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# Assessing Middle School Students' Grasp of Definitions and Postulates in Euclid's Elements Volume I

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***Abstract.*** *Many junior high school students lack exposure to the fundamental definitions and postulates of Euclid's Elements, leading to gaps in their understanding of basic geometric concepts. This research addresses the need to assess students' comprehension levels of Euclid's Elements in mathematics education. This research is qualitative and uses case study methods. The case study was conducted on eight junior high school students (six females and two males), from Padang, Indonesia. The instruments used in this research were open interviews and written tests related to the definition and postulates of Euclid's Elements Volume I. The data analysis was analyzed using Categorization Analysis (Content Analysis). The results assess the extent of junior high school students' knowledge and understanding of the definitions and postulates in Euclid's Elements Volume I. The findings of this research highlight the need to strengthen the teaching of Euclid's definitions and postulates in junior high school curricula to improve students' understanding of basic geometric concepts. Additionally, it emphasizes the importance of historical mathematical texts in education and suggests further studies on effective instructional methods to enhance students' comprehension and reasoning skills.*

***Keywords****: definitions, Euclid's elements, geometry, postulates, volume I*

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## Introduction

The most well-known mathematical work from the ancient era is Euclid's Elements, the oldest mathematics textbook still used today (Wilck, 2020; Laskowska, 2021). Euclid's Elements was written by the Greek mathematician Euclid in Alexandria, Egypt, in the early 3rd century B.C. and was first published in 1482 in Europe (Nugraheni, 2021). This book consists of 13 volumes containing a collection of definitions, postulates, theorems, and mathematical proofs of these theorems. The thirteen volumes cover the fields of geometry, comparison, and number theory (Akhsani et al., 2022; Laskowska, 2021). The Elements can be called a work of complication of all the mathematical knowledge that had been obtained by Pythagoras and his school, Plato, Hippocrates from Chios, Theaetetus from Athens, and Eudoxus from Cnidos, which Euclid then united (Marsigit, 2012).

One of the topics of Euclid's Elements is geometry. Geometry is a branch of mathematics that explains the properties of lines, angles, planes, and space. Geometry is a field of mathematics that studies points, lines, planes, shapes, properties, sizes, and relationships (Nur'aini et al., 2017).

Euclid's geometry (Parabolic geometry) is a geometry that was first created by a Greek mathematician named Euclid and consists of 5 postulates (Akhsani et al., 2022).

The Elements in the book do not lie in Euclid's statement of personal formulas. Almost all the theories discussed in this book have been written about before by other scientists, and the truth has been proven. Euclid's contribution lies in the way materials and problems are arranged and their overall formulation in planning the book's preparation (Wilck, 2020). Reviewing the definitions and postulates in Euclid's Elements is critical as these postulates are the basis for the development and derivation of theorems contained in mathematics, especially in geometry (Melisa, 2020). Considering its importance, this study provided a brief overview of Euclid's Elements and reviewed the junior high school students' understanding of the definitions and postulates contained in Euclid's Elements Volume I.

The importance of studying Eclulid elements theoretically is because Euclidean elements are: (1) the basis of mathematics and logic, where Euclidean geometry is the foundation of the axiomatic system in mathematics and teaches deductive and logical thinking through postulates and theorems, (2) the foundation for other geometries, namely: being the basis for understanding non-Euclidean geometry (for example hyperbolic and elliptic geometry) and is used in theory development space and other mathematical structures, (3) applications in other sciences such as physics: the concepts of space and motion often use the principles of Euclidean geometry as well as engineering science and engineering Euclidean elements can be used for building structure calculations, bridges, and other designs (Corry, 2021; Evers, 2021; Kourniatis, 2020; Shabel, 2020). Furthermore, practically the benefits of Euclidean elements can be applied in: (1) Architecture and civil engineering, namely they can be used in building design, angle calculations, and space layout, (2) navigation and cartography, namely they can help in making maps and coordinate-based navigation systems, graphic design, and animation which can be used in digital image processing. Euclidean geometry can also be used for perspective and object transformation, (3) robotics and artificial intelligence, which can be used in programming robot movements and shape recognition in AI (Bo, 2023; De Risi, 2021b; Jenkins, 2020).

