

Students' Learning Obstacles on Sequences and Series Viewed by Pirie Kieren's Theory

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Abstract. *Mathematical understanding develops through eight layers, as described by Pirie and Kieren: primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventizing. However, evidence indicates that many students encounter obstacles that hinder their progression through these layers. This study aims to identify and describe the learning obstacles students face in understanding sequences and series, utilizing Pirie and Kieren's theoretical framework. A descriptive qualitative research design was employed, purposive sampling to select participants from 30 Year 11 students in Malang, Indonesia. Data were gathered through tests and interviews and analyzed based on indicators of learning obstacles and the corresponding layers of mathematical understanding outlined by Pirie and Kieren. The findings reveal that many students experience significant difficulties in noticing and formalizing layers within the property. These challenges are attributed to inadequate foundational knowledge (ontogenic conceptual obstacles) and a lack of structured opportunities for developing deeper mathematical understanding (ontogenic instrumental, didactical, and epistemological obstacles). The results underscore the need for further research to address these learning barriers by focusing on enhancing students' foundational knowledge and designing educational experiences that foster the growth of mathematical understanding.*

Keywords: *learning obstacles, mathematical understanding, Pirie-Kieren, sequences and series*

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Introduction

Mathematical understanding refers to the ability to organize and articulate the relationships between ideas, facts, and procedures within mathematics (Hiebert et al., 1997). It encompasses not only the comprehension of mathematical concepts and their interconnections but also the capacity to apply procedures accurately and appropriately (Pirie & Schwarzenberger, 1988). Consequently, understanding concepts and procedures forms an inseparable unit in mathematical learning (Hurrell, 2021; Usiskin, 2012).

Sa'dijah et al. (2018) suggested that understanding a mathematical concept entails grasping it thoroughly and being able to explain it in one's own words without changing its essential meaning. According to Kilpatrick et al. (2001), students must meet several indicators to demonstrate mathematical understanding: accurately defining concepts, creating different representations of a concept, identifying examples and non-examples, making connections between concepts, and applying these concepts through procedures in an efficient, flexible, and

accurate manner. These indicators are crucial benchmarks of students' learning success, as they signify a strong understanding of mathematics (Andamon & Tan, 2018).

Mathematical understanding is not static but can be developed and enhanced through learning (Gulkilik et al., 2020; Putri & Susiswo, 2020; Yao, 2020). Pirie and Kieren's theory provides a valuable framework for observing the growth of students' mathematical understanding. It is widely used to describe students' mathematical understanding profiles (Donuata & Pratama, 2021; Muliawati, 2020; Sagala, 2016, 2017). According to Pirie and Kieren's model, mathematical understanding evolves through eight distinct layers: primitive doing (primitive knowing), image making, image having, property noticing, formalizing, observing, structuring, and inventizing (Patmaniar et al., 2021; Pirie & Kieren, 1989, 1994; Yao, 2020). Pirie and Kieren (1989) described the development of mathematical understanding as dynamic, multilayered, and recursive. When students encounter problems that cannot be resolved at a given level of understanding, they may return to earlier layers to reorganize and expand their comprehension a process referred to as "folding back" (Mustikaningtyas & Susiswo, 2020). Parameswaran (2010) emphasized that students' understanding evolves throughout the learning process, highlighting the importance of instructional approaches that stimulate and promote the growth of mathematical understanding (Gulkilik et al., 2020).

Despite these theoretical insights, many students face significant challenges in developing mathematical understanding (Ariyanto et al., 2019; Hoiriyah et al., 2019; Tsurayya & Ningrum, 2021). Learning obstacles often impede this growth. Jannah et al. (2019) defined learning obstacles as factors that hinder students' progress in learning. These obstacles can manifest as difficulties in identifying solutions to problems or grasping mathematical concepts. Brousseau (2002) categorized learning obstacles into epistemological, didactical, and ontogenic. Epistemological obstacles, as described by Jannah et al. (2019), arise from the inherent nature of mathematical concepts, such as misunderstandings in defining or interpreting a concept. Didactical obstacles are associated with the use of inappropriate teaching methods or instructional materials (Suryadi, 2019). In contrast, ontogenic obstacles, defined by Musyriyah et al. (2022), pertain to students' mental readiness and maturity in acquiring knowledge. Suryadi (2019) classified ontogenic obstacles into three subtypes: instrumental, psychological, and conceptual.

