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How High School Students Solve Proportional Reasoning Problem?

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Abstract. *This research is motivated by the phenomenon of students' difficulties in solving proportional reasoning problems and the lack of proportional reasoning research at the high school level. This research aims to describe in depth the various strategies high school students use in solving proportional reasoning problems. The method used is qualitative with a case study approach. The research subjects involved were 34 students from grades 10 to 12. The instruments used were tests and interview guides. The research results show that the strategy used by high school students in solving proportional reasoning problems is the factor of change strategy as a form of using the multiplication strategy, and the addition strategy is the dominant strategy used by students. Other findings indicate that students experience difficulties in solving proportional reasoning problems, such as (1) difficulty distinguishing the use of addition strategies from multiplication strategies, (2) difficulty solving problems related to the concept of inverse proportion, and (3) difficulty comparing the 2 ratios. The research findings can be used as a basis for further research designing learning flows and/or learning designs to develop students' proportional reasoning abilities.*

Keywords: *proportion, proportional reasoning, ratio*

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Introduction

Proportional reasoning abilities are critical for students. This ability is essential in understanding mathematical concepts and solving mathematical problems (Lundberg, 2022; Lutfi et al., 2022). Viewed from its constituent terms, proportional reasoning is explicitly used in understanding and solving problems related to the concept of ratio and proportion. Besides that, it is also useful for other concepts in mathematics, such as linear equations, probability, trigonometry, and geometry, as well as other concepts outside mathematics, such as physics and geography (Akatugba & Wallace, 1999; Dole et al., 2012; Lamon, 2020). Therefore, proportional reasoning is the key to understanding high-level mathematical concepts (Casler-Failing, 2018). Proportional reasoning bridges basic mathematical concepts with higher-level mathematical concepts. The ability to reason about proportions is also used in various contexts in the real world, such as scaling objects, determining the best choice when shopping, exchanging money, unit conversions, and travel planning (Im & Jitendra, 2020; Small, 2015). Proportional reasoning provides students with the underlying skills of making relationships between quantities, solving problems efficiently, and applying mathematical concepts to real-life situations.

In mathematics education research, proportional reasoning has become a topic studied by many researchers. Lutfi et al. (2023) explored various research that focused on mathematics learning environment interventions in developing students' proportional reasoning abilities and found that in the 2018-2023 time period, there were 44 studies related to this focus. Apart from that, there are also various other studies (Lutfi et al., 2022; Wijayanti & Winsløw, 2022), focusing on the analysis of books used by teachers and students in studying the concepts of ratio and proportion and other research (Cabero-Fayos et al., 2020; Díaz & Aravena, 2021; Jitendra et al., 2022; Weiland et al., 2019, 2021), studying the proportional reasoning abilities of teachers and prospective teachers. These various types of research attempt to overcome students' difficulty developing this ability, as seen from the perspective of teachers and teaching materials.

From the aspect of the students themselves, several studies have directly examined these difficulties, and it indicates that students experience difficulties in solving proportional reasoning problems. The research results of Yuliani et al. (2021) showed that students' difficulties in solving the proportional reasoning problems given lead to the conclusion that the proportional reasoning abilities of junior high school students are relatively low and are seen as underdeveloped among students in the schools studied. Husain et al. (2021) reported that the low mastery of proportional reasoning skills was due to students' lack of familiarity with proportional reasoning problems. This shows that students are not used to being presented with problems that require proportional reasoning, as the results of research by Mardika & Mahmudi (2021) reported that the difficulties experienced by students were due to the limited types of problems presented by teachers in learning. The limited presentation of various problems in the learning process impacts the limited experience students gain in solving proportional reasoning problems. Furthermore, this causes student failure and mistakes. Some of the students' mistakes that occurred were: (1) using a cross-multiplication strategy on problems in the form of non-proportional problems; (2) using the addition strategy in proportional problems; and (3) difficulty in distinguishing direct and indirect proportion problems (Ayan & Isiksal-Bostan, 2019; Gündoğdu & Tunç, 2022; Karli & Yildiz, 2022).

The various studies on how students solve the proportional reasoning problems mostly take junior high school students as research subjects. This is corroborated by search results from Lutfi et al. (2023), which reported that there was 55% proportional reasoning research at the junior high school level and only 7% proportional reasoning research with high school students as research subjects from 2018 to 2023. This research shows that the high school level received the least attention from researchers. This can be understood because mastery of proportional reasoning skills is the focus of its development at junior high school age (Kahraman et al., 2019). However, by examining the strategies of high school students in solving proportional reasoning problems, information will be obtained regarding indications regarding the accumulation of students' learning experiences towards developing their proportional reasoning abilities.

