ISSN 2355-4185(p), 2548-8546(e)
DOI: https://doi.org/10.24815/jdm.v11i1.36729

Kotobee-Assisted Means-End Analysis Learning: Impact on Algebraic Reasoning Abilities Reviewed From Students' Cognitive Stage

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Received: 9 January 2024; Revised: 30 March 2024; Accepted: 27 April 2024

Abstract. *The Indonesian curriculum emphasizes tailored learning based on students' developmental stages and achievement levels, focusing on cognitive development, particularly in algebraic reasoning, which is very important for students' overall cognitive maturation. This study aimed to determine the effect of the implementation of the learning model and students' cognitive stage on algebraic reasoning ability. The study was conducted using quantitative research with a post-test design to achieve the research objective. Two classes were taken as samples from one of the high schools in West Bandung Regency. The instruments used were the Test of Logical Thinking (TOLT) to categorize students into students' cognitive stage groups and algebraic reasoning test instruments to determine the achievement of algebraic reasoning ability. Data analysis was done using statistical tests, including a normality test, homogeneity test, and two-way ANOVA test. The results are: there is an effect of implementing the Means-End Analysis (MEA) learning model supported by Kotobee on algebraic reasoning; there is an effect of students' cognitive stage on algebraic reasoning; and there is no interaction effect of students' cognitive stage and implementation learning model on algebraic reasoning ability. Further research on other mathematical skills using the Kotobee-based MEA learning model is needed.*

Keywords*: means-end analysis learning, software kotobee, algebraic reasoning ability*

How to cite: Fitrianna, A. Y., Yuliani, A., Farida, A., & Rasid, N. S. M. (2024). Kotobee-assisted means-end analysis learning: impact on algebraic reasoning abilities reviewed from students' cognitive stage. *Jurnal Didaktik Matematika*, *11*(1), 74-87. DOI: https://doi.org/10.24815/jdm.v11i1.36729

Introduction

The current curriculum in Indonesia is based on the principle that learning is designed with attention to the stage of development and the current level of achievement of students through learning needs and reflects the diversity of characteristics and development. This approach ensures learning becomes meaningful and enjoyable (Kemdikbudristek, 2021). Concerning the student's stage of development, the stage of cognitive development is also a factor that needs to be considered by the teacher. Inhelder and Piaget (1958) classified the development of children's cognitive structures into four main stages: the sensory-motor operational stage, the pre-concrete operational stage, the concrete operational stage, and the formal operational stage. Learning designed to meet the characteristics of the independent curriculum can pay attention to students' cognitive development.

The capacity to reason logically and abstractly concerning fundamental mathematical concepts profoundly impacts students' cognitive development. Tobin and Capie (1981) emphasize a robust correlation between students' reasoning abilities, particularly in the context of algebra, and their cognitive maturation. Algebra, the foundation of mathematical thinking beyond basic arithmetic, involves students generalizing ideas, establishing justifications through argument, and expressing these understandings in increasingly formal ways appropriate for their age (Blanton & Kaput, 2005). Kieran (2004) defines algebraic reasoning as the ability to think by developing mathematical thinking through symbol and operation definitions. Carpenter and Levi (2000) define algebraic reasoning as making generalizations and using symbols to describe mathematical concepts and solve problems. Key aspects of algebraic reasoning include representation, generalization, and justification (Kieran, 2004; Lepak, Wernet, & Ayieko, 2018).

Students' difficulties in algebraic reasoning can be observed in several contexts. First, students' comprehension of the concepts of equality and variables affects their performance in problem-solving tasks (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). Second, students struggle to transition from arithmetic to algebra (Herutomo & Saputro, 2014). Third, students cannot recognize the pattern of a mathematical problem due to difficulties in obtaining the necessary information (Cahyaningtyas, Novita, & Toto, 2018). This research highlights the necessity of developing algebraic reasoning abilities in students through the learning process.