Research by Anzani et al. (2022) indicated that students frequently encounter didactical, ontogenic, and epistemological obstacles when learning sequences and series. This topic is particularly significant due to its numerous real-world applications, including investment systems, loan calculations, asset depreciation, and medical dosage administration (Grigorieva, 2016). Moreover, studying sequences and series fosters computational thinking, which enhances students' creativity and problem-solving abilities (Angeli & Giannakos, 2020; Fauzi et al., 2022).

Consequently, sequences and series are featured as a domain in the Trends in International Mathematics and Science Study (TIMSS) (Sari et al., 2020). Mastery of this topic is essential for developing computational thinking skills.

Previous studies have shown that many students struggle with sequences and series, as indicated by their difficulties in solving related problems (Annisa & Kartini, 2021; Maryani & Chotimah, 2021). These findings suggest that students' abilities in this area are generally weak (Earls, 2017; Rahmatillah et al., 2017). Therefore, a comprehensive analysis of students' understanding of sequence and series concepts and the obstacles that hinder their learning is necessary.

Several studies have addressed mathematical understanding, including efforts to enhance students' comprehension through the development of instructional materials and educational media (Ariyanto et al., 2019; Fasaenjori et al., 2023; Hussein et al., 2022; Putri & Susiswo, 2020; Putri et al., 2023). Research specifically focusing on the understanding of sequences and series has examined aspects such as error analysis, mathematical connections, reasoning, and critical thinking (Annisa & Kartini, 2021; Masuda et al., 2021; Rahmatillah et al., 2017; Rohmah & Warmi, 2021). In contrast, studies on learning obstacles have predominantly concentrated on geometry (Aisah & Kusnandi, 2018; Cesaria & Herman, 2019; Rosita et al., 2020) and functions (Jannah et al., 2019). Regarding sequences and series, research on learning obstacles has primarily addressed three main types: didactical, epistemological, and ontogenic (Anzani et al., 2022).

However, previous studies have revealed research gaps that warrant further investigation. Existing research has primarily focused on students' learning obstacles within specific mathematical topics (Aisah & Kusnandi, 2018; Anzani et al., 2022; Cesaria & Herman, 2019; Jannah et al., 2019; Rosita et al., 2020), while other studies have described students' mathematical understanding profiles based on Pirie and Kieren's theory (Donuata & Pratama, 2021; Muliawati, 2020; Sagala, 2016, 2017). Yet, no research has examined students' learning obstacles in relation to the layers of mathematical understanding, as outlined by Pirie and Kieren. Given that the growth of students' mathematical understanding occurs throughout the learning process (Parameswaran, 2010), learning obstacles significantly hinder optimal development. Therefore, this study aims to fill this gap by describing students' learning obstacles in sequences and series through the lens of Pirie and Kieren's theory.

Methods

This research uses qualitative methods with a descriptive type (Creswell, 2012; Fraenkel et al., 2012). Based on the steps of qualitative descriptive research, according to Cresswell, researchers summarized the process in three stages: planning, implementation, and conclusion. In

the planning stage, observations were conducted, and relevant literature on students' learning obstacles was reviewed. Research instruments, including test questions and interview guidelines, were prepared. Four test questions were designed to assess students' mathematical understanding based on established indicators adapted from Kilpatrick et al. (2001). The five indicators used to evaluate student performance were the ability to (1) restate concepts, (2) provide examples and non-examples, (3) represent concepts in various forms, (4) connect concepts with other concepts, and (5) apply procedures correctly. The interview guidelines included questions to clarify the students' test responses and gain deeper insights into their understanding of sequences and series. All research instruments were validated by a mathematics lecturer prior to use.

