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Defragmentation of Students' Conceptual Understanding in Solving Non-Routine Mathematics Problems

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Abstract. *This study investigated the impact of concept fragmentation on students' understanding of mathematics and examines efforts to reduce it. Concept fragmentation hinders problem-solving abilities, often occurring when students struggle to create meaningful connections or new representations from existing ones. Limited interventions, such as cognitive conflict and scaffolding, are suggested to address this issue. This study employed a qualitative descriptive approach, focusing on two seventh-grade students in Sukoharjo, Indonesia, who exhibited concept fragmentation. Data collection involved tests, interviews, and observations, with analysis following qualitative methods. The findings indicate two main types of fragmentation: translational thinking fragmentation and meaningless connection fragmentation. These arise when students attempt to build new representations but make errors due to disconnected prior knowledge. Interventions revealed a pattern of developing schemas, where students knit together concepts to minimize problem-solving errors. Techniques such as rereading problems and substituting information into formulas improved concept comprehension. The study concludes that defragmentation aids students in connecting existing knowledge with new information, enhancing their problem-solving strategies. Future research should investigate other fragmentation types and effective interventions for reducing fragmentation in mathematics learning.*

***Keywords**: conceptual understanding, defragmentation, fragmentation, limited interventions*

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Introduction

Conceptual understanding is the ability to explain and formulate something, give examples and diagrams, use more creative explanations to understand words and symbols, know ideas and thoughts, and interpret them without changing the concept's meaning (Kholid et al., 2021). Students have a comprehensive understanding of concepts if they can develop solving strategies, apply simple calculations, represent concepts using symbols, and change one form to another without changing the concept's meaning. Students often struggle with mathematics because they do not actively construct their understanding of mathematical concepts. Instead, they tend to memorize these concepts without comprehending their underlying meaning. Consequently, they frequently make errors when faced with mathematical problems and fail to identify the solutions (Khoerunnisa & Hidayati, 2022).

Conceptual understanding is important in mathematics because it helps develop problem-solving skills, transfer knowledge, identify misunderstandings, and build a strong foundation for further learning (Damayanti, 2023). It enables students to understand mathematical concepts in depth and apply their knowledge effectively in various fields of economics, engineering, physics, statistics and other fields. In problem-solving, students must develop an understanding of concepts, which is the basis for solving problems meaningfully and developing them (Yulianty, 2019). Usually, students who use their intuition will experience fragmentation of concept understanding in solving math problems.

Problems in mathematics include two types: routine problems and non-routine problems. In solving routine problems, students involve the algorithm that has been learned and apply it step by step using a specific formula or formulas (Surif et al., 2014). Non-routine mathematical problems refer to problems that cannot be solved using procedures or formulas that have been known beforehand or routine methods. These problems often require a more creative, analytical approach and higher critical thinking skills. In mathematics education, non-routine problems challenge students to search for various solving strategies, explore the relationships between mathematical concepts, and build a deeper understanding (Sternberg, 2021)

Fragmentation is a failure of construction in memory because the concepts learned are not well connected, so students have difficulty in mathematical modeling (Wibawa et al., 2020). Subanji (2016) proposed five types of fragmentation in mathematical solutions: (1) pseudo-construction fragmentation, (2) fragmentation of construction holes, (3) fragmentation of misconnection, (4) fragmentation of logical thinking errors, (5) fragmentation of analogy thinking errors.

Fragmentation in solving mathematical problems in students' concept understanding is called concept understanding fragmentation. The fragmentation experienced by students can be minimized or overcome with various efforts. One of the efforts in question is defragmentation, which is the provision of limited interventions. According to Hidayanto et al. (2017), defragmenting refers to a facilitator's (in this case, the researcher’s) limited intervention aimed at helping a student organize their fragmented thinking into a coherent structure. This intervention enables the student to appropriately solve problems using the mathematical concepts they have previously learned. Limited interventions are carried out by creating disequilibration, providing cognitive conflict, and providing scaffolding. The three can be free from each other, intersecting or gradually. In other definitions of defragmentation, Wibawa et al. (2020) stated that it is to restructure a wrong concept or idea so that it can correct and reduce mistakes and maintain a correct understanding.

Disequilibration is a condition in which a person experiences difficulties/difficulties reflecting an imbalance between assimilation and accommodation. Wibawa et al. (2020) said that being curilibated is a limited intervention step by asking questions that can raise suspicion or a gap in thinking in the problem solver so that they will reflect on the answers given, such as: (1) Are you sure of the answer?, (2) Why is the formula that way? and so on.