The Means-End Analysis (MEA) learning model, supported by Kotobee software, is highly compatible with the core element of the current curriculum in Indonesia. As Kemdikbudristek (2021) emphasized, such a curriculum necessitates pertinent and contextual learning and engages parents and the community. MEA exemplifies this vision by commencing with everyday life problems and encouraging students to actively construct their understanding through teacher-guided problem-solving tasks. Research by Fitrianna and Dahlan (2022) indicates that MEA effectively contributes to the development of algebraic reasoning abilities due to its multifaced approach to addressing this ability. Based on Suherman (2008), the definition of MEA can be seen as a specific type of problem-solving learning with three main stages: The first step is to decompose the problem into smaller sub-problems. The second step is to identify how the sub-problems are related. The third step is to choose the appropriate solution strategy. As outlined by Umar (2016), the implementation of MEA has four key steps: (1) presenting material through a problem-solving approach informed by heuristics, (2) simplifying the material into sub-problems, (3) organizing these sub-problems in an interconnected manner, and (4) choosing an appropriate settlement strategy. The MEA learning model has been identified as having limitations in the literature, prompting research into the development of learning models such as SEA-MEA to enhance problem-solving skills (Setiawan, Wasis, & Jatmiko, 2020). These references highlight the acknowledgment of the weakness of the MEA model and the ongoing efforts to address this limitation through the development of learning models and the application of MEA in different contexts.

Kotobee software is a software program designed to develop interactive teaching materials. Kotobee software is one of the interactive e-book authoring software programs developed by Vijua and can be utilized in education. This software program enables the insertion of video, audio, and quizzes that students can access directly through smartphones. There are two versions of Kotobee software: Kotobee Author for writers and Kotobee Reader for readers (users). This software can be accessed on a laptop or Android smartphone. In addition to offering several advantages that complement learning models, Kotobee software supports problem-based learning (PBL). This is achieved by integrating PBL into the software's learning modeling software design, emphasizing problems as the starting point of student learning, collaboration systems, and student-directed learning (Bahar, Wibawa, & Situmorang, 2019). Furthermore, Kotobee can create a supportive learning environment and an engaging learning process, actively involving students in the learning models (Rozi & Panjaitan, 2020). The findings of Fitrianna and Dahlan (2022) research indicated that Kotobee software could be a solution as a learning medium, thereby encouraging students to become more active during the learning process. One advantage of this software is that it can be used to insert video, audio, images, and quizzes into a module or teaching material, which students can access directly. The results of this research also indicate that learning media with Kotobee software can be used to enhance the students' learning experience.

Several studies related to algebraic reasoning (Akmal, Fitriyanto, & Sumaatmaja, 2020; Kramarski, 2008; Lee, Capraro, Capraro, & Bicer, 2018) demonstrated that algebraic reasoning is challenging for students. Other research (Authary & Nazariah, 2019; Damayanti, Purwanto, Parta, & Chandra, 2019) has indicated that several indicators of algebraic reasoning have not been achieved optimally in terms of the SOLO Taxonomy, students' cognitive styles, and learning barriers in understanding certain mathematical content. This existing research suggests that MEA's effectiveness in enhancing student learning is evident. Studies by Rizal and Mubarika (2023) demonstrate its positive impact on mathematical connections, while Aisyah (2023) and Permana (2023) report improved learning outcomes in grade VII mathematics. The use of Kotobee affects students' mathematics achievement in class VIII, as indicated by the significant difference between pretest and post-test scores (Otico, Pasion, & Dones, 2023). Studies related to cognitive stages examine the impact of inductive and deductive approaches on reasoning abilities, focusing on cognitive stages (Maesaroh, Sumarmo, & Hidayat, 2020). Another line of inquiry examines the relationship between mathematical understanding and reasoning abilities, particularly emphasizing students' cognitive stages (Rohaeti, Putra, & Primandhika, 2019). Romsih, Yuhana, and Nindiasari (2019) also explored the impact of the problem-posing approach on mathematical reasoning abilities in terms of students' cognitive development stages.

A review of the description and literature indicates a need to investigate the impact of implementing learning with the Means-End Analysis (MEA) learning model, assisted by Kotobee software, on algebraic reasoning from the cognitive stage. The objective of the research is to determine the impact of implementing the MEA learning model assisted by Kotobee on algebraic reasoning, the influence of students' cognitive stage on algebraic reasoning, and the interaction effect of students' cognitive stage and implementation of the learning model on algebraic reasoning ability.