Several studies have discussed the Euclidean element. Akhsani et al. (2022) discuss Euclid as a figure in the development of mathematical philosophy. Kourniatis (2020), who comes from Yunani in his research, has described the journey of the development of science and the way humans understand the world, from the era of logic and geometry (Euclid) to the era of visuals and modern technology. Ackerberg-Hastings (2023) has analyzed and synthesized the elements of Euclid. Banionis (2021), in His research, has discussed the publication of Euclid's famous work, Elements, in Vilnius in 1807. This article also highlights that the publication of Elements in Vilnius enabled communities in the former Polish-Lithuanian Commonwealth to access ancient heritage. However, because it was translated and published later than in other European countries, Elements did not become the only geometry textbook of its time. Next, the research De Risi (2022) discusses the Fourth Postulate in Euclid's work, Elements, which states that all right angles are equal. This postulate has long been considered problematic in the deductive structure of the work, and even ancient interpreters were confused about its mathematical role and epistemological status. This research aims to analyze junior high school students' understanding of the definitions and postulates contained in Euclid's Elements Volume I. The level of understanding in this context refers to the degree to which junior high school students comprehend and apply the definitions and postulates found in Euclid's Elements Volume I. It categorizes students based on their ability to recall, explain, and use these mathematical concepts in problem-solving (Andamon & Tan, 2018; Gerace, 2001).

## Method

This qualitative research uses a descriptive approach using case study methods with Categorization Analysis (Content Analysis) (Fathoni, 2006; Wahidmurni, 2017). A case study is a study carried out by researchers conducting an in-depth study of a particular case, the conclusions of which are limited to that specific case.

This research used open interview instruments and three questions regarding the definition and postulates of Euclid's Elements regarding angles, triangles, and circles. These questions are valid based on an expert review by three mathematics education experts. These questions are directly used without a field test because the questions are only basic questions to know how far the students' knowledge and understanding are. The questions asked in the interview are as follows: The interview guidelines consist of six interview questions: 1) What do you know about points in geometry?; 2) Can a line be formed through two points? ; 3) What is meant by a line? ; 4) Can the line be extended infinitely?; 5) What is the meaning of a circle? ; and 6) What is a right angle? Furthermore, the three case study questions given are: 1) Do you know the formula for the circumference and area of a circle? If yes, write it down!; 2) Calculate the area and circumference of the circle if its diameter is 14 cm! ; 3) Look at the following picture (Figure 1), determine the size of the angle BAC, the perimeter, and the area of the shape (in cm)!

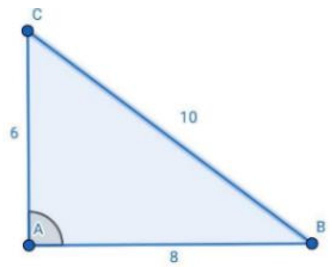



Figure 1. Case study questions

The level of students' understanding can be categorized into three levels based on their comprehension and application of definitions and postulates. High-level understanding is indicated by the ability to accurately recall and explain Euclidean concepts, apply postulates to solve complex problems, and justify answers logically. Middle-level understanding is marked by partial recall of definitions, basic application of postulates, and occasional reasoning errors that require guidance. Low-level understanding is characterized by difficulty recalling concepts, vague or incorrect explanations, struggles applying postulates even to simple problems, and misconceptions in logical reasoning. These indicators help assess students' comprehension and provide insights for improving geometry education (Desita, 2021).

The data used in this research were obtained from a case study conducted on eight junior high school students in Padang, Indonesia. The students selected were high, medium, and low-ability students. Students who are given interview questions have different answers when answering written test questions. Therefore, three students were interviewed. They are DDF, VY, LM, RAP, and MAC (Sitasari, 2022; Surata et al., 2020).

## Results and Discussion

After conducting research on eight junior high school students, various answers were found by conducting open interviews and asking several case study questions related to the definition. Postulates of Euclid's Elements in Volume I. Below are presented the results of interviews and answers from five out of eight students who were respondents who were given interview questions and the students who had different answers in answering written test questions.

The results of the interview with DDF show that his understanding of geometric concepts still needs to be clarified. DDF initially thought that a point in geometry meant a multiplication sign, but he understood that a line could be formed by connecting two points. The definition of a line given is still simple, namely two connected points, and DDF thinks that a line can be extended indefinitely if a long piece of paper is used. Regarding circles, DDF mentions several important elements such as radius, center point, diameter, and arc. However, when asked about right angles, DDF incorrectly refers to them as angles measuring 180°. This interview shows that although DDF has a basic understanding of some geometric concepts, misconceptions still need to be cleared up, especially in the definitions of points, lines, and angles.