Based on search results using the keywords "proportional reasoning" and "high school," no research has been found that aimed to examine high school students' strategies in solving proportional reasoning problems. This research is important because it can provide an overview of how students' accumulated learning experiences from middle to high school lead to the development of proportional reasoning abilities. In addition, this is a reference for improving the quality of learning in high school, as is the role of this ability in understanding higher mathematics topics. Therefore, this research focuses on efforts to describe the various strategies high school students use in solving proportional reasoning problems. To achieve this goal, the research question is: What strategies do high school students use to solve proportional reasoning problems?

Method

This research uses a qualitative method with a case study approach. The research stages carried out were: (1) reviewing literature regarding proportional reasoning abilities, (2) developing research supporting instruments, namely tests and interview guidelines, (3) carrying out tests and interviews with students, (4) data analysis, and (5) reporting research results. The research subjects involved at the start of the research consisted of 34 students from different grades, namely grades 10, 11, and 12, at a private high school in Bandung Regency, Indonesia. Previous researchers have researched subjects with different grades. This is based on the need to achieve research objectives.

The differences in the research objectives to be achieved cause differences in the characteristics of the research subjects required, as seen in the following research. Previous research involved subjects of different grades because the research aimed to describe differences in students' proportional reasoning abilities at each grade level (Jiang et al., 2016; Martínez-Juste et al., 2023; Ojose, 2015). Especially in research by Jiang et al. (2016), different grade students were used because this research also attempted to describe the developmental trajectory of students' abilities from additive reasoning to proportional reasoning. Another research used research subjects with different grades, especially with different ages (Lundberg, 2022; Vanluydt et al., 2019). This research subject is used in these studies because it aims to describe how students who have never learned about proportionality handle tasks related to proportional reasoning based on age.

Different from previous research, which involved research subjects with different grades to describe proportional reasoning abilities at each grade, research by I et al. (2018) used it to obtain more general conclusions. This research involved students in Grade 6 to 9 as research subjects because this research aimed to explore students' misconceptions at the middle school level in solving proportion problems. There are similarities in the research subjects used, namely that the research subjects involved have gained learning experience regarding ratios and their application. In line with I et al. (2018) research, this research aims to describe in depth the various strategies high school students use in solving proportional reasoning problems. Therefore, in this study, the research subjects involved came from different grades at the high school level because the research subjects had both studied the concepts of ratio and proportion at the middle school level.

From all these students, five students were selected based on the characteristics of the strategies used in solving the test items. These 5 students were involved in this research from the beginning to the end. Based on experience in class (such as activeness in learning and mathematics test scores in class), the researcher, who is also the mathematics teacher for these 5 students, believes these students are classified as having good mathematical abilities. The role of the researcher, who is also the mathematics teacher of the research subject, has created an emotional closeness between the subject and the researcher. This is very necessary in qualitative research, especially in the interview process. Students can explain their ideas and opinions openly to obtain in-depth information.

For research purposes, the identities of the five students are coded as S1, S2, S3, S4, and S5. The following are the characteristics of these five students. S1 is a grade 10^{th} student with good mathematical skills and easily understands the lessons but tends to be quiet and passive in learning. In solving proportional reasoning problems, S1 tends to still use additive relationship strategies, one of which is using repeated subtraction operations to solve problems involving multiplicative relationships. S2 is an 11^{th}-grade student with good mathematical skills and is active in learning but less careful in solving problems. S2 could differentiate between using multiplicative and additive relationships in solving proportional reasoning problems, but S2 had difficulty solving inverse proportion problems. S3 is a grade 12^{th} student who has relatively low mathematical abilities. S3 often experiences difficulties understanding the material and solving problems, but S3 has a good fighting spirit, dares to try, and often asks questions. In solving proportional reasoning problems, S3 used an intuitive strategy for the problem of comparing values in quantitative form, but S3 succeeded in solving problems in the form of qualitative comparisons. S4 is a grade 12^{th} student who has good mathematics skills. S4 can understand the material but tends to be passive in learning and less thorough in solving problems. S4 could solve proportional reasoning problems with a strategy involving multiplicative relationships, but S4 could not differentiate the use of these relationships, so S4 used multiplicative relationships in non-proportional problems. S5 is a grade 12^{th} student who has good math skills, is active in learning, and can often solve problems in class faster than other students. In solving proportional reasoning problems, S5 could differentiate multiplicative relationships from additive relationships, but S5 used a different strategy in solving numerical comparisons; S5 answered that the ratio was equal.