Cognitive conflict is given to the subject when the subject experiences a mistake. This requires an example that can be used to form a conflict so that, eventually, the subject will rethink the answer. Cognitive conflict is a limited intervention step that provides information or questions that contradict the information that the problem solver has, creatingconflicts in their minds (Wibawa et al., 2020). Cognitive conflicts are created to stimulate problem solvers to rethink the truth of their statements. If they can accommodate the cognitive conflicts that have been created, they will succeed in solving math problems and vice versa.

Scaffolding means providing help to a person during the early stages of learning, then removing the support and asking for greater responsibility as he or she can (Wibawa et al., 2020). Meanwhile, scaffolding provides sufficient (minimal) assistance from external sources so that their understanding reaches the correct answer. Provided in a way that allows troubleshooters to correct errors.

Defragmentation of conceptual thinking refers to the reunification of faulty schemas within an individual's cognitive structure. It can be understood as cognitive restructuring (Wulandari & Gusteti, 2021). The defragmentation method aims to minimize student errors in problem-solving by helping to reorganize their cognitive processes. Cognitive restructuring involves rearranging thought patterns and eliminating irrational beliefs that cause tension and anxiety, which can negatively affect emotions and behavior (Faisal et al., 2022).

Research by Wulandari and Gusteti (2021) highlighted the importance of defragmentation in enhancing students' reflective thinking. Another study by Kholid et al. (2022) focused on the defragmentation of students' metacognitive structures, suggesting that more organized thinking structures improve metacognitive understanding. Putri and Indrawatiningsih (2023) examined the impact of cognitive fragmentation on students' conceptualization abilities, identifying that fragmented conceptual understanding often arises when solving complex mathematical problems. Additionally, Kholid et al. (2024) demonstrated that the fragmentation of understanding can affect concept construction, although their research was limited to problems involving absolute value equations.

From previous studies, it appears that each study focuses on specific aspects of fragmentation, such as reflective thinking, absolute value equations, and metacognitive structures, but no one has comprehensively examined the fragmentation of understanding mathematical concepts in general. In addition, most studies have not formulated a defragmentation strategy that can be applied to different materials and mathematical problems. This research is expected to fill this gap by exploring fragmentation in understanding mathematical concepts and developing a more adaptive defragmentation approach for non-routine mathematical problem-solving.

From the studies that discuss the fragmentation experienced by students in solving mathematics, no research has discussed the fragmentation of students' understanding of concepts. The research questions are: (1) What are the types and forms of fragmentation of students' conceptual understanding in solving non-routine mathematical problems? (2) What efforts can be made to define students' understanding of concepts according to each type and form of fragmentation? and (3) Does a conceptual understanding of defragmentation work for students who experience fragmentation at each level?

Method

This study involved 30 grade 7 students from an Islamic junior high school in Sukoharjo, Indonesia, who experienced fragmentation in understanding concepts and communication. The study focused on two subjects: those experiencing fragmentation of translational thinking structures and meaningless connections. Limited interventions were provided to help students organize their thinking structure and solve problems appropriately. The main instruments used in the study were researchers as planners and implementers of data collection and analysis, while companion instruments included algebraic problem-solving tests, interview guides, and audiovisual recording tools. These instruments were validated by experts in qualitative research in mathematics education and mathematical cognitive problems.

This study described the fragmentation of students in solving non-routine problems and the defragmentation of conceptual understanding. The study used five indicators, that the students are being able to: (1) restating a concept, (2) identifying the characteristics of a concept and recognize the conditions that determine a concept, (3) presenting the concept in the form of representation of mathematics, (4) developing sufficient requirements and necessary conditions of a concept, (5) using, utilizing and choosing certain procedures or operations.

Data collection techniques included written tests to measure students' understanding of concepts and interview guidelines to confirm students' problem-solving abilities. Interviews were conducted to confirm students' understanding of concepts and defragmentation processes. Data analysis techniques followed Miles and Hubberman's flow, including data reduction, presentation, and conclusion drawing. Data reduction involved summarizing and sorting important data, presenting it into narrative text, and drawing conclusions. Validity was achieved through triangulation, comparing data on students' written test results and interview results. The test questions as Figure 1.