Based on the results of the researcher's interview with DDF, it can be concluded that DDF does not yet understand the definitions and postulates of 1-4 of Euclid's Elements. This is because it has not been taught to junior high school students, but for right angles, DDF said that it had been studied at the elementary school level. However, he forgot how big it was.

Figure 2 shows the answers from DDF in solving three questions related to angles, perimeter, area of triangles, and circles.

<table>
<tr><th>Students's Writing</th><th>Translate</th></tr>
<tr><td>1. Tau, yaitu<br>Rumus keliling lingkaran : $2 \times \pi \times r$ , $\pi \times d$<br>Rumus luas lingkaran : $\pi \times r^2$<br><br>2. Luas lingkaran : $\pi \times r^2$<br>$= \frac{22}{7} \times 7 \times 7$<br>$= 22 \times 7$<br>$= 154 \ cm^2$<br><br>keliling lingkaran :<br>1. $2 \times \pi \times r$    2. $\pi \times d$<br>$2 \times \frac{22}{7} \times 7$    $\frac{22}{7} \times 14$<br>$2 \times 22$    $= 44 \ cm$<br>$44 \ cm$<br><br>3. a. Besar sudut siku-siku : $90^\circ$<br>b. keliling : $ab + bc + AC$<br>$: 8 + 10 + 6 = 24 \ cm$<br>c. luas : $\frac{1}{2} \times a \times t$<br>$: \frac{1}{2} \times 8 \times 6$<br>$: \frac{1}{2} \times 48 = 24 \ cm^2$</td><td>1. Yes,<br>Circle circumference formula<br>$= 2 \ x \ \pi \ x \ r, \ \pi \ x \ d$<br>Circle area formula = $\pi \ x \ r^2$<br>2. Circle area $= \pi \ x \ r^2$<br>$= \frac{22}{7} \ x \ 7 \ x \ 7$<br>$= 22 \ x \ 7$<br>$= 154 \ cm^2$<br>Circle circumference :<br>1. $2 \ x \ \pi \ x \ r = 2 \ x \ \frac{22}{7} \ x \ 7$<br>$= 2 \ x \ 22 = 44 \ cm$<br>2. $\pi \ x \ d = \frac{22}{7} x \ 14 = 44 \ cm$<br>3. a. Large right angle = 90º<br>b. circumference = ab + bc + AC<br>= 8+10+6<br>= 24 cm<br>c. Area $= \frac{1}{2} \ x \ a \ x \ t$<br>$= \frac{1}{2} x \ 8 \ x \ 6$<br>$= \frac{1}{2} x \ 48$<br>$= 24 \ cm^2$</td></tr>
</table>

Figure 2. DDF's answers

Based on Figure 2, it is known that DDF answered all questions. In the first question, DDF answered the formula for the circumference and area of a circle correctly, and DDF even wrote the procedure for the rim of a process in two different forms. In the second question, DDF solved the problem in a structured and systematic manner. DDF first wrote down the formula for the circumference and area of a circle, then substituted the known values; DDF solved the second question correctly and precisely. DDF solved the third question in a structured manner and answered all questions correctly and precisely.

Furthermore, the interview results with VY indicate that their understanding of geometric concepts is still limited and needs further refinement. VYperceives a point as an object but does not provide a more specific definition. They understand that a line can be formed through two points and define it as something straight, though their explanation lacks depth. When asked if a line can extend infinitely, VY believes it can, but only as far as the edge of a book. Their understanding of a circle is also basic, describing it as round. Additionally, there is a misconception about a right angle, as VY incorrectly states that it measures 100°. Overall, the interview suggests that while VY has some foundational knowledge, further guidance is needed, particularly regarding lines, circles, and angles.

In Figure 3, VY's answers are shown when solving three questions about angles, perimeter, and area of triangles and circles.