Data was collected using a test, interviews, observation, and documentation. Data collection was done through the test aiming to obtain information related to the strategies used by students in solving proportional reasoning problems. The test consisted of five problems grouped into four categories: missing value problems (MVP), numerical comparison problems (NCP), non-proportional problems (NPP), and qualitative reasoning problems (QRP). The problems used were adapted from research by Gündoğdu & Tunç (2022), called the Proportional Reasoning Test. Table 1 presents an explanation of the proportional reasoning problem used.

Table 1. Types of problems used in test support instruments

Types of Problems	**Aim**	**Completion strategy**	**Number of Problem**
Missing value problem (MVP)	To find out the strategies used by students in solving proportion problems	• Cross-multiplication • Factor of change • Scaling-up and scaling-down • Unit rate • Addition strategy	1,2
Numerical comparison problem (NCP)	To find out the strategies used by students in comparing ratio	• Factor of change • Unit rate • Addition strategy	4
Non-proportional problem (NPP)	To determine students' ability to differentiate the context of proportion and non-proportion problems	• Addition strategy so that the answer is correct • Multiplication strategy so that the answer is wrong	3
Qualitative reasoning problem (QRP)	To determine students' reasoning abilities, which can be expressed verbally, visually, and numerically	• Construct multiplicative comparisons so that the answer is correct • Unable to build multiplicative comparisons, so the answer is wrong	5

From the four proportional reasoning problems presented in Table 1, test items were then prepared to represent each type of problem, each represented by one problem. For problems of the missing value type, there is an exception by being represented by 2 problem s in the form of direct proportion and indirect proportion. For more details, the following test items have been compiled in Figure 1.

The research supporting instrument were test items, which students then work on, analyze, and explore using in-depth interview techniques. Interviews with students aim to explore students' thinking processes in solving proportional reasoning problems. Interviews were also conducted to confirm the suitability of the various meanings made by researchers in students' answers. Additionally, observation was also used when students did the test. This technique is used to record student responses when working on test items. Another technique used to obtain research data was the documentation technique. This technique obtains data in documents, videos, theoretical reference sources, and other research sources (Yin, 2016). After the data was obtained, the data was analyzed using data triangulation techniques. The data triangulation technique combines various data to analyze data from the same source (Creswell, 2014).

Instruction:

Answer the questions below independently and each question can be done in more than one way!

1. Ridwan and Sandi ride their motorbikes at the same speed. If Ridwan has driven 4 km in 6 minutes, how many times has Sandi driven 20 km?
2. A house reconstruction job is estimated to take 30 days if done by two workers. If the owner wants the work to be completed within 10 days, how many workers should be involved in the work? (Assumed that the workers' abilities are considered the same)
3. When the mother is 35 years old, the child is 10 years old. How old will the daughter be if the mother is 70 years old?
4. Two large jugs are provided, namely Jug A and Jug B, for making orange juice with different mixtures. Jug A is filled with 2 glasses of orange syrup and 3 liters of water. Jug B is filled with 3 glasses of orange syrup with 4 liters of water. Which jug has sweeter orange juice?
5. Every morning Rendi's father always drinks sweet tea. This morning Rendi's father drank sweet tea in a bigger glass and with less sugar than yesterday. How does this morning's sweet tea taste compare to yesterday?

Figure 1. The test items

Results and Discussion

In this section, various findings obtained during the research will be discussed. To obtain data regarding students' strategies in solving proportional reasoning problems, supporting instruments were a test consisting of five problems, as presented in Figure 1, and in-depth interviews. In explaining the strategies students use on each problem, the identities of the 5 students will be represented by codes S1, S2, S3, S4, and S5. The following is a presentation of various research findings based on the order of problems students worked on.

Student strategies in answering Problem 1

In Problem 1, an MVP-type problem is presented. Students must find the fourth unknown quantity based on the three known quantities. This problem represents a proportional reasoning problem related to direct proportion. The context used is regarding the relationship between distance and time. The farther the distance traveled, the longer it takes. In the solution process carried out by students, most students use the strategy of looking for factors of change. Students look for factors to obtain the quantity 20 km from the known value of 4 km. This factor is also called covariance (Lamon, 2020). The following is one of the answers of students who use a strategy by looking for covariance values.

