using the Trigonometric Adaptive Worksheet. However, the effect of the Single Subject Research intervention is viewed as ungeneralizable or sample localized applicable case if it does not meet the replication standard, which is the basis for internal validity (Zanuttini, 2020). Generalizations are typically recognized when an intervention's effects can be replicated consistently with the same dependability across various study samples or circumstances that are more varied than the initial circumstances (Lanovaz, Turgeon, Cardinal, & Wheatley, 2019). This is a study's shortcomings, and there are suggestions to make future research more effective. The single-subject research technique offers a wide range of study alternatives, including integrating statistical analysis into visual inspection.

Conclusion

Based on the results of the within-phase analysis, which includes the level, trend direction, and trend stability of each research subject, and the between-phase analysis consisting of the immediacy effect, consistency of data patterns, and non-overlapping data, it can be concluded that the Trigonometric Adaptive Worksheet affects changing the behavior of the research subjects, specifically in terms of trigonometric thinking abilities. Consistency in change is experienced by subjects with moderate and low mathematical abilities. However, subjects with high mathematical ability did not exhibit consistent changes. Additionally, the research effect, measured through the percentage of non-overlapping data, indicates that the intervention with subjects with high mathematical ability is at 20%, categorizing it as unreliable, while for subjects with moderate and low mathematical ability, it reaches 100%, placing it in the effective category. These facts indicate that subjects with moderate and low mathematical abilities have a higher likelihood than subjects with high mathematical ability to optimize their trigonometric thinking abilities using the Trigonometric Adaptive Worksheet. Additionally, the fact that structured worksheets have a greater potential of enhancing students' cognitive abilities

enables teachers to appreciate the value of offering these assistances, particularly for difficult mathematical ideas. Furthermore, it is required to carry out replication processes, either direct or systematic replication, given that research using the single-subject research approach has serious problems with the generalization of research outcomes.

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References

- Albahboh, H., Gingold, H., & Quaintance, J. (2019). Trigonometry in complex inner product spaces. *Linear Algebra and Its Applications*, 575, 216–234. <https://doi.org/10.1016/J.LAA.2019.04.014>
- Bedada, T. B., & Machaba, F. (2022). The effect of GeoGebra on STEM students learning trigonometric functions. *Http://Www.Editorialmanager.Com/Cogentedu*, 9(1), 2034240. <https://doi.org/10.1080/2331186X.2022.2034240>
- Bhoke, W., & Bara, F. E. (2021). Pengembangan Lembar Kerja Siswa (LKS) berbasis pendekatan penemuan terbimbing berbantuan GeoGebra untuk membelajarkan materi trigonometri pada siswa kelas X SMA. *Jurnal Pendidikan Tambusai*, 5(3), 10429-10443. <https://jptam.org/index.php/jptam/article/view/2648/2300>
- Byers, T., Leighton, V., Leggett, J., Krzensk, A., Adamson, R., Pollock, C., & MacMahon, S. (2022). The impact of a preparatory science of learning intervention in secondary schools contexts in Australia. *Review of Education*, 10(1), 1-46. <https://doi.org/10.1002/REV3.3340>
- Catagnus, R. M., Garcia, Y. A., & Zhang, D. X. (2023). Single-subject design studies of mindfulness interventions for behavior change: A scoping review and behavioral conceptualization. *Advances in Neurodevelopmental Disorders*, 7(2), 133–164. <https://doi.org/10.1007/S41252-022-00294-2/METRICS>
- Çekmez, E. (2020). What generalizations do students achieve with respect to trigonometric functions in the transition from angles in degrees to real numbers?. *The Journal of Mathematical Behavior*, 58, 100778. <https://doi.org/10.1016/J.JMATHB.2020.100778>
- Chiang, I.-C. A., Jhangiani, R. S., & Price, P. C. (2015). *Single-subject research designs*. BCcampus. Retrieved June 12, 2023, from <https://opentextbc.ca/researchmethods/chapter/single-subject-research-designs/>