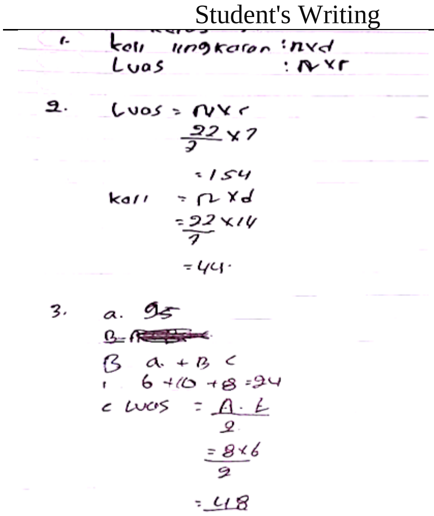
| Student's Writing | Translate |
|---|---|
|  | 1. Circle circumference $= \pi\ x\ d$<br>Area = $\pi\ x\ r$<br>2. Area $= \pi \text{ x } r = \frac{22}{7}\ x\ 7$<br>$= 154$<br>Circle circumference $= \pi\ x\ d$<br>$= \frac{22}{7} x\ 14$<br>$= 44$<br>3. a. 95<br>b. a + b + c = 6 + 10 + 8 = 24<br>c. Area $= \frac{A\ x\ t}{2} = \frac{8\ x\ 6}{2} = 48$ |

Figure 3. VY's answer

Based on Figure 3, it can be seen that VY answered all the questions given. In the first question, VY answered the formula for the circumference of a circle correctly but incorrectly answered the recipe for the area of a circle; VY wrote the procedure for the circle's location as $\pi$r, which should be $\pi r^2$. In the second question, VY answered correctly the circumference of a process but did not write down the units. VY was wrong in writing the formula for the area of a circle, but the results he obtained were correct; this shows that VY was not careful in carrying out multiplication and division operations. In the third question, VY correctly answered the perimeter of the triangle, but VY incorrectly answered the angle BAC and the area of the triangle.

Furthermore, the interview with LM shows a partial understanding of geometric concepts with some misconceptions. LM correctly identifies a point as the smallest part with no specific size and understands that a line can be formed through two points. However, their definition of a line as "a point that has no ending" is unclear, and they are unsure whether a line can extend infinitely. Similarly, LM describes a circle as "a point that has no ending," which suggests confusion between the concepts of a point, a line, and a circle. They correctly state that a right angle measures 90°, though their wording implies some misunderstanding. Overall, LM has a basic grasp of geometry but requires further clarification, especially regarding the distinctions between points, lines, and circles. Based on the researcher's interview with LM, it can be concluded that LM knows some of the definitions and postulates 1-4 contained in Euclid's Elements. Still, LM admits that he did not study this material at the junior high school level.

In Figure 4, LM's answers are presented in solving three questions related to angles, perimeter, and area of triangles and circles.