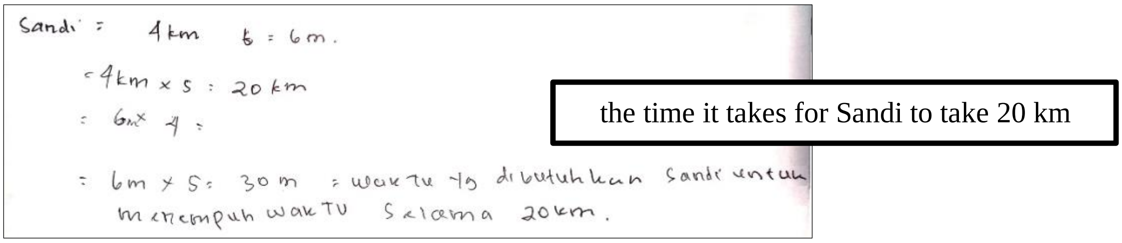


Figure 2. S2's answer to Problem 1

From the answers in Figure 2, students found the covariance values from 4 km to 20 km was 5. S4 and S5 also used the same method. The following is a part of the interview with S2.

Researcher	:	*How do you solve this problem?*
S2	:	*So initially, it was 4 (km) multiplied by how much it took to get 20 (km), which means 4 times 5. This means that for the time, 6 minutes multiplied by 5 equals 30 minutes*

Based on the interview results, students could see the multiplicative relationship between the values 4 and 20. The ability to see this relationship is the key to mastering proportional reasoning skills. However, a student answers this problem with a subtraction strategy to obtain these factors, as shown in Figure 3.

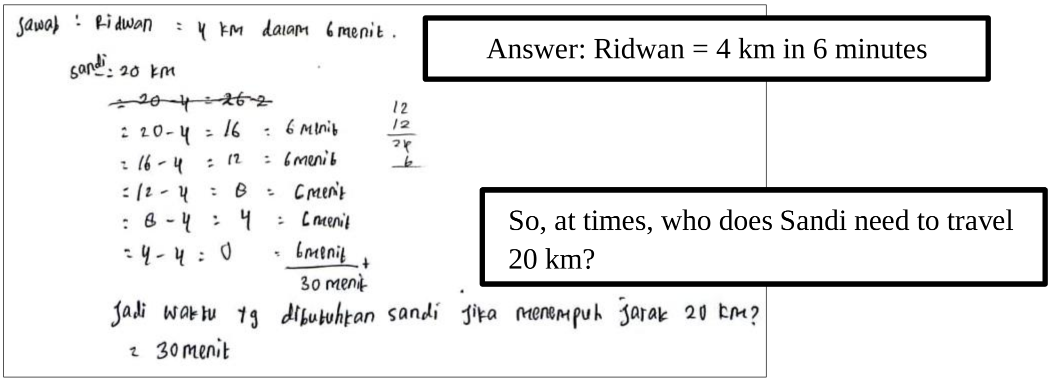


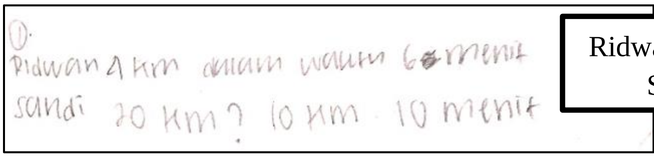
Figure 3. S1's answer to Problem 1

Figure 3 shows that students subtract 4 repeatedly five times. Based on the interview results, it was concluded that students build multiplicative relationships through subtraction strategies, which are closely related to addition strategies. Students repeated subtraction on the distance part and added the time on the right side. Students have not been able to find the value 5 as a covariance value. Below is a part of the interview with the student.

Researcher	:	*Now, how do you solve problem 1?*
S1	:	*eee the way is…*
Researcher	:	*How?*
S1	:	*20 minus 4 equals 16, which is 6 minutes. That's the first step. 16 minus 4 equals 12, which is the remainder. Yes, sir, and so on until 0. So, the result is 30 minutes.*

From this student's work, it can be seen that students still think rather than think relatively. Students still view changes in quantity in an addition relationship rather than in a multiplication relationship. This absolute thinking process can support the development of students' proportional reasoning abilities, but if left unchecked, there is a risk of causing mistakes in students (Lamon, 2020).

In addition, some students failed to solve this problem correctly. This happened in S3 with a solution that involved calculations without being based on good arguments. S3 answered with the connection that 10 km takes 10 minutes. The following is S3's answer to Problem 1.