- Chin, S. L., Choy, B. H., & Leong, Y. H. (2021). *Adapting curriculum materials in secondary school mathematics: A case study of a Singapore teacher's lesson design*. Proceedings of the 43rd Annual Conference of the Mathematics Education Research Group of Australasia. Retrieved Juni 12, 2023, from <https://repository.nie.edu.sg/handle/10497/23245>
- Collini, S. A. (2019). *An experimental comparison of visual and statistical analyses*. [Doctoral dissertation, Western Michigan University] Retrieved June 12, 2023, <https://scholarworks.wmich.edu/dissertations/3515>
- Edelman, A., & Wang, Y. (2020). The GSVD: Where are the Ellipses?, Matrix trigonometry, and more. *Society for Industrial and Applied Mathematics*, 41(4), 1826–1856. <https://doi.org/10.1137/18M1234412>
- Hamzah, N., Maat, S. M., & Ikhsan, Z. (2021). A systematic review on pupils' misconceptions and errors in trigonometry. *Pegem Journal of Education and Instruction*, 11(4), 209–218. <https://doi.org/10.47750/PEGEGOG.11.04.20>
- Hayuningrat, S., & Rosnawati, R. (2022). Development of learning tools based on realistic mathematics approach that oriented to high school students' mathematical generalization ability. *Jurnal Riset Pendidikan Matematika*, 9(2), 191–200. <https://doi.org/10.21831/JRPM.V9I2.52197>
- Kadarisma, G., Sari, I. P., & Senjayawati, E. (2020). Pengembangan lembar kerja siswa berbasis inkuri untuk meningkatkan hots siswa sma pada materi trigonometri. *Teorema: Teori Dan Riset Matematika*, 5(2), 239–248. <https://doi.org/10.25157/TEOREMA.V5I2.3725>
- Kemp, A. H., & Fisher, Z. (2021). Application of single-case research designs in undergraduate student reports: An example from wellbeing science. *Teaching of Psychology*, 50(1), 86–92. <https://doi.org/10.1177/00986283211029929>
- Körei, A., Szilágyi, S., & Török, Z. (2021). Integrating didactic games in higher education: Benefits and challenges. *Teaching Mathematics and Computer Science*, 19(1), 1–15. <https://doi.org/10.5485/TMCS.2021.0517>
- Kratochwill, T. R., Horner, R. H., Levin, J. R., Machalicek, W., Ferron, J., & Johnson, A. (2023). Single-case intervention research design standards: Additional proposed upgrades and future directions. *Journal of School Psychology*, 97, 192–216. <https://doi.org/10.1016/J.JSP.2022.12.002>
- Lane, J. D., & Gast, D. L. (2014). Visual analysis in single case experimental design studies: Brief review and guidelines. *Neuropsychological Rehabilitation*, 24(3–4), 445–463. <https://doi.org/10.1080/09602011.2013.815636>
- Lanovaz, M. J., & Primiani, R. (2023). Waiting for baseline stability in single-case designs: Is it worth the time and effort? *Behavior Research Methods*, 55(2), 843–854. <https://doi.org/10.3758/S13428-022-01858-9/TABLES/11>
- Lanovaz, M. J., Turgeon, S., Cardinal, P., & Wheatley, T. L. (2019). Using single-case designs in practical settings: Is within-subject replication always necessary? *Perspectives on Behavior Science*, 42(1), 153–162. <https://doi.org/10.1007/S40614-018-0138-9/METRICS>
- Lobo, M. A., Moeyaert, M., Cunha, A. B., & Babik, I. (2017). Single-case design, analysis, and quality assessment for intervention research. *Journal of Neurologic Physical Therapy: JNPT*, 41(3), 187. <https://doi.org/10.1097/NPT.0000000000000187>