Method

The research method employed is a quasi-experimental design with a posttest-only control group. The posttest-only control group was selected because this study will compare the effect of the experimental group, who received learning with the MEA learning model assisted by Kotobee, with the control group, who received learning with the usual learning. Thus, the dependent variable in this study is algebraic reasoning ability, and the independent variable is the learning model. The students ' cognitive stage is another independent variable influencing the dependent variable. According to Campbell and Stanley (2012), this study employs a quasi-experimental design due to practical, ethical, and methodological considerations regarding sample selection.

The population of this study consisted of all students in Grade 11 in West Bandung Regency. The sample was selected using a cluster random sampling technique, and two classes from one of the high schools in West Bandung were selected as samples. The samples consisted of 68 students, divided into two groups: 33 students in the experimental class, who were taught using the Kotobee-assisted MEA learning model, and 35 students in the control class, who were taught using the usual methods employed by the teacher. This research was conducted over six meetings, one dedicated to the algebraic reasoning test and another to the cognitive stage categorization test.

The data were collected by administering an algebraic reasoning test for algebraic reasoning ability, which included seven questions on inverse functions. These questions were developed based on indicators of competency achievement in composition and inverse functions, as well as indicators of algebraic reasoning ability. The Algebraic reasoning questions were prepared based on indicators of algebraic reasoning adapted from Lepak et al. (2018) and validated by mathematics teachers. The aspects of ARA that are measured include the following:

1) Representation: This aspect encompasses the ability of students to identify relevant information, relate the information, and model relationships through algebraic representation; students are expected to be able to present the given problem in the form of an algebraic representation, which may include coordinating graphs and equations that connect the context with the information given; students can provide initial representations or add to existing representations by carrying out calculation procedures in algebraic or arithmetic forms and changing the form of expressions or equations into other equivalent forms. 2) Generalization encompasses students' ability to make conclusions from the existing context and transform specific forms into general forms. 3) Justification, which includes the capacity of students to provide explanations or justifications.

The determination of student categories at the cognitive stage is based on Inhelder and Piaget's theory of cognitive development. The student's cognitive stage test was derived from an instrument developed by Tobin and Capie (1981) called the logical thinking test (TOLT), which has been translated into Indonesian and validated based on research (Tobin & Capie, 1981). The TOLT was selected for cognitive stage classification because it measures five reasoning abilities that are indicators of logical thinking: control variable reasoning, proportional reasoning, probabilistic reasoning, correlational reasoning, and combinatorial reasoning. The TOLT comprises 10 test items in the form of multiple-choice questions accompanied by reasons for selecting answers. Guidelines for scoring and classification of cognitive stages are described in Tables 1 and 2.

Table 1. TOLT scoring guidelines

Question number	Criteria	Skor
1-8	The answers and reasons are correct	1
	Answers and reasons are wrong or otherwise	0
9 dan 10	The combination and number of correct combinations	1
	The combination is repeated, and the answer contains all the correct combinations	1
	The number of correct answers was 24, and the students answered the number of combinations 25, and 24 combinations were correct.	1
	There is one wrong combination	1

(Source: Sumarmo, 1987)

Table 2. Cognitive stage classification

Test score acquisition	Cognitive Stage Classification
0 – 3	Concrete
4 – 5	Transition
6 – 10	Formal

(Source: Sumarmo, 1987)

The grouping of students based on their cognitive stage is based on scoring guidelines. The post-test scores obtained from the algebraic reasoning ability test were analyzed using the normality test, homogeneity test, and two-way ANOVA test with the help of SPSS software.

Results and Discussion

Prior to the commencement of the learning process, students were grouped according to their cognitive stage. This was achieved through the administration of the TOLT, which enabled the identification of the extent to which students had developed logical thinking skills, particularly in the context of algebra. Following the conclusion of the learning process, the students were given the post-test. This post-test was designed to assess the extent to which students had developed algebraic reasoning skills due to their participation in the learning process. Consequently, the integration of TOLT before learning and post-test after learning offers a comprehensive representation of students' cognitive development in terms of algebraic reasoning ability.

Table 3. Algebraic reasoning test results of experimental and control class students

Class	Lowest score	Highest score	Mean	Median	Var	Std
Experiment	18	96	60.91	62	461.460	21.482
Control	5	81	58.09	52	446.793	21.137

Table 3 reveals a discrepancy between the average and highest scores of the experimental and control classes. The elevated average score in the experimental class suggests that the treatment provided to the experimental group yields superior outcomes in developing algebraic reasoning abilities. Moreover, the highest score attained by students in the experimental class indicates the potential for attaining higher algebraic reasoning abilities through the MEA Learning Model, which is facilitated by Kotobee and implemented in the group.