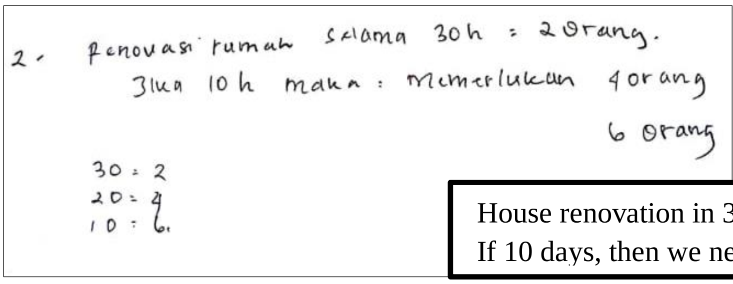
Ridwan : 4 km in 6 minutes
Sandi : 20 km? 10 km 10 minutes

Figure 4. S3's answer to Problem 1

Students who do not know the correct solution strategy often use strategies like this. They attempt to involve various operations and statements that do not explain the logical reasons for using the strategy.

Student strategies in answering Problem 2

Problem 2 presents an MVP problem where the relationship between quantities is not equivalent. This problem represents a proportional reasoning problem on the concept of indirect proportion. In the context of this problem, the less processing time, the more workers must be involved. From the students' answers, most students realized that the change in quantity reversed direction. However, most students have difficulty getting the right answer. Even though the answer obtained is correct, students use an addition strategy to obtain the answer.



House renovation in 30 days = 2 workers
If 10 days, then we need 4 workers

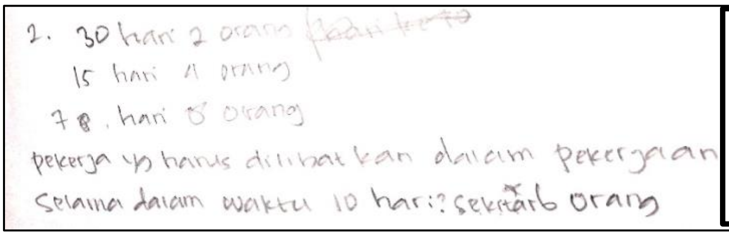
Figure 5. S2's answer to Problem 2

The student's answer to this problem is correct. For a house renovation that takes 10 days, the total number of workers needed is 6 people. Unfortunately, the process for obtaining these answers is not precise. Below is a part of the interview with the student.

Researcher	:	*How do you get to solve this problem?*
S2	:	*So if 30 days need 2 workers, then 20 days need 4 workers*
Researcher	:	*How do you get 4?*
S2	:	*multiplication of 2*
Researcher	:	*How do you get this multiplication? From this? So if 2 times 2 is 4, but 4 times 2 is…*
S2	:	*it means 4 add by 2*
Researcher	:	*So add it. And then how about 30 become 20?*
S2	:	*It is subtraction by 10*

From the interview, it is known that the student reduced the number of days by 10 and increased the number of workers by 2. The student also could not explain why he believed this step was correct, as in S1 and S5’s answers. This answer indicates that students cannot construct a solution to this problem and still rely on addition strategies.

Another answer to this problem appears in the work process by S4 and S3. In answering this problem, students can use multiplicative relationships in reverse values. However, students have difficulty getting 10 days worth, as seen in Figure 6.



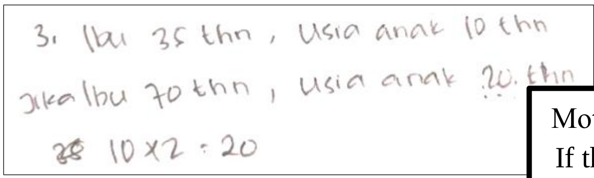
30 days 2 people
15 days 4 people
7 days 8 people
Workers who must be involved in work for 10 days? about 6 people

Figure 6. S4's answer to Problem 2

Students can find the relationship if the work is completed in 15 days, then 4 workers are needed. However, students began to have difficulties when continuing the relationship. Students mistakenly stated that the relationship should be 7.5 days, equal to 8 people; students wrote 7 days, equal to 8 people. As a result, the student answered that if the work had to be completed within 10 days, the workers needed would be "around 6 people". From this answer, it can be seen that the value changes made by students only refer to multiples of 2. If students can change the value by other factors, such as 3, students will get a solution based on correct calculations.

Student strategies in answering Problem 3

Problem 3 presents an NPP-type problem, which aims to see students' ability to differentiate between proportional and non-proportional situations. The context used in this problem is regarding the age difference between the mother and her child. Although the change in the mother's age from 35 to 70 years can be interpreted as doubling, this meaning does not apply to changes in the child's age. Some students were able to solve this problem correctly. However, some students answer incorrectly. Following are the answers of students who did not answer correctly.