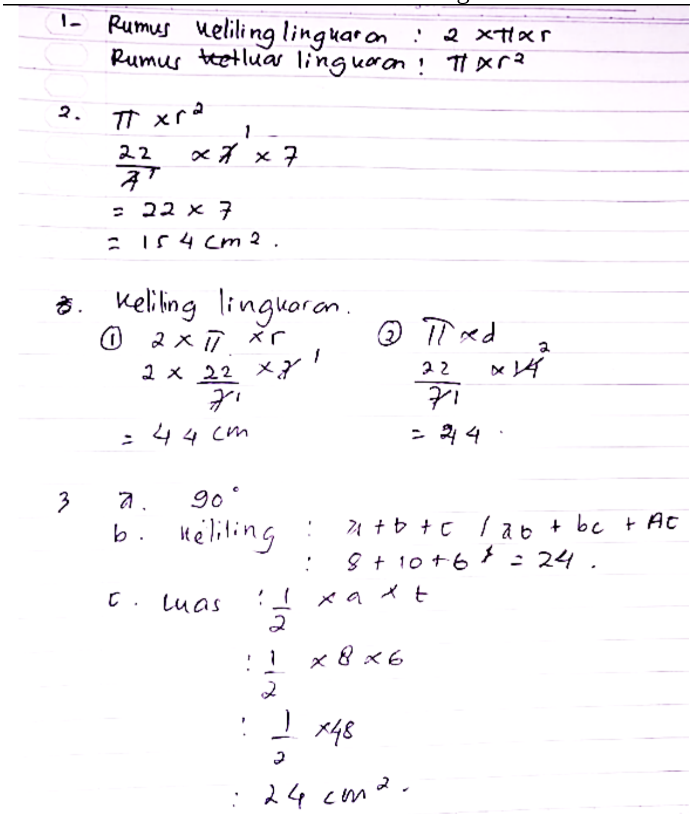
| Student's Writing | Translate |
|---|---|
| <br>1- Rumus keliling lingkaran : 2 x π x r<br>Rumus ~~ke~~luas lingkaran : π x r²<br>2. π x r²<br>22/7 x 7 x 7<br>= 22 x 7<br>= 154 cm².<br>3. Keliling lingkaran.<br>(1) 2 x π x r (2) π x d<br>2 x 22/7 x 7 22/7 x 14<br>= 44 cm = 44.<br>3 a. 90°<br>b. keliling : a + b + c / ab + bc + Ac<br>: 8 + 10 + 6 = 24.<br>c. Luas : 1/2 x a x t<br>: 1/2 x 8 x 6<br>: 1/2 x48<br>: 24 cm². | 1. Circle circumference formula $= 2\ x\ \pi\ x$ r<br>Circle area formula = $\pi\ x$ r$^2$<br>2. $\pi$ x r$^2$ $= \frac{22}{7}\ x\ 7\ x\ 7$<br>$= 22\ x\ 7$<br>$= 154\ cm^2$<br>Circle circumference :<br>1. $2\ x\ \pi\ x$ r $= 2$ x $\frac{22}{7}\ x\ 7$<br>$= 44\ cm$<br>2. $\pi\ x\ d = \frac{22}{7} x\ 14 = 44$<br>3. a. 90°<br>b. Circumference = a + b + c / ab + bc + Ac<br>= 8+10+6<br>= 24<br>c. Area $= \frac{1}{2}\ x\ a\ x\ t$<br>$= \frac{1}{2} x\ 8\ x\ 6$<br>$= \frac{1}{2} x\ 48$<br>$= 24\ cm^2$ |

Figure 4. LM's answer

Based on Figure 4, it is known that LM answered all questions. In the first question, LM correctly replied to the formula for the circumference and area of a circle. In the second question, LM did not write the location of the process equal to ..., but LM immediately wrote the formula and got the correct answer. LM also answered correctly for the circumference of the circle. LM solved the third question in a structured manner and answered all questions correctly and precisely.

The following are presented the results of the researcher's interview with RAP:

| | | |
|---|---|---|
| *Researcher* | : | *What do you know about points in geometry?* |
| *RAP* | : | *I think a point is an object* |
| *Researcher* | : | *Do you think a line can be formed through two points?* |
| *RAP* | : | Yes, I do |
| *Researcher* | : | *So what is meant by that line?* |
| *RAP* | : | *A line is an elongated image.* |
| *Researcher* | : | *Can the line be extended infinitely?* |
| *RAP* | : | *Yes, if you draw continuously until the length of the paper* |
| *Researcher* | : | *What do you think is the meaning of a circle?* |
| *RAP* | : | *A circle is round, has no corners, and has one side* |
| *Researcher* | : | *Do you know what a right angle is?* |
| *RAP* | : | *An angle whose measure is 50°* |
| *Researcher* | : | *OK, please do these three questions on a piece of paper* |
| *RAP* | : | *OK* |

Based on the results of the researcher's interview with RAP, it can be concluded that RAP does not know at all the definitions and postulates 1-4 contained in Euclid's Elements, and RAP admits that he has not studied this material at the junior high school level. In Figure 5, the answers from RAP are presented in solving three questions related to angles, perimeter, and area of triangles and circles.