During the implementation stage, tests were administered to 30 Year 11 students at a high school in Malang, Indonesia, who had previously studied sequences and series. Purposive sampling was used to select students for follow-up interviews based on their test results. The selection process involved analyzing the types of errors made on each test item. From these observations, one to two students were chosen to represent different error types for each question, allowing for in-depth interviews. For the first question, SZ and DA were selected as representatives of subjects who both experienced problems identifying arithmetic sequences. However, the two subjects had different understandings of number patterns. For the second question, AN and DA were selected as representatives of subjects who experienced difficulty identifying geometric sequences, but both had different reasons. For the third question, RS was selected who had the most distinct method from his friends, namely using the n th-term formula. Meanwhile, for the last question, DAS was selected as a student who solved the question using a trial-and-error.

The conclusion stage began with data analysis, which involved data reduction, data presentation, and drawing conclusions. To ensure the validity of the data, triangulation was conducted using test results and recorded interviews as data sources. The indicators for the layers of mathematical understanding in the Pirie-Kieren framework were adapted from Putri and Susiswo (2020) and are provided in Table 1. Indicators for learning obstacles were based on Suryadi (2019) and are shown in Table 2.

Table 1. Pirie-Kieren's layers of mathematical understanding indicators

Layers	Indicator
Primitive Knowing	Students possess the foundational knowledge necessary to construct a concept.
Image Making	Students engage in activities to create mental images about a topic by leveraging their prior knowledge and applying it to new learning experiences.
Image Having	Students comprehend a topic without performing activities or relying on examples to trigger their understanding.
Property Noticing	Students are able to connect and combine various aspects of a topic to identify specific characteristics of the constructed image.
Formalizing	Students generalize the characteristics obtained from the previous layer into formal concepts.

Layers	Indicator
Observing	Students utilize the formal concepts they have acquired.
Structuring	Students organize and link existing formal concepts into a coherent theoretical framework.
Inventizing	Students create new questions that lead to the development of novel concepts.

Tabel 2 Learning obstacles indicator

Learning Obstacles	Indicator
Ontogenic Obstacles: Conceptual	A lack of prerequisite knowledge hinders students.
Ontogenic Obstacles: Psychological	Students lack interest and motivation to engage with the material.
Ontogenic Obstacles: Instrumental	Students face challenges because the instruction provided by the teacher is either too difficult or too easy.
Epistemological Obstacles	Students encounter difficulties due to limited knowledge of a specific context or subject matter.
Didactical Obstacles	Students experience challenges arising from the methods or media used in teaching.

Results and Discussion

In the implementation phase, the researchers evaluated the performance of 30 Year 11 students who had previously studied sequences and series. The results of the student assessments are summarized in Table 3.

Table 3. Mathematical understanding test results

The Number of Students	Maximum Value	Minimum Value	Average	Standard Deviation
30	78	12	45.63	18.56

Based on the test results, the researchers categorized the students into three groups: high, medium, and low. This classification was based on Arikunto's (2018) three-rank grouping technique. The results of this grouping are presented in Table 4 below.

Table 4. Level of mathematical understanding

Category	Provision	Number of Students	Percentage
High	≥ 64.2	5	16.67%
Medium	$27.06 < \text{Score} < 64.2$	19	63.33%
Low	≤ 27.06	6	20 %

Table 4 indicates that only 16.67% of students achieved scores in the high category. The majority of students scored within the medium and low categories. Table 5 summarizes the distribution of student performance on each test item.

Table 5. Achievement on each question

Number 1	Number 2	Number 3	Number 4
19.05%	30.24%	66.55%	55.42%

The researcher selected several students as research subjects based on the achievement data presented in Table 5, their responses recorded on the answer sheets, and their adequate communication skills. The following sections present the results of the data analysis.

Analysis of Question Number 1

Question number 1 presents two sequences: 1, 2, 4, 8, 16, 32, ... and 1, 1, 1, 1, 1. Students must determine whether each sequence is an arithmetic sequence and provide relevant justifications for their answers. This question assesses students' mathematical understanding, specifically measuring their ability to restate concepts correctly and to provide appropriate examples and non-examples.

Table 5 indicates that question number 1 had the lowest percentage of correct answers compared to the other questions. Only 5 out of 30 students answered it correctly, and among them, only 3 provided relevant justifications for their answers. While some students understand that a constant difference between terms characterizes an arithmetic sequence, many struggle to grasp the significance of this difference. This issue is evident in SZ's responses (see Figure 1).