- Lu, C., Maknun, il, Rosjanuardi, R., & Jupri, A. (2020). Didactical design on drawing and analysing trigonometric functions graph through a unit circle approach. *International Electronic Journal of Mathematics Education*, 15(3), 614. <https://doi.org/10.29333/iejme/9275>
- Maki, K. E., & Adams, S. R. (2019). Specific learning disabilities identification: Do the identification methods and data matter? *Learning Disability Quarterly*, 43(2), 63–74. <https://doi.org/10.1177/0731948719826296>
- Maknun, C. L., Rosjanuardi, R., & Jupri, A. (2022). Epistemological obstacle in learning trigonometry. *Mathematics Teaching Research Journal*, 14(2), 5–25. Retrieved June 12, 2023, from <https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2022/08/v14n2-Epistemological-Obstacle-in-Trigonometry.pdf>
- Manikmaya, P., Charitas, R., & Prahmana, I. (2021). Single subject research: Pembelajaran perbandingan senilai dan berbalik nilai berpendekatan contextual teaching and learning untuk siswa slow learner. *Journal of Honai Math*, 4(1), 35–48. <https://doi.org/10.30862/JHM.V4I1.172>
- Manolov, R., & Onghena, P. (2022). Defining and assessing immediacy in single-case experimental designs. *Journal of the Experimental Analysis of Behavior*, 118(3), 462–492. <https://onlinelibrary.wiley.com/doi/epdf/10.1002/jeab.799>
- Miller, C. J., Smith, S. N., & Pugatch, M. (2020). Experimental and quasi-experimental designs in implementation research. *Psychiatry Research*, 283, 112452. <https://www.sciencedirect.com/science/article/pii/S0165178119306833?via%3Dihub>
- Mumcu, H. Y., & Aktürk, T. (2020). Mathematics teachers' understanding of the concept of radian. *Hacettepe University Journal of Education*, 35(2), 320–337. [doi: 10.16986/HUJE.2019053683](https://doi.org/10.16986/HUJE.2019053683)
- Nasution, D. H., & Yerizon, Y. (2019). Development of student worksheets based on discovery learning to improve student mathematical problem solving ability in class X senior high school. *International Journal of Scientific and Technology Research*, 8(6), 228–231. Retrieved June 12, 2023, from <https://www.ijstr.org/paper-references.php?ref=IJSTR-0619-20582>
- Natesan Batley, P., Minka, T., & Hedges, L. V. (2020). Investigating immediacy in multiple-phase-change single-case experimental designs using a Bayesian unknown change-points model. *Behavior Research Methods*, 52(4), 1714–1728. <https://doi.org/10.3758/S13428-020-01345-Z/FIGURES/4>
- Ngu, B. H., & Phan, H. P. (2023). Differential instructional effectiveness: Overcoming the challenge of learning to solve trigonometry problems that involved algebraic transformation skills. *European Journal of Psychology of Education*, 1–21. <https://doi.org/10.1007/S10212-022-00670-5/TABLES/2>
- Nuari, L. F., Charitas, R., Prahmana, I., & Fatmawati, I. (2019). Learning of division operation for mental retardations' student through math GASING. *Journal on Mathematics Education*, 10(1), 127–142. <https://ejournal.unsri.ac.id/index.php/jme/article/view/6913/pdf>
- Nurdiyanto, T., Hartono, Y., & Studi Pendidikan Matematika, P. (2020). Pengembangan lkpd materi trigonometri berbasis generative learning di kelas X. *Jurnal Pendidikan Matematika*, 14(1), 51–66. <https://doi.org/10.22342/JPM.14.1.6798.51-66>
- Oktaviyanthi, R., & Sholahudin, U. (2023). Phet assisted trigonometric worksheet for students' trigonometric adaptive thinking. *Mosharafa: Jurnal Pendidikan Matematika*, 12(2), 229–242. <https://doi.org/10.31980/MOSHARAF.A.V12I2.2569>

- Schlatter, E., Molenaar, I., & Lazonder, A. W. (2022). Adapting scientific reasoning instruction to children's needs: Effects on learning processes and learning outcomes. *International Journal of Science Education*, 44(17), 2589–2612. https://doi.org/10.1080/09500693.2022.2140019/SUPPL_FILE/TSED_A_2140019_SM9941.PDF
- Sen, N. (2021). Investigation of regression-based effect size methods developed in single-subject studies. *Behavior Modification*, 46(6), 1346–1382. <https://doi.org/10.1177/01454455211054018>
- Spangenberg, E. D. (2021). Manifesting of pedagogical content knowledge on trigonometry in teachers' practice. *Journal of Pedagogical Research*, 5(3), 135–163. <https://doi.org/10.33902/JPR.2021371325>
- Stovner, R. B., & Klette, K. (2022). Teacher feedback on procedural skills, conceptual understanding, and mathematical practices: A video study in lower secondary mathematics classrooms. *Teaching and Teacher Education*, 110, 103593. <https://doi.org/10.1016/J.TATE.2021.103593>
- Urrutia, F. Z., Urrutia, F. Z., Loyola, C. C., & Marín, M. H. (2019). A tangible user interface to facilitate learning of trigonometry. *International Journal of Emerging Technologies in Learning (IJET)*, 14(23), 152–164. <https://online-journals.org/index.php/ijet/article/view/11433/6199>
- Virues-Ortega, J., Moeyaert, M., Sivaraman, M., Rodríguez, A. T., & Castilla, B. F. (2023). Quantifying outcomes in applied behavior analysis through visual and statistical analyses: A synthesis. *Handbook of Applied Behavior Analysis: Integrating Research into Practice*, 515–537. https://doi.org/10.1007/978-3-031-19964-6_28
- Widodo, S. A., Cahyani, E. R., & Istiqomah, I. (2020). Single subject research: Learning algebra operations in introverted students. *Formatif: Jurnal Ilmiah Pendidikan MIPA*, 10(2), 133–142. <https://doi.org/10.30998/FORMATIF.V10I2.6300>
- Zanuttini, J. Z. (2020). Using a systematic review to explore single-subject design use within special education over time. *Australasian Journal of Special and Inclusive Education*, 44(1), 1–14. <https://doi.org/10.1017/JSI.2020.4>
- Zhang, D. jun, & Zhang, F. (2023). Two trigonometric function solutions of the mKdV equations. *Applied Mathematics Letters*, 146, 108839. <https://doi.org/10.1016/J.AML.2023.108839>