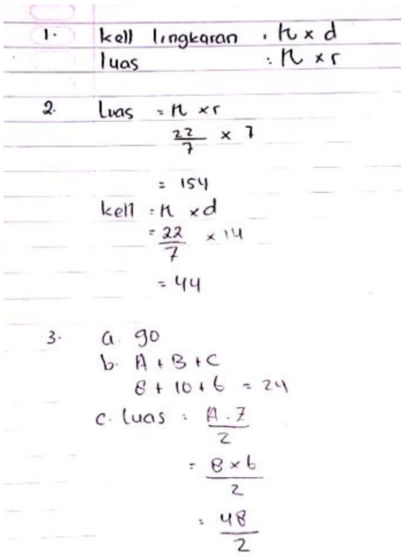
| Student's Writing | Translate |
|---|---|
| <br> | 1. Circle circumference $= \pi\ x$ d<br>Area $= \pi\ x\ r$<br>2. Area = $\pi$ x r $= \frac{22}{7}\ x\ 7$<br>$= 154$<br>Circumference $= \pi\ x\ d$<br>$= \frac{22}{7}\ x\ 14$<br>$= 44$<br>3. a. 90<br>b. A + B + C<br>8+10+6 = 24<br>c. Area $= \frac{A\ x\ t}{2}$<br>$= \frac{8\ x\ 6}{2}$<br>$= \frac{48}{2}$ |

Figure 5. RAP's answers

Based on Figure 5, it is known that RAP answers all the questions given. In the first question, RAP answered the formula for the circumference of a circle correctly but incorrectly answered the recipe for the area of a circle; RAP wrote the procedure for the size of a circle as $\pi$r, which should be $\pi r^2$. In the second question, RAP answered the circumference of a circle correctly but did not write down the units. RAP was wrong in writing the formula for the area of a process, but the results he obtained were correct; this shows that RAP was not careful in carrying out multiplication and division operations. In the third question, RAP answered correctly the size of the angle BAC but did not write the degrees. RAP answered the circumference of the triangle correctly, but RAP did not write the units (cm); for the area of the triangle, RAP answered correctly, but for writing the formula, RAP wrote $\frac{A\,x\,t}{2}$, it should be $\frac{a\,.t}{2}$.

Furthermore, the interview with MAC shows a basic understanding of geometry but includes some misconceptions. MAC describes a point as a small circle, which suggests confusion between a point and a physical dot. They correctly state that a line can be formed through two points and define it as an image connecting one point to another, though their explanation lacks mathematical precision. They believe a line can extend infinitely because it can be drawn continuously. Their understanding of a circle is vague, mentioning that it is round and "has fingers," likely referring to its radius. However, a clear misconception about a right angle exists, as MAC incorrectly states that it measures 80°. While MAC grasps some fundamental geometric concepts, further clarification is needed, particularly regarding points, circles, and angles. Based on the researcher's interview with MAC, it can be concluded that MAC does not know the definitions and postulates 1-4 contained in Euclid's Elements. Figure 6 shows MAC's answers to three questions related to angles, perimeter, area of triangles, and circles.

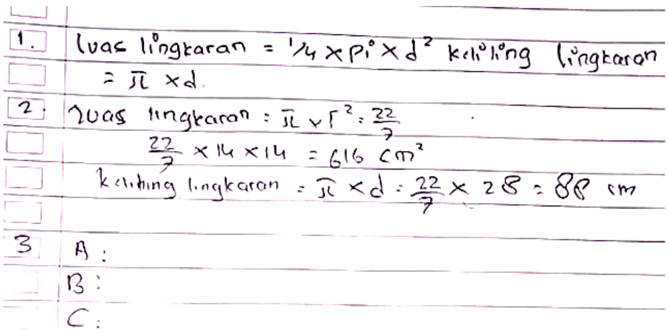
| Student's Writing | Translate |
|---|---|
|  | 1. Circle area $= \frac{1}{4}\, x\, \pi\, x\, d^2$<br>Circle circumference $= \pi\, x\, d$<br>2. Circle area $= \pi\, x\, r^2$<br>$= \frac{22}{7}$ x 14 x 14 = 616 cm$^2$<br>Circle circumference<br>$= \pi$ x d $= \frac{22}{7}$ x 28<br>= 88 cm<br>3. A :<br>B :<br>C : |

Figure 6. MAC answers

Based on Figure 6, it can be seen that MAC only answered two questions; the third question was not answered. In the first question, MAC correctly answered the formula for the area and circumference of a circle. What's unique is that MAC wrote the recipe for the location of a process not in the general form, namely $\frac{1}{4}\ x\ phi\ x\ d^2$. In the second question, MAC incorrectly answered the area and circumference of a circle; it seems MAC thought 14 cm was the radius, even though it was clearly stated that the diameter was 14 cm, meaning MAC was not careful in reading the question. On the third question, MAC did not answer any questions.