1. a. barisan aritmetika karena barisan yang mempunyai pola tertentu, selisih 2 suku barisan sama dan tetap. b. bukan karena barisan tersebut tidak memiliki selisih 2 suku barisan sama dan tetap.	Kelas: XI IPA 2. Translation a. It's an arithmetic sequence because it has a certain pattern, where the difference between two consecutive terms is the same and constant. b. No, because this sequence does not have the same and constant difference between two consecutive terms.
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Figure 1. SZ's answer to question number 1

In problem 1.a, SZ identifies the sequence as an arithmetic sequence because of the constant difference between consecutive terms. However, in problem 1.b, SZ correctly recognizes that the sequence 1, 1, 1, 1, 1... does not exhibit a constant difference between consecutive terms and, therefore, is not an arithmetic sequence. To gain further insight into students' comprehension of these concepts, the researchers conducted an interview with SZ. The following is an excerpt from that interview between researcher (R) and research subject.

- R : In question number 1.a, you stated that 1, 2, 4, 8, 16, 32,... is an arithmetic sequence because the difference between the terms is the same and constant. Are you sure about your answer?
- SZ.1 : Yes
- R : What is the difference between each term?
- SZ.2 : From 1 to 2 is one; from 2 to 4 is two; from 4 to 8 is four
- R : This means that the difference between the terms is the same and remains constant. How does that work?
- SZ.3 : Maybe it means it's always increasing, ma'am, because from 2 to 4 and from 4 to 8, the difference keeps getting larger
- R : Now, in number 1.b, the sequence doesn't have the same constant difference between consecutive terms. What's the difference from 1 to 1?
- SZ.4 : It's always 0
- R : Is it an arithmetic sequence?
- SZ.5 : I don't think so.
- R : Why?
- SZ.6 : Because it stays the same; it doesn't increase.

According to observations and interviews, students appear to lack a solid understanding of the differences between terms in an arithmetic sequence, as evidenced by statements from SZ.3 and SZ.6. While SZ can identify the differences in a sequence, they struggle to grasp the definition of an arithmetic sequence itself. Furthermore, their understanding seems to be confined to the property noticing layer. Although the students should have already mastered the concept of arithmetic sequences, it is clear that SZ recognizes the key characteristic of such sequences having a constant difference between terms. However, SZ has not yet attained the formalizing layer of understanding, as demonstrated by their inability to restate the concept accurately. Statements from SZ.3 and SZ.6 indicate a lack of comprehension regarding a fixed difference. The learning obstacles indicator suggests that students may be experiencing epistemological obstacles, such as limited knowledge of arithmetic sequence material. This limitation hampers their ability to determine whether a sequence is arithmetic and affects their capability to accurately restate concepts.

Another case was identified during the analysis of question number 1, specifically with student DA. According to DA, the sequence 1, 1, 1, 1, 1, ... is not considered an arithmetic sequence due to the absence of a discernible number pattern or difference. To investigate this further, the researchers conducted interviews with DA. The following is an excerpt from that interview.

- R : You stated that problem 1.b is not an arithmetic sequence. Can you explain why?*
- DA.1 : Arithmetic sequences have a consistent difference between consecutive terms. Since problem 1.b does not exhibit a number pattern, it is not considered an arithmetic sequence*
- R : What is the difference between the terms?*
- DA.2 : It's zero*
- R : If the difference is still zero, why isn't it considered an arithmetic sequence?*
- DA.3 : Because there is no discernible pattern*
- R : Okay, let's revisit number patterns. In your opinion, do the terms in the same sequence have a different or similar pattern?*
- DA.4 : I'm not clear on the terms 'pattern,' 'series,' and 'sequence,' ma'am. But if I know the formula, I would know how to solve it.*

Based on the interviews, it appears that DA struggles to differentiate between examples and non-examples of arithmetic sequences. However, upon closer examination, DA demonstrates an improved understanding of mathematics, particularly in formalization. DA can now generalize that an arithmetic sequence is characterized by a constant difference between terms, as articulated in DA.1. Despite this progress, DA's understanding falls short at the observing layer. This is evidenced by the inability to accurately identify examples and non-examples of arithmetic sequences. This challenge may stem from DA's insufficient exposure to number patterns, sequences, and series. Alternatively, it can be argued that students cannot expand their

understanding when revisiting the primitive knowing layer due to inadequate foundational knowledge. This situation is partly a result of DA's emphasis on knowing, memorizing, and applying formulas, as indicated in statement DA.4.