Table 4. Description of observed data on the cognitive stage of experimental and control class students

Learning_model	Cognitive_stage	Mean	Std. Deviation	N
Experiment	Formal	83.60	15.126	5
	Transition	65.00	12.987	22
	Concrete	27.00	9.675	6
	Total	60.91	21.482	33
Control	Formal	59.00	21.781	6
	Transition	61.26	12.096	19
	Concrete	26.10	13.337	10
	Total	50.83	21.137	35
Total	Formal	70.18	22.221	11
	Transition	63.27	12.568	41
	Concrete	26.44	11.753	16
	Total	55.72	21.746	68

Table 4 indicates that the average student score in the formal category in the experimental class is greater than the transition stage. In contrast, the average student score in the formal category in the control class is smaller than the transition stage. The results of the tests above were then subjected to statistical analysis to determine the impact of the implementation of the MEA learning model assisted by Kotobee on algebraic reasoning ability, the effect of students' cognitive stage on algebraic reasoning ability, and the effect of the interaction of the implementation of learning model and students cognitive stage on algebraic reasoning ability.

The results of the algebraic reasoning test, which are based on data from each learning model and cognitive stage group, are obtained. Subsequently, data analysis is conducted to test the normality of the residual data using the Shapiro-Wilk test with the assistance of SPSS. The hypotheses of the normality test are as follows.

H_0 : The sample was collected from a population that is normally distributed

H_1 : The sample was collected from a population that is not normally distributed

Table 5. Normality test results

No	Group	Sig.	Test Decision
1	Experiment	0,330	H_0 accepted
2	Control	0,170	H_0 accepted
3	Formal	0,202	H_0 accepted
4	Transition	0,175	H_0 accepted
5	Concrete	0,062	H_0 accepted

The results of the Shapiro-Wilk test indicated that all null hypotheses (H_0) test decisions were accepted. This implies that the data collected from each group was drawn from a normally distributed population. Furthermore, the homogeneity test was carried out using the Levene test with the help of SPSS. The results of the Levene test indicated that the significance value was 0.741, which signifies that the population exhibited a similar variance. Consequently, the assumption of homogeneity of variance was validated.

The two-way ANOVA test was then carried out using SPSS by the previously established assumptions of normality and homogeneity. The objective was to evaluate the effects of implementation learning models and students' cognitive stage on algebraic reasoning ability.

Table 6. Tests of between-subjects effects
Dependent Variable: Skor

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	20151.907[a]	5	4030.381	21.669	.000	.636
Intercept	141724.798	1	141724.798	761.976	.000	.925
Learning_model	1168.677	1	1168.677	6.283	.015	.092
Cognitive_stage	17633.485	2	8816.743	47.403	.000	.605
Learning_model * Cognitive_stage	1088.920	2	544.460	2.927	.061	.086
Error	11531.784	62	185.997			
Total	242809.000	68				
Corrected Total	31683.691	67				

R Squared = .636 (Adjusted R Squared = .607)

Table 6 indicates a significant main effect on the implementation of the learning model, with a statistical value of $F(1, 62) = 6.283$ and $p = 0.015$. Additionally, the partial $\eta 2$ value of 0.092 suggests that this variable exerts a considerable influence in the context of this study. Moreover, the results demonstrated the impact of implementing the Kotobee-assisted MEA learning model on algebraic reasoning ability. This finding indicated a notable discrepancy in the algebraic reasoning ability test outcomes between the two classes that received disparate treatments. The average algebraic reasoning ability of the experimental class, which utilized Kotobee-assisted MEA learning, was 60.91, while the average for the control class, which received regular learning, was 58.09. These findings indicate that Kotobee-assisted MEA learning has a significant positive impact on students' algebraic reasoning ability.

The results demonstrated a highly significant main effect on the student’s cognitive stage, with a statistical value of $F(2, 62) = 47.403$ and $p = 0.000$. This indicates that the student’s cognitive stage variable considerably influences algebraic reasoning ability, as evidenced by the partial η2 value of 0.605. Moreover, the research findings strongly correlate students' cognitive stage and algebraic reasoning ability. The emergence of three distinct cognitive stages, concrete, transitional, and formal, suggests that each stage uniquely influences algebraic reasoning ability. To gain further insight, additional tests were conducted to elucidate the specific impact of each student's cognitive stage on algebraic reasoning ability.