Based on variations in answers from eight students as respondents, three categories of students' understanding of the definitions and postulates of The Element Volume I were obtained regarding angles, circles, and triangles. The categories can be seen in Table 1.

Table 1. Categories of middle school students' level of understanding in solving questions related to the definition and postulates of Euclid's Elements Volume I

| No. | Category | Information |
|---|---|---|
| 1 | High-Level Comprehension and Knowledge | Students who know the formulas of Euclid's Elements and can solve problems correctly |
| 2 | Middle-Level Comprehension and Knowledge | Students who know the formula of Euclid's Elements and are unable to solve the problem correctly |
| 3 | Low-Level Comprehension and Knowledge | Students who don't know the formula of Euclid's Elements and are unable to solve the problem correctly |

Based on the result, the level of understanding of junior high school students regarding the definitions and postulates of Euclid's Elements Volume I can be categorized into three categories, namely: high-level comprehension and knowledge, middle-level comprehension and knowledge, and low-level comprehension and knowledge.

This research provides insights into students' levels of understanding in geometry, categorizing them into low, middle, and high comprehension levels. This classification aligns with previous research in mathematics education, which has identified similar hierarchical structures in mathematical reasoning (Rohati et al., 2022), conceptual understanding (Gunawan et al., 2019), and problem-solving behavior (Muir et al., 2008). However, beyond categorization, it is essential to analyze why students fall into these different levels and how instructional strategies can bridge the gaps in their understanding.

One of the key factors influencing students' varying levels of comprehension is constructivist learning theory (Vygotsky, 2011), which emphasizes that knowledge is actively constructed through experiences and interactions. Students with high comprehension levels likely benefit from prior knowledge and scaffolded learning experiences that allow them to make connections between definitions, postulates, and their applications. Meanwhile, students at the middle and low levels may struggle due to insufficient scaffolding, a lack of conceptual reinforcement, or difficulties in translating abstract mathematical principles into concrete understanding (Vollmeyer et al., 1996) These findings align with (Mcleod, 2023) Cognitive Load Theory, which suggests that students with low comprehension levels may experience excessive cognitive load, making it difficult for them to process and retain information effectively.

## Conclusion

The most well-known mathematical work from the ancient era is Euclid's Elements, the oldest mathematics textbook still used today. This book contains 13 volumes containing 131 definitions, five postulates, five general notes, and 465 propositions. The work of earlier Greek mathematicians like Pythagoras (and his school), Hippocrates of Chios, Theaetetus of Athens, and Eudoxus of Cnidos produced the majority of the theorems that appear in The Elements. Apart from being a development of the strict field of geometry, The Element also explains parts of extensive algebra problems along with addition theory. Research that has been conducted shows that the majority of junior high school students do not know the definitions and postulates contained in Euclid's Elements Volume I; this is because the material on definitions and postulates of Euclid's Elements has not been taught at the junior high school level. There are three categories of students in solving problems regarding the purpose and postulates of Euclid's Elements Volume

I regarding angles, triangles, and circles, namely High-Level Comprehension and Knowledge, Middle-Level Comprehension and Knowledge, and Low-Level Comprehension and Knowledge.

The findings of this research highlight a gap in the mathematical curriculum at the junior high school level, where fundamental definitions and postulates from Euclid's Elements are not explicitly taught. As a result, students struggle with understanding key geometric concepts related to angles, triangles, and circles. This suggests the need for curriculum development that integrates Euclidean geometry more effectively at an earlier stage. Teachers and educators could incorporate more structured lessons on Euclid's postulates and definitions, ensuring a stronger foundation in geometry. Additionally, differentiated teaching strategies can be implemented to address students' varying levels of comprehension, such as interactive visual aids for low-level learners and advanced problem-solving tasks for high-level learners. From a theoretical perspective, the research reinforces the significance of Euclid's Elements as a foundational text in mathematics education. Classifying students into different comprehension levels suggests that cognitive development in geometric reasoning varies widely among learners, supporting existing educational theories on mathematical cognition. Furthermore, the findings emphasize the importance of explicit instruction in axiomatic systems, demonstrating how the absence of formal exposure to Euclid's postulates can lead to fragmented understanding. This study also opens avenues for further research on how historical mathematical texts can be effectively integrated into modern teaching methodologies to enhance students' conceptual understanding and logical reasoning skills.

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