Sebuah kerangka balok yang terbuat dari kawat mempunyai ukuran panjang $(2x+3)$ cm , lebar $(3x-1)$ cm serta tingginya x cm. Jika panjang kawatnya adalah 42 cm.

a. Gambarkan balok tersebut dengan ukurannya
b. Tentukan nilai x
c. Tentukan ukuran sebenarnya panjang, lebar dan tinggi

Figure 1. Non-routine math problems

Results and Discussion

Description of Conceptual Understanding of Subject 1 (S1) Before Defragmentation

Student S1 must read the question several times when given a question like Figure 1. The interview results also stated that S1 had to read the questions repeatedly to understand the given problem. In addition, S1 also underlines words that are considered keywords in understanding the problem. This is shown by underlining keywords, namely length $(2x+3)$ cm, width $(3x-1)$ cm and height x cm as in Figure 2.

Sebuah kerangka balok yang terbuat dari kawat memiliki panjang $(2x+3)$ cm, lebar $(3x-1)$ cm, dan tingginya x cm. Jika panjang kawat adalah 42 cm.

a. Gambar balok berdasarkan ukurannya
b. Tentukan nilai x
c. Tentukan ukuran panjang, lebar, dan tinggi yang sebenarnya

Translation

A frame of blocks made of wire has a length $(2x+3)$ cm , wide $(3x-1)$ cm and as well as its height x cm. If the length of the wire is 42 cm.

a. Draw the block by its size
b. Specify the x value
c. Determine the actual size of length, width and height

Figure 2. S1 evidence underlining keywords

In answering the question of drawing cuboids with their size, S1 tried to remember the concepts of length, width and height by drawing them even though they feel unsure. This was found in the interview, "I'm not sure how to draw the cuboids". Then, to answer the question of determining x, S1 understood the concept of the relationship between wire length and the formula for the number of rib lengths. That is when S1 wrote down the concepts he understood into a mathematical model. S1 wrote the wire length equation $= 4(p+l+t) = 42$. Then, p, l and t were substituted to the known problem to be $4\ ((2x+3) + (3x-1) + x = 42$. However, S1 stopped solving the problem because she found it difficult to continue the operation of algebraic forms. This was confirmed in the interview, when S1 said, *"Right. As I remember, the total wire length to make the beam frame is equal to the number of rib lengths. Isn't it? but I am a bit confused of how to determine the x."*. And by dabbling, fiddling with answers, S1 enters values $x = \frac{17}{12}$, to answer the question of actual size of length, width and height. This was confirmed when asked:

R-01 : *How do you determine x?*

S1-01 : *Hmm.. I don't know. I tried and found $x = \frac{17}{12}$. I can't write down and explain why I get $x = \frac{17}{12}$, ... just logic.. and it means $x = \frac{17}{12} = t$. So, to determine p and l, I substitute $x = \frac{17}{12}$ into algebraic measures on the problem, but I'm confused about how to write so let's go straight to it. (there is fragmentation of translational thinking)*

The answer sheet for solving question S1 is shown in Figure 3.