Therefore, it can be concluded that DA experiences both ontogenic conceptual and didactical obstacles. Ontogenic conceptual obstacles arise from a lack of mastery over the prerequisite material for arithmetic sequences, specifically number patterns and sequences. Meanwhile, didactical obstacles are present because students are accustomed to rote memorization of formulas rather than developing a deeper understanding of the concepts.

Analysis of Question Number 2

Question number 2 presents two sequences: $2, 2, 2, 2, 2, \dots$ and $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots$. Students are tasked with identifying these sequences as examples and non-examples of geometric sequences while providing relevant justifications for their classifications. This question assesses students' mathematical understanding based on indicators requiring them to accurately restate concepts and provide appropriate examples and non-examples.

Table 5 indicates that achievement in question number 2 was low, with only 11 out of 30 students answering correctly and only four providing relevant justifications for their answers. The researcher observed several common mistakes among students, including a fundamental understanding that a constant ratio between its terms characterizes a geometric sequence. However, many students struggled to grasp the significance of ratios in the context of geometric sequences. Figure 2 illustrates the responses provided by DA and AN for question number 2.

	Translation a. Yes, because it has a ratio of 1. b. Yes, because it has a ratio of $\frac{1}{2}$.
(a) AN's Answers	
Translation a. It is a geometric sequence because of the multiplication of the same ratio, "1". b. It is neither a geometric nor arithmetic sequence because the pattern of the numbers decreases, not in the form of a ratio or multiplication.	(b) DA's Answers

Figure 2. DA and AN's answers to question Number 2

In question 2.b, DA and AN provided differing response. DA believes that the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots$ is not a geometric but rather an arithmetic sequence, as the decreasing number pattern does not represent a constant ratio. In contrast, AN assert that the same sequence is geometric with a common ratio of $\frac{1}{2}$. The researchers conducted interviews with both students to

further investigate their understanding as they worked through the problem. The following is an excerpt from those interviews.

- R : For question 2.a, you stated that the sequence 2,2,2,2,.. is a geometric sequence because the ratio is 1. Does this mean that it is indeed a geometric sequence?
- AN.1 : It uses multiplication and division
- R : Can you provide an example?
- AN : (writes 3,6,12,24,...)
- R : What is the ratio?
- AN.2 : I forgot
- R : Then what do you mean by a ratio of 1?
- AN.3 : If you multiply the result by 1, for example, 2,2,2,2,2
- R : Then In part b, you mentioned that it has a ratio $\frac{1}{2}$. What do you mean by a ratio of $\frac{1}{2}$?
- AN.4 : If you multiply, half of the results follow. Hmm, but that might be wrong
- R : How should you express it?
- AN : (writing $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$)

Based on the interviews with AN, it is evident that AN recognizes a defining characteristic of a geometric sequence as one in which the terms are derived through multiplication and division by a fixed number, as indicated in AN.1. However, AN struggle to generalize this concept, which suggests difficulty in accurately articulating formal mathematical concepts. Consequently, it can be concluded that the student's understanding has reached the property-noticing layer but has not yet progressed to the formalization layer. This condition is attributed to the epistemological obstacles the student experiences. AN's lack of understanding regarding fixed ratios exemplifies this limitation, as it prevents them from expressing the concept in a formal manner. As a result, AN can only identify the special characteristics of geometric sequences without fully grasping their underlying principles. In addition to interviewing AN, the researcher also conducted an interview with DA to further explore their response to question number 2. The following is an excerpt from the interview with DA.