Mother’s age 35 years, child’s age 10 years
If the mother's age is 70 years, the child's

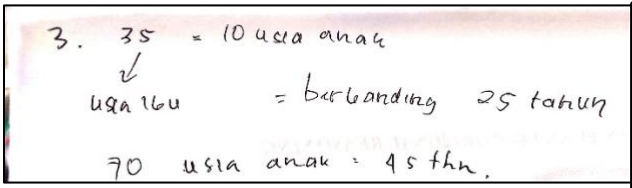
Figure 7. S4's answer to Problem 3

The answers in Figure 7 show that students view changes in quantity in the problem with a proportional relationship. Based on the interview results, students view the change in the mother's age to 70 as twice the change from 35. Therefore, the child's age is multiplied by 2 to 20 years. Below is a part of the interview with S4.

Researcher	:	*How is it done?*
S4	:	*35 multiplied by 2 equals 70 years. The child's age is 10 times 2, too, so the child is 20 years old.*

Students do not recheck regarding the accuracy and logic of the answers obtained. Therefore, it is suspected that students cannot differentiate between proportion and non-proportion problems. This results in students being unable to differentiate between contexts that use addition and multiplication strategies.

Students can answer correctly for other students' answers. Below is one of the answers of a student (S2) who solved the problem correctly.



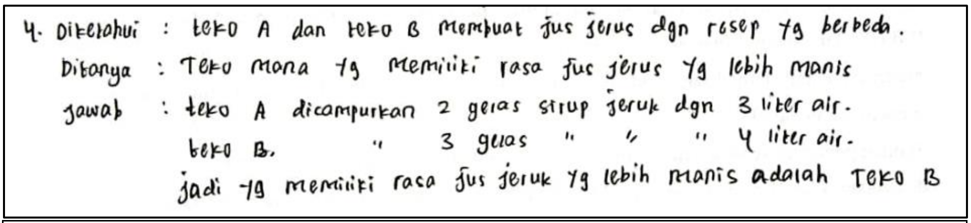
35 years = 10 years
(Mother’s age) (child’s age)
= 25 years difference
70 (years) = 45 years
(Mother’s age) (child’s age)

Figure 8. S2's answer to Problem 3

Students solve this problem by finding the difference between the ages of the mother and child from the information provided, which is 25 years. Because this age difference is fixed, when the mother reaches 70, the child's age will be $70 - 25 = 45$.

Student strategies in answering Problem 4

Problem 4 presents an NCP-type problem, which aims to see students' ability to compare two or more available ratios. In this problem, students must compare which jug has the sweeter taste of orange juice. Most students answered that jug B tasted sweeter than jug A. This answer was based on more syrup volume in jug B than in jug A. However, students could not explain the larger water volume's effect on the sweet taste of the orange syrup in the jug B. The following is the student's answer to the problem.


4. Diketahui : teko A dan teko B membuat jus jeruk dgn resep yg berbeda.
Ditanya : Teko mana yg memiliki rasa jus jeruk yg lebih manis
Jawab : teko A dicampurkan 2 gelas sirup jeruk dgn 3 liter air.
teko B. " 3 gelas " " " 4 liter air.
jadi yg memiliki rasa jus jeruk yg lebih manis adalah Teko B

Known: Jug A and Jug B for making orange juice with different mixtures
Question: Which jug has sweeter orange juice?
Answer: Jug A is filled with 2 glasses of orange syrup and 3 liters of water
Jug B is filled with 3 glasses of orange syrup with 4 liters of water
So the one that has a sweeter orange juice taste is Jug B

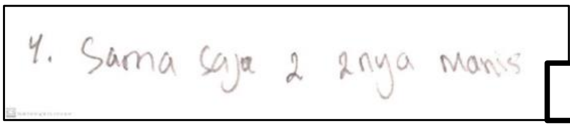
Figure 9. S1's answer to Problem 4

The students' answers in Figure 9 show that the students tried to compare the composition of the syrup and water in each teapot. Even though the student's answer was correct, based on interviews, the student could not explain the effect of water volume differences on sweetness. The following is part of an interview with students.

Researcher : *Why did you choose Jug B?*
S1 : *Because it contains more orange*
Researcher : *It is correct, but it also contains more water. How about that?*
S1 : *Hmm… My feeling tells me that the orange juice has the same sweetness, but because the problem asked which jug was sweeter, the answer was jug B.*

After being interviewed, the student revealed that the student's answer was the same sweetness, but because the problem required choosing one, the student chose jug B because it had more syrup. This shows that students do not believe the answers given. This lack of confidence is caused by students' inability to adequately provide arguments (reasoning) based on the concept of ratio.