Table 7. Multiple comparisons
Dependent Variable: Skor
Tukey HSD

(I) Cognitive_stage	(J) Cognitive_stage	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Formal	Transition	6.91	4.631	.301	-4.21	18.03
	Concrete	43.74*	5.342	.000	30.92	56.57
Transition	Formal	-6.91	4.631	.301	-18.03	4.21
	Concrete	36.83*	4.020	.000	27.18	46.48
Concrete	Formal	-43.74*	5.342	.000	-56.57	-30.92
	Transition	-36.83*	4.020	.000	-46.48	-27.18

Based on observed means.
The error term is Mean Square(Error) = 185.997.
*. The mean difference is significant at the 05 level.

A Post hoc analysis employing the Tukey HSD method revealed no statistically significant difference in algebraic reasoning ability between students at the formal and transitional stages ($p=0.301$). Nevertheless, the test results indicated significant differences between students at the concrete stage and students at the transitional stage ($p = 0.000$) and formal stage ($p = 0.000$) regarding algebraic reasoning ability. This suggests that the cognitive stage experienced by students can influence their algebraic reasoning ability, particularly during the transition from the concrete stage to the transitional and formal stages.

However, the analysis demonstrated no significant interaction between the implementation of the learning model and students' cognitive stage, with a statistical value of $F(2, 62) = 2.927$ and $p = 0.061$. The partial η2 value of 0.086 indicates that this variable exerts a minor influence on the interaction between the learning model and category students' cognitive stage on students' algebraic reasoning ability. This finding corroborates the conclusion that no significant interaction effect exists between the learning model and students' cognitive stage on algebraic reasoning ability. In other words, the combination of the implemented learning model and students' cognitive stage cannot significantly enhance students' algebraic reasoning ability in this study.

The study's results confirmed the positive impact of implementing the learning model on algebraic reasoning ability. This research underscores the importance of incorporating diverse teaching methodologies and tools in the educational context, contributing to the variation of student learning outcomes. A comparative analysis of the algebraic reasoning ability test results between the experimental and control classes revealed a statistically significant difference. The experimental class, which applied the MEA learning model supported by Kotobee software, achieved a higher average score than the control class (Otico et al., 2023). This indicates that integrating technology and diverse learning approaches can enhance students' reasoning abilities.

The efficacy of the MEA learning model in enhancing algebraic reasoning ability is contingent upon its intrinsic characteristics that stimulate students to cultivate a spectrum of cognitive abilities. These encompass the capacity to engage in critical thinking, reflection, logical reasoning, systematic problem-solving, and the generation of creative solutions. This finding is supported by a series of previous studies demonstrating that the MEA learning model, particularly when utilized in conjunction with Kotobee software, has a significant positive impact on the development of students' reasoning abilities (Hidayat, Surya, Sukarno, & Poerwanti, 2020; Otico et al., 2023). Moreover, other studies' evidence confirms that interactive learning modules created using Kotobee software can effectively improve students' algebraic reasoning ability (Fitrianna & Dahlan, 2022). Therefore, combining the MEA learning model and Kotobee software provides a holistic learning approach, facilitating the development of higher-order thinking skills essential for students' academic and professional progress.

The consistency of findings from various studies supports the argument that the MEA learning model is an effective approach to improving algebraic reasoning ability. These studies also highlight the crucial role of educational technology, such as Kotobee software, in improving the quality of learning and student achievement (Otico et al., 2023). This learning approach, which emphasizes developing critical thinking and problem-solving skills, contributes significantly to more effective learning outcomes when compared to conventional teaching methods. Consequently, integrating technology into the learning process not only introduces variety to teaching methods but also facilitates substantial advancement in students' academic achievement.

The results of this study provide valuable insights into the impact of implementing a learning model on algebraic reasoning ability and the influence of students’ cognitive stage on algebraic reasoning ability. The study demonstrates a significant relationship between cognitive stages and algebraic reasoning ability, underscoring the necessity of employing a learning approach that aligns with students' characteristics and cognitive needs. The findings indicate statistically significant differences in mean algebraic reasoning ability scores across different cognitive stages, suggesting variations in mathematical reasoning proficiency among students at different stages of development. Specifically, students at the formal operational stage exhibit superior algebraic reasoning ability compared to students at the concrete and transitional stages, per the theory of cognitive development proposed by Inhelder and Piaget (1958).