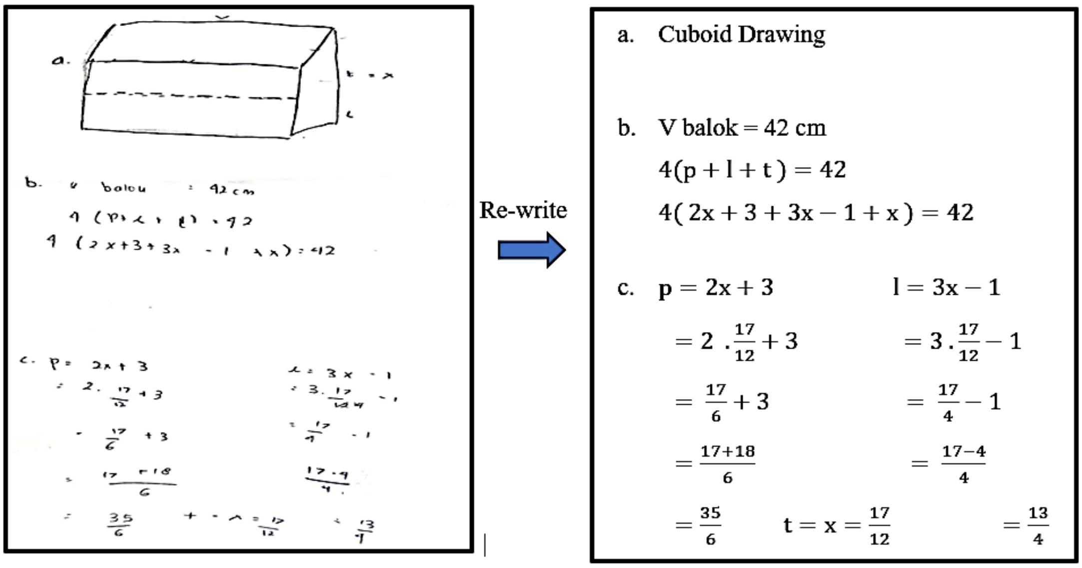


Figure 3. S1 answer sheet before defragmentation

Based on Figure 3, the analysis of test answers and interview results shows that S1 was able to draw the frame of the beam and determine the actual sizes of p, l, and t. However, S1 had not yet mastered the correct way to write systematic mathematical sentences. S1 has not developed an effective approach to solving non-routine problems, likely because S1 was more accustomed to routine problems. In contrast, S1 was not well-trained in solving non-routine problems, which require multiple stages of thinking. This observation aligns with Yunianta and Selan (2020), who found that S1 struggles with concept determination and has not rechecked whether the solution method used is correct.

S1 was unable to immediately solve mathematical problems, particularly non-routine ones, and therefore requires repeated reflection on the problem to arrive at an appropriate solution. However, given the conceptual ability of each student, non-routine problem-solving can still be achieved, albeit suboptimally. This is consistent with the findings of Susanti (2015). Furthermore, a study by Ziegler and Heller (2000) revealed that subjects with the ability to understand concepts were able to classify objects according to specific characteristics of partially fulfilled concepts, though mistakes still occurred during problem-solving.

The misunderstanding of concepts experienced by S1 is what is meant by fragmentation of conceptual understanding. S1 experiences fragmentation of translational thinking structures, where subjects find it difficult to logically present images to mathematical models or complete algebraic operations. So, efforts are needed to defragment the understanding of the S1 concept.

Defragmentation of S1 Conceptual Understanding

After knowing the location of the fragmentation of the translational thinking structure experienced by S1 in solving the problem of algebraic operations, a defragmentation process was carried out to overcome it. It was done during an interview session with S1. In the interview session, S1 was cognitive conflicts to make the subject aware of the mistakes experienced by the subject, and the scheme was brought up through scaffolding. Here are the results of interviews with the subject.

R-02	:	*Do you think it is correct to draw this cuboid frame, as in Figure 3.? (Facilitates the occurrence of cognitive conflict)*
S1-02	:	*It's wrong.. (replied quickly with a smile) ehm... (while thinking for a long time) ... If the beam has a length, width and height, huh... Just draw it? (While doodling and sketching)*
R-03	:	*Try to remember it first..* *What is the p, l and t positions? (facilitate disequilibration)*
S1-03	:	*Ehm…as I recall.. I see... Isn't that right...? (While thinking because of hesitation in answering)*
R-04	:	*Why do you write the length of the wire equal to 4 times the sum of p, l and t..? (In figure 3)*
S1-04	:	*Ehm…as I recall.. I see... Isn't right...? (While thinking because of hesitation in answering)*
R-05	:	*If the length of the wire is equal to 4 times the sum of p, l and t, then the length of the wireframe of the beam is the same as what the beam.? (Facilitate disequilibration)*
S1-05	:	*(thinking for a 3 up to 4 second) ... ehm... What's... wait... the amount of rib length...?"*
R-06	:	*So... the length of the beam frame wire is equal to...? (Provide scaffolding)*
S1-06	:	*Ooo.. so, the length of the wireframe beam is equal to the sum of the rib lengths huh? (Indication of the occurrence of knitting schemes)*
R-07	:	*How do you find the length, width and height of a beam...?*
S1-07	:	*Yes... Just change the x value. to find the actual length... so p equals $2x + 3$,.... its x was replaced $\frac{17}{12}$, so 2 times by $\frac{17}{12}$. Then the result is plus 3. Because of fractions, yes... should be equated to the denominator, and the result is $\frac{35}{6}$ hence the measure of length $\frac{35}{6}$ cm. (already able to explain algebraic operations verbally)*

R-08	:	*I.. Then in the same way to find the size of the width and height, yeah... (providing scaffolding)*
S1-08	:	*Yes... If the width is size $3x - 1$, then the then the value $x = \frac{17}{12}$ multiplied by 3, the result will be reduced by 1, so that the width is found $\frac{13}{4}$ cm. And since it is known that height is equal to x, so the actual height is $\frac{17}{12}$ cm.*

Defragmentation that has been carried out to minimize fragmentation of understanding the concept of S1 based on the interview can be illustrated in Figure 4 on the following subject completion sheet.

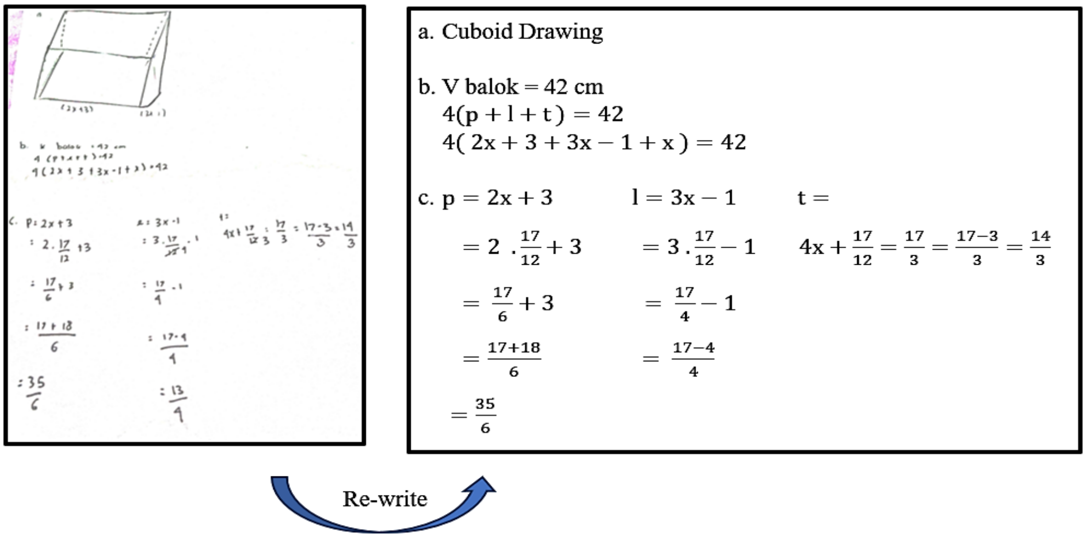


Figure 4. S1 answer sheet after defragmentation

From the analysis of students' answers in Figure 4, after a defragmentation attempt, S1 felt more confident in drawing cuboids and in verbally answering what he had according to his conceptual understanding. What researchers do is facilitate cognitive conflict and disequilibration and provide scaffolding assistance, making subjects indicated by the occurrence of scheme knitting in understanding concepts and connecting with concepts that are already owned to minimize errors in solving mathematical problems.

Description of Conceptual Understanding of Subject 2 (S2) Before Defragmentation

S2, when given a question like Table 1, must read the question several times. The interview results stated that S2 had to read the questions repeatedly to understand the problem. In addition, the subjects also circled and put question marks on words that were considered difficult. This is indicated by circling and giving question marks to words that are considered difficult, namely in length (2x + 3) cm, width (3x-1) cm and height x cm as shown in Figure 5.

In answering the question of drawing cuboids to their size, S2 felt confused. S2 feels that he remembers the concept of length, width and height by drawing it, until several times drawing cuboids, with no kink. This can be seen when interviewed, "I think the picture of the cuboid is like this, Ehm...

like this... or is it like this...?" (Several times S2 sketched the image hesitantly). Then, to answer the question of determining x, S2 found it difficult to understand the concept of rib length. That is when S2 writes down the algebraic operations it understands into a mathematical model. S2 writes $p = 2x + 3$, $l = 3x - 1$, $t = x$, then multiplied by 4 respectively, then sums all the variables into 9x and 8. However, S2 wrote down the addition method with many scribbles and question marks because he found it very difficult to operate algebra and immediately found a value $x = \frac{17}{12}$. When confirmed in the interview, S2 said: *"right... the length, width and height of the beam were 4. So, I multiplied it by 4..but I then confused... and I wrote the final result 9x and 8.., is it, true ma'am..? But I'm confused, huh... is this the case... sorry, I put it this way... confused...! Then... How to find the x value...?"*

Sebuah kerangka balok yang terbuat dari kawat memiliki panjang (2x + 3) cm, lebar (3x − 1) cm, dan tingginya x cm. Jika panjang kawat adalah 42 cm.

a. Gambar balok berdasarkan ukurannya
b. Tentukan nilai x
c. Tentukan ukuran panjang, lebar, dan tinggi yang sebenarnya

Translation

A frame of blocks made of wire has a length $(2x + 3)$ cm , wide $(3x - 1)$ cm and as well as its height x cm. If the length of the wire is 42 cm.

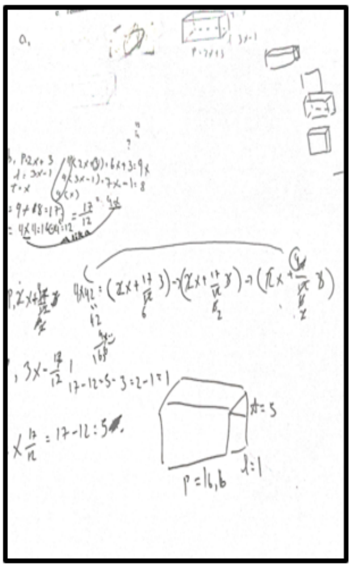
a. Draw the block by its size
b. Specify the x value
c. Determine the actual size of length, width and height

Figure 5. Proof S2 circles and puts question marks on words that are considered difficult

By trial and error, S2 enters values $x = \frac{17}{12}$, to answer the question of actual size of length, width and height. This was confirmed when asked as the following the interview script:

R-01	:	*How to find the value of x ?*
S2-01	:	*Ehm. I don't know..I replaced $x = \frac{17}{12}$ into the size of length, width and height... but... I can't.., um... I'm trying to replace the value $x = \frac{17}{12}$ into the size of the length, which is p = 2x + 3, even found not round which is 16.8 cm but it seems not true ..., Then I enter the value $x = \frac{17}{12}$ into the size of the width, l = 3x – 1, the result is 1 cm and the height size is found 3 cm.... how come... even though the height is the same as $x = \frac{17}{12}$ ... I don't understand at all and I think I was all wrong. I can't matter this, ma'am... I really don't know..." (there is meaningless connection fragmentation)*

The answer sheet for solving the S2 question is shown in Figure 6. Based on the analysis of test and interview answer results, S2 has not been able to draw a cuboid frame because many sketches of images made by S2 are impossible. This can be seen in Figure 6, the sketches of small cuboids, showing S2's difficulty in understanding the concept of drawing cuboids. In determining the value of x, S2 also finds it difficult to solve algebraic operations. This is because S2 is unable to connect mathematical concepts. A low ability to connect mathematics can be seen when students can name mathematical concepts but have difficulty using them (Khomsatun et al., 2022). In fact, if students can relate mathematical concepts, their understanding of mathematics will be deep and last longer because they can see the relationship between topics in mathematics, with contexts other than mathematics, and everyday life experiences. S2 has not been able to build an understanding between mathematical concepts (connections) of algebraic operation concepts and their application. This misconception of concepts experienced by S2 is what is meant by fragmentation of conceptual understanding. S2 experiences fragmentation of meaningless connections; S2 finds it difficult to generate and make mathematical connections and develop conceptual ideas in solving algebraic operations. S2 finds it difficult to understand procedural and conceptual mathematical concepts. So, efforts are needed to defragment the understanding of the S2 concept.



a. Drawing Cuboid

b. $p = 2x + 3 \quad 4(2x + 3) = 6x + 3 = 9x$
$l = 3x - 1 \quad 4(3x - 1) = 7x - 1 = 8$
$t = x \quad 4(x) = 4x$

$= 9 + 18 = 17 \quad = \frac{17}{12}$

$= 4 \text{ x } 4 = 16 - 4 = 12$

$p = 2x + \frac{17}{12}\,.3 \quad 4 \text{ x } 42 = (2x + \frac{17}{12}) \rightarrow \left(2x + \frac{17}{12}\,.3\right) \rightarrow \left(2x + \frac{17}{12}.3\right)$

$, 3x - \frac{17}{12}$

$17 - 12 = 5 - 3 = 2 - 1 = 1$

$x\frac{17}{12} = 17 - 12 = 5$

Re-write

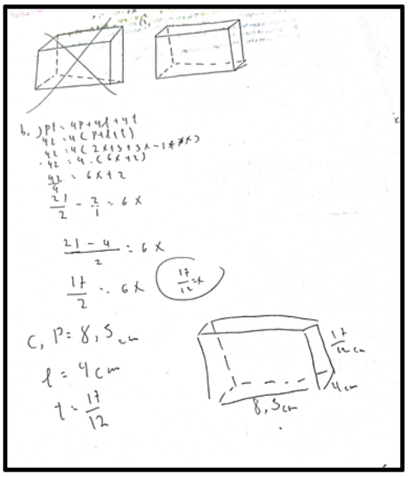
Figure 6. S2 answer sheet before defragmentation

Defragmentation of S2 Conceptual Understanding

After knowing the location of meaningless connection fragmentation experienced by S2 in solving the problem of algebraic operations, a defragmentation process will be carried out to overcome it. Defragmentation was carried out during an interview session with S2. In the interview session, S2 will be given cognitive conflict and will facilitate disequilibration to make S2 aware of the mistakes experienced by the subject. Then, the scheme will be brought up through scaffolding. Here are the results of the interview with S2.

R-02 : *Why do you draw so many small cuboid frames…? as in figure 6.? (Facilitates the occurrence of cognitive conflict)*
S2-02 : *Sorry..I'm confused about drawing the cuboid." (Answered in doubt) ehm… (while thinking for a long time) ... Actually, if the beam has a length, width and height, huh... Just draw it... (while doodling to make small and many sketches)*
R-03 : *Try to remember it first... What are the p, l and t positions? (facilitate disequilibration)*
S2-03 : *Ehm…The length is the front side, the width is the sloping side, and the height is the upright side, huh...? (Indication of the occurrence of knitting schemes) Ehm.. but for a while..I drew it wrong again.... I redrew it.*
R-04 : *Why do you write the length of the wire equal to 4 times the sum of p, l and t..?*
S2-04 : *Ehm….I saw a picture of the frame. There are 4 lengths, 4 widths and 4 heights... I see…Isn't right...? (Feel a bit sure)*
R-05 : *If the length of the wire is equal to 4 times the sum of p, l and t, then the length of the wireframe of the beam is the same as what the beam...? (Facilitate disequilibration)*
S1-05 : *(think for a long time) ... ehmm... What's…wait... the amount of rib length huh...?*
R-06 : *So... the length of the beam frame wire is equal to...? (Provide scaffolding)*
S2-06 : *Ooo.. So, the length of the wireframe of the beam is equal to the sum of the rib lengths, huh? (Indication of the occurrence of knitting schemes)*
R-07 : *How to find the size of the length, width and height of the beam...?*
S2-07 : *Yeah...that's what I can't write. I'm confused... I wrote it on paper and found it was 8.5 cm long. (not yet able to explain algebraic operations verbally)*
R-08 : *Ooo.. then in the same way to find the size of the width and height, huh..? (providing scaffolding)*
S2-08 : *Yes... then width 4 cm... This is also I can't write... (not yet able to explain algebraic operations verbally) But if it's high, it's actually because it's the same as x, so I can write ,* $t = x = \frac{17}{12}$ *cm ...and I draw it again… for actual p, l and t sizes....*

The defragmentation that has been carried out as an effort to minimize the fragmentation of students' understanding of concepts based on such interviews can be illustrated on the following subject completion sheet.



Re-write

a. Drawing Cuboid

b. $Jpr = 4p + 4l + 4t$
$42 = 4(p + l + t)$
$42 = 4(2x + 3 + 3x - 1 + x)$
$42 = 4(6x + 2)$
$\frac{42}{4} = 6x + 2$
$\frac{21}{2} - \frac{2}{1} = 6x$
$\frac{21-4}{2} = 6x \qquad \frac{17}{12} = x$

c. $p = 8{,}5$ cm
$l = 4$ cm
$t = \frac{17}{12}$ cm

Figure 7. S2 answer sheet after defragmentation

From the analysis of students' answers after a defragmentation attempt, S2 felt more confident in drawing cuboids and in verbally answering what according to their understanding of concepts. However, S2 had difficulty in explaining algebraic operations verbally, which needs to be subjected to further defragmentation. Researchers facilitated cognitive conflicts, facilitate disequilibration, and provide scaffolding assistance, making S2 indicate the occurrence of scheme knitting in understanding concepts and connecting with concepts that are already owned to minimize errors in solving math problems.

The analysis of S1 student answer sheets, which displayed fragmentation in