In contrast to other students' answers, S5 said the taste of orange juice from both jugs was the same for a different reason. Here is S5's answer to this problem.


4. Sama saja 2 2nya manis

Both are equally sweet.

Figure 10. S5's answer to Problem 4

From these answers, an interview was then conducted with S5. The interview results showed that students answered that the sweetness of the two jugs was the same because the difference in the value of the ratio of syrup and water in each jug was the same; that is, they both had a difference of 1. The following is part of an interview with students.

Researcher	:	*What is the answer?*
S5	:	*Both are equally sweet.*
Researcher	:	*Why do both jugs taste the same?*
S5	:	*Because it's compared to 1*
Researcher	:	*What is compared to 1?*
S5	:	*(in jug A) this is 2 (glass of syrup) and 3 (liter of water). (in jug B) this is 3 (glasses of syrup), and this is 4 (liters of water), so* both are equally sweet

From the interview excerpt, it can be seen that students saw the difference in the value of syrup and water in each jug. This reason is different from S1, which saw the similarity of additions with a value of 1 to both syrup and water problems, encouraging students to be able to build two ratios that are formed and compare them. Therefore, students' answers to this problem indicate that students do not use strategies to compare the 2 ratio. In addition, it is indicated that students are still using the addition strategy to solve this problem. This indication was seen when students answered that the taste of orange juice was the same because the quantity of syrup and water increased by 1 or had a difference of 1.

Student strategies in answering Problem 5

Problem 5 presents a QRP-type problem, which aims to see students' ability to reason about proportions without involving numbers. In this problem, students are asked to determine the taste of sweet tea this morning compared to yesterday if the glass used this morning was larger and had less sugar. Students can answer this problem correctly. All students answered that the taste of sweet tea this morning was not as sweet as yesterday. Students can compare the taste of sweet tea even without involving numbers. The following is one student's answer to Problem 5.

5. rasa tg kemarin cenderung lebih manis dari pada teh manis yg sekarang

Yesterday's sweet tea tasted sweeter than today's sweet tea

Figure 11. S2’s answers to Problem 5

This student initially had difficulty understanding the problem because the student thought that the measurements for sweet tea yesterday morning were unknown. However, after understanding more, the students used yesterday morning's sweet tea as a benchmark in determining the level of sweetness of sweet tea.

From this answer, it can be seen that students have relatively no difficulty solving qualitative comparison problems. Students can solve this problem by determining a qualitative measure as a reference. In this problem, one measure used as a reference is that the quantity of sugar used today is less than yesterday. From this reference, students can conclude that the quantity of sugar yesterday was greater than that of sugar today. Students can solve this problem by making consequence statements related to the information in the problem. Studying students' strategies in solving proportional reasoning problems is an effort to deepen the proportional reasoning process and understand the concept of proportion. In connection with the process of students building knowledge, Harel introduced the idea of a triadic model: Mental Act of Thinking (WoT)-Ways of Understanding (WoU) (Suryadi, 2019). The efforts made by students to solve the problems they face are initially in the form of mental action as a response to obtain a solution to the problem. However, the mental actions that appear at the beginning are random and not single (Suryadi, 2019). The mental actions are then arranged to obtain a continuous flow of thought (WoT). Efforts to compile WoT will not be separated from the involvement of specific contexts, such as concepts, principles, and facts previously formed by students (WoU).

Based on the presentation of the research results, the strategies used by high school students in solving proportional reasoning problems consist of addition strategies and multiplication strategies, namely looking for factors of change. This strategy tends to be the same as junior high school students (Ayan & Isiksal-Bostan, 2019; Karli & Yildiz, 2022; Öztürk et al., 2021). The use of addition strategies is more dominant than multiplication strategies. The addition strategy is a strategy that looks at changes in values that occur by looking at the differences between values (Ayan & Isiksal-Bostan, 2019). As a result, students are more likely to involve addition or subtraction operations in solving problems than multiplication operations. In fact, in proportional reasoning, building multiplicative relationships is the main indicator of this ability (Lamon, 2020).

The addition strategy can be an alternative for students to build multiplicative strategies to solve proportional reasoning problems. As seen in Figure 3, students attempt to build multiplicative relationships by repeatedly carrying out subtraction (or addition). However, using this strategy will also result in errors in answers and strategy. One of them is shown in Figure 9. Students see that the change in each value in the NCP problem is that they both increase by 1, so students answer that the two teapots taste the same as sweet.