- R : First, regarding question number 1, you said it wasn't an arithmetic sequence but a geometric sequence. Why?
- DA.5 : Because we use ratios
- R : What is the ratio?
- DA.6 : Ratios are like differences, but they are called ratios in geometry. In arithmetic sequences, we refer to them differently.
- R : Let's try an example. What would the sequence be if I have a geometric sequence with the first term 4 and the ratio of one-half?
- DA : (tries to calculate using a formula $U_n = ar^{n-1}$ and write 4,2,1...)
- R : For question 2b, you stated it was not a geometric but an arithmetic sequence. Why?
- DA.7 : Because it doesn't use ratios. For example, if you use a ratio of one-half, that gives you $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$. There isn't a $\frac{1}{6}$

- R : Then why do you consider it an arithmetic sequence? What is the difference?*
DA.8 : I haven't calculated it yet, ma'am, but I think it's not a geometric sequence; it must be an arithmetic sequence

After analyzing the answers and interviews, it was confirmed that DA understands that a geometric sequence has a constant ratio between terms, while an arithmetic sequence has a constant difference between terms. DA even attempted to use the formula for the n th term of a geometric series. Therefore, it can be concluded that DA has formalized its understanding (formalizing layer) but struggles with the observing layer. This is evident in their response to question 1, where DA fails to apply formal concepts to distinguish between examples and non-examples of geometric sequences. This difficulty is attributed to epistemological obstacles experienced by students. This is illustrated in statement DA.8, where DA indicates that students perceive sequences as belonging to only two categories: arithmetic sequences and geometric sequences. Thus, students demonstrate limited knowledge in this specific context or material.

Analysis of Question Number 3

In addition to questions 1 and 2, students faced challenges with question 3. This question assessed students' ability to articulate concepts in various forms and apply procedures flexibly, accurately, and efficiently when solving problems. The researcher presented an example problem involving bacterial growth that follows a geometric sequence. Based on the provided illustrations, students were instructed to create observation reports on bacterial development using sets of ordered pairs, arrow diagrams, graphs, tables, or other suitable forms. Furthermore, students were required to determine the number of bacteria at specific time intervals.

According to the conducted tests, question number 3 had the highest percentage of correct responses compared to the other questions. However, the overall percentage of achievement remains relatively low. After an in-depth analysis, it was observed that most students presented their reports exclusively in tabular form, with none attempting to create alternative representations. Additionally, out of the 30 students, only 18 were able to correctly plan a solution to determine the number of bacteria at a specific time, and only 4 students solved it accurately. Overall, there were two types of student responses: some registered their calculations manually, while others employed the formula for the n th term of a geometric series to address the questions. However, both methods contained errors. Figure 3 illustrates examples of the students' mistakes.

Based on the answer snippet above, it is evident that students using the registration method encountered calculation errors due to inaccuracies. Meanwhile, RS attempted to apply a formula but made a critical error, particularly in determining the value of n . To gather more insights, the researchers conducted additional interviews with RS. The following is an excerpt from the interview with RS.

- R : For question number 3, please explain how you approached it. How did you determine the number of bacteria at 16:00?
- RS.1 : The bacteria started the experiment at 13:15. First, I found the time difference from 13:15 to 16:00 in minutes. Then I multiplied the number of bacteria every 15 minutes. So, I divided the total minutes by 15. The result is 11, so 11 is the n
- R : What does n represent?
- RS.2 : The term
- R : Now, how many minutes are there from 13:15 to 16:00, divided by 15?
- RS.3 : There are quite a few divisions
- R : Are the number of divisions and n in the formula the same?
- RS.4 : Yes
- R : We were instructed to present our findings in ordered pairs, arrow diagrams, graphs, tables, or other forms. Do you remember what concept this relates to?
- RS.5 : Hmm, I don't know
- R : That's a function, right? Do you think a sequence is a function?
- RS.6 : Hmm, maybe it is a function, ma'am, but I'm not sure
- R : You've studied number patterns before, right? What do you think is the difference between number patterns and sequences?
- RS.7 : In sequences, there's only one pattern, ma'am. In number patterns, you can have two patterns. For example, in one problem, you can add 2 and multiply by 2. If you add 2 to a sequence, you add 2 to everything.

3. tabel hasil pengamatan

waktu	banyak bakteri
13.15 WIB	1. 3 bakteri
14.00 WIB	2. 6 bakteri
14.15 WIB	3. 12 bakteri
14.30 WIB	4. 24 bakteri

b. banyak bakteri pada pukul 16.00 WIB

$a = 3$
 $r = 2$
 $n =$ membetah setiap 15 menit
 maka pukul 16.00 membetah
 sebanyak 11 kali

$u_n = a \cdot r^{(n-1)}$
 $= 3 \cdot 2^{(11-1)}$
 $= 3 \cdot 2^{10}$
 $= 3 \cdot 1024$
 $= 3072$

Translation
 Observation table

Time	Number of Bacteria
13.15 pm	3 bacteria
14.00 pm	6 bacteria
14.15 pm	12 bacteria
14.30 pm	24 bacteria

b: number of bacteria at 16:00 pm
 $a = 3$
 $r = 2$
 n : divides every 15 minutes, so by 16:00 it has divided 11 times

(a) Using formulas U_n geometry

b) waktu awal = 13.15

Observation 1 Pengamatan 1 = 13.15 (3)

Observation 2 Pengamatan 2 = 13.30 (6)

Observation 3 Pengamatan 3 = 13.45 (12)

4 14.00 24

5 14.15 48

6 14.30 96

7 14.45 192

8 15.00 384

9 15.15 768

10 15.30 1536

11 15.45 3072

12 16.00 6144

384

(b) Using the manual method (by listing)

Figure 3. Example of student answers for question number 3

This problem generally highlights the application of geometric sequences in real-life scenarios. However, students frequently overlook the meanings of terms associated with geometric sequences. Statement RS.1 indicates that bacteria divide according to a specific pattern, suggesting students have reached the formalizing layer of understanding. However, their

application of this concept remains insufficient at the observing layer. Research has shown that students struggle to differentiate between various divisions and terms.

Additional interviews were conducted to further investigate students' abilities regarding prerequisite materials. Students were found to lack a solid understanding of functions and number patterns. This gap leads to difficulties connecting the concepts of functions and sequences, which hampers their ability to express ideas in multiple representations. These issues represent ontogenic conceptual obstacles. The challenges faced by students were further highlighted when researchers assessed RS on algebraic operations, specifically in determining a value

The researcher presented the algebraic equation $56 = \frac{20(r^2-1)}{(r-1)}$. RS and instructed RS to determine the correct value of r . However, RS could not find a value of r that satisfies this equation, as illustrated in Figure 4.

Handwritten work showing the student's attempt to solve the equation $56 = \frac{20(r^2-1)}{(r-1)}$. The student incorrectly simplifies the right-hand side to $56 = 20$ by canceling $(r-1)$ from the numerator and denominator. This leads to the incorrect equation $(r-1) = 56 - 20 = 36$.

Figure 4. RS' answers in algebraic operations

The evidence suggests RS has not reached the observing layer due to ontogenic conceptual obstacles. This is evident from RS's inadequate prerequisite skills and the misunderstandings that arise. Figure 4 illustrates the shortcomings in the students' primitive understanding during their work processes. Furthermore, statement RS.7 highlights a misunderstanding in students' foundational knowledge.

Analysis of Question Number 4

Like questions 1, 2, and 3, students faced challenges with question 4. This question assessed the students' ability to connect concepts and apply procedures accurately and effectively. The context of the problem presented in question 4 involved tart cakes shaped like circles, which were cut according to arithmetic sequences. Students were required to calculate the base area of a piece of cake.

During the assessment, most students in one class struggled to comprehend the questions. Despite the clear indication that the problems pertained to sequences and arithmetic series concepts, many students could not devise the correct solutions. This difficulty stemmed from their

unfamiliarity with the question format, as they had not encountered similar problems in prior learning experiences. Consequently, researchers found it necessary to assist by explaining the meaning of the questions. After receiving this clarification, some students could answer correctly; however, others remained unable to respond until the end of the test. Additionally, some students resorted to trial-and-error methods to attempt a solution, as illustrated in Figure 5.

$$b \cdot 10^\circ + 20^\circ + 30^\circ + 40^\circ + 50^\circ + 60^\circ + 70^\circ + 80^\circ = 360^\circ$$

$$\frac{60}{360} \times 616 = 102,56$$

$$\pi r^2$$

$$\frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$$

Figure 5. DAS' answer to question number 4

The excerpt from DAS's response, as shown in Figure 5, indicates that DAS recognizes the potential solution to this problem lies within the concepts of arithmetic series and the area of a circle. To gain further insights into DAS's understanding, researchers conducted interviews with DAS. The following is an excerpt from the interview.

- R : What is your strategy for question number 4?
- DAS.1 : I approached it by first looking for the formula for the area of a circle, which is πr^2 . I calculated the area of the circle to be 616. For the smallest piece of cake, which I assumed was 10° ?. I don't know, it just happened to be random, I thought there might be a pattern like $10^\circ, 20^\circ, 30^\circ, \dots, 80^\circ$
- R : For example, if the smallest piece is 20° , would you approach it the same way?
- DAS.2 : I don't think so
- R : Then what method would you use?
- DAS.3 : Maybe I would use the formula for the n th term (U_n).
- R : What difficulties do you face when learning about sequences and series
- DAS.4 : I think it requires a lot of memorization of formulas, and you have to be very careful

Based on the interviews, DAS approached the given questions through trial and error rather than relying on memorized formulas. Further examination indicates that DAS struggled at the observing layer, failing to recognize that the problem pertained to an arithmetic series, as evidenced by DAS's responses in DAS.1 and DAS.3. This difficulty is attributed to instrumental ontogenic obstacles. Moreover, students may find it challenging to solve questions about sequences and arithmetic series when they are related to other mathematical concepts. In this case, incorporating circles in the problem may have hindered students' ability to connect the relevant concepts effectively.

Based on the research, researchers have identified several findings related to students' learning obstacles within the context of these layers of mathematical understanding, based on research that referenced indicators of mathematical understanding layers (Putri & Susiswo, 2021) and strategies to eliminate obstacles (Suryadi, 2019). Based on the analysis, researchers found that students' mathematical understanding did not improve due to ontogenic, didactical, and epistemological obstacles. Ontogenic conceptual obstacles arise when students lack mastery of prerequisite material, as observed in the cases of DA and RS, who demonstrated inadequate knowledge and failed to reach the observing layer. This condition reinforces Yao's (2020) findings that insufficient initial understanding of mathematical concepts can hinder the development of mathematical comprehension, as noted in Putri and Susiswo's (2020) research. Furthermore, students often struggle to grasp mathematics in a comprehensive and integrated manner (Martin & Towers, 2016). For instance, RS encountered difficulties connecting the concepts of functions and sequences. Although such connections are crucial for developing mathematical skills (Kenedi et al., 2019), students frequently face challenges in understanding these concepts.

Addressing these obstacles is vital to ensuring robust mathematical understanding. These barriers can stem from unfamiliar contexts and an emphasis on memorizing formulas without grasping their underlying meanings. Typically, students rely heavily on textbooks for their studies, lacking additional teaching materials that could enhance their mathematical comprehension. Thus, developing learning designs that effectively stimulate students' mathematical growth is essential (Gulkilik et al., 2020). Hiebert et al. (1997) further emphasize the importance of creating classrooms that facilitate and support students' understanding. Maximizing five dimensions is crucial for effective learning: the nature of the learning task, the role of the teacher, the social culture of the classroom, the types of mathematical tools available, and the accessibility of mathematics for each student. Based on the research results, it is imperative to design learning experiences that stimulate students' mathematical understanding and expand upon their primitive knowledge. This approach will promote the growth of students' mathematical comprehension.

While this research has identified several obstacles to student learning in developing their mathematical understanding, it primarily highlights obstacles related to conceptual abilities. These challenges are linked to Pirie and Kieren's theory of growth in mathematical understanding. Additionally, the limited sample size may restrict the generalizability of the findings. Therefore, further research is necessary to explore other obstacles students encounter to develop their mathematical understanding more comprehensively.

Conclusion

The research results indicate that students' understanding of mathematical sequences and series is insufficient, particularly in progressing beyond the property noticing and formalizing stages, even though they are expected to reach the observing stage. This issue is attributed to learning obstacles, which stem from inadequate or inappropriate primitive knowledge (ontogenic conceptual obstacles) and a lack of systematic opportunities for students to develop their mathematical understanding (ontogenic instrumental, didactical, and epistemological obstacles). Therefore, it is crucial to conduct further research to identify and address these learning obstacles, emphasizing the development of foundational knowledge and designing learning experiences that foster the growth of students' mathematical understanding.

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