The research conducted by Gunawan, Prawoto, and Sumarmo (2019) and Romdon, Puspowati, and Sumarmo (2018) provides substantial evidence in support of the relationship between the cognitive stage and more technical mathematical skills (hard skills). This further underscores the urgency of considering students' cognitive readiness when designing learning strategies. In contrast, a study conducted by Fuchs (2012) made an important contribution to understanding cognitive resources and their direct impact on understanding pre-algebra concepts. This study emphasized the important role of nonverbal reasoning and spoken language in mathematical reasoning. This suggests that an in-depth understanding of the linkages between cognitive developmental stages and specific aspects of mathematical ability is critical in designing effective learning approaches.

The interaction between the implementation of the learning model and students' cognitive stages in influencing algebraic reasoning ability is a complex issue. The statistical analysis revealed no significant interaction effect between the implementation of the learning model and students' cognitive stage on algebraic reasoning ability, challenging the assumption that the combined implementation of the learning model and students' cognitive stage considerations would synergistically improve students' algebraic reasoning ability. This finding is consistent with Maesaroh et al. (2020), who also found no interaction between teaching approaches and cognitive stages on students' reasoning ability. Furthermore, the study revealed that the average algebraic reasoning ability of transition-stage students in the control class was superior to that of students at the formal and concrete stages. This indicates the significance of considering individual differences in cognitive readiness and designing learning interventions that effectively support students' mathematical reasoning skills at different stages of development. The absence of a significant interaction effect between the implementation of the learning model and students' cognitive stage on algebraic reasoning ability indicates that these factors may independently contribute to students' mathematical reasoning skills but do not necessarily complement each other in a combined learning approach. This underscores the need for more targeted and effective learning strategies tailored to students' cognitive developmental needs. Additionally, the study identified symbol sense and basic skills as crucial elements. Symbol sense encompasses strategic work with a global view and an emphasis on algebraic reasoning. At the same time, basic skills involve procedural work with a local focus and an emphasis on algebraic calculations.

Furthermore, the study's findings underscore the significance of recognizing individual differences in cognitive readiness and designing learning interventions that effectively support students' mathematical reasoning skills at various stages of development. This aligns with the notion that the readiness of the workplace to provide opportunities for individuals to engage in work activities and access direct and indirect support is a pivotal determinant of the quality of learning in workplaces (Kop, Janssen, Drijvers, & van Driel, 2021). In conclusion, the interaction between implementing the learning model and students' cognitive stage in influencing algebraic reasoning ability is multifaceted. The absence of a significant interaction effect suggests the need for tailored learning strategies that consider individual differences in cognitive readiness. This underscores the importance of understanding the diverse factors that influence algebraic reasoning ability and developing interventions that effectively support students' mathematical reasoning skills at different stages of cognitive development.

Conclusion

This research aims to ascertain the impact of learning models and students' cognitive stage on algebraic reasoning ability. The findings of this study indicate that: 1) implementation of the learning model exerts an influence on algebraic reasoning ability, with the MEA learning model assisted by Kotobee resulting in higher algebraic reasoning test scores than the algebraic reasoning ability test scores for students receiving conventional learning. 2) students' cognitive stage exerts an influence on algebraic reasoning ability. Students at the formal and transition stages demonstrate superior algebraic reasoning abilities, as evidenced by their test results, compared to students at the concrete stage. 3) There was no interaction or effect between the implementation of the learning model and students' cognitive stage when used to develop algebraic reasoning ability. Both factors exerted distinct influences on algebraic reasoning ability. Based on these findings, mathematics educators are encouraged to prioritize factors that can impact mathematical learning outcomes or the mathematical abilities of other students. The factors considered include the use of the MEA learning model assisted by Kotobee and the category of students' cognitive stages, which allows for the creation of learning designs that align with the characteristics of students.

Acknowledgment

The author would like to thank IKIP Siliwangi and the Kartika Jaya Foundation for providing support in the form of funding for this research through the IKIP Siliwangi Internal Lecturer Research Grant.

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