The dominance of the use of addition strategies was also found in research (Karli & Yildiz, 2022; Mardika & Mahmudi, 2021; Yuliani et al., 2021) which found that junior high school students used addition relationships in problems that required multiplication relationships. In this study, students had difficulty distinguishing the use of these two relationships or strategies. Students also experience difficulty in building multiplicative relationships. These findings indicate that the high school students involved in this research could not release the absolute thinking process (involving the addition strategy) to the relative thinking process (involving the multiplication strategy). This will impact students' difficulties in understanding material that involves proportional reasoning skills in high school. On the other hand, the dominance of using the addition strategy impacts students' success in solving NPP problems. Students did not experience significant difficulties in solving these problems. Even if students are asked to differentiate problems involving additive relationships from multiplicative relationships, students may not necessarily be able to differentiate between them.

Another research finding is the difficulty students have in building strategies for MVP problems related to the concept of inverse proportion. The same findings were found in research (Ayan & Isiksal-Bostan, 2019; Mardika & Mahmudi, 2021; Yuliani et al., 2021), where only a small percentage of students could solve this problem correctly. In this study, most students could answer correctly but with inappropriate strategies. Students use an addition strategy to solve this problem. From the characteristics of the problem and the strategies used by students, there are at least 2 reasons why this difficulty occurs. First, students lack flexibility in changing the form of the equation. As seen in Figure 6, students only try to change the equation with a factor of 2. Students who try to use other factors will easily get the correct results. Second, students cannot find the constant value of the proportion. Lamon (2020) explains that this is the total value formed from the ratio relationship. In the context of Problem 2 in this research, the value of the proportion constant is the total workload that must be completed. This value is obtained by multiplying the length of work time by the total number of workers involved.

In addition, another finding in this research is students' difficulties in building strategies for NCP problems. This failure was caused by students having difficulty building 2 ratios and comparing the two ratios. This indicates that students have limitations regarding the concept of ratio (Pebrianti et al., 2023). The absence of students who can develop appropriate strategies indicates that this type of problem is the most challenging problem for students.

In general, the strategies high school students use in solving proportion problems in proportion material tend to be the same as those used by middle school students. Such factors as the factor of change strategy as a manifestation of the multiplication strategy and the dominance of the addition strategy in solving proportional reasoning problems also occur in junior high school students. Likewise, with the difficulties experienced by students, both middle and high school students tend to experience the same difficulties in solving proportional reasoning problems. The difficulties referred to include difficulties in building multiplicative relationships and differentiating their use from additive relationships or strategies, difficulties in solving problems related to inverse proportion, and difficulties in comparing 2 ratios. Therefore, ongoing efforts are needed to develop students' proportional reasoning abilities. By providing a variety of proportional reasoning problems that are oriented towards students' ability to build multiplicative relationships, especially at the junior high school level, it is hoped that it can release students from the pattern of solving using the addition strategy, which is often used to solve various problems at the elementary school level. This is important because if a solution pattern using a multiplicative strategy is not formed in students, students will experience difficulties understanding higher mathematical concepts (Casler-Failing, 2018).

Conclusion

Based on the explanation, high school students can solve proportional reasoning problems. All students answered various problems based on the results of their reasoning without involving writing ratio notation and cross-multiplication strategies in constructing their answers. This shows that reasoning ability is the key to students' thinking processes. With reasoning skills, students can try to construct answers even if they forget the concepts used. In solving MVP-type problems, the correct strategy used by students is the factor of change strategy, and the incorrect strategy uses the addition strategy. In solving NCP-type problems, no students used the correct strategy. Students use the wrong strategy, namely by using the addition strategy. In NPP problems, most students use the correct strategy, namely the addition strategy. In this problem, it was also found that students used multiplication (relationship) strategies. In QCP problems, all students were able to answer the problems correctly. Students can compare quantities qualitatively.

The addition strategy is the dominant strategy used by students. This indicates that students have difficulty distinguishing addition strategies from multiplication strategies. Another difficulty experienced by students is solving problems related to inverse proportion and comparing 2 ratios. The results of this research can be used as a basis for further research in the form of research that designs learning flows and/or learning designs that can develop students' proportional reasoning abilities. It is hoped that this research can encourage teachers, researchers, and relevant parties to design proportional reasoning problem contexts that are varied and sequential according to students' abilities to facilitate the development of a variety of strategies developed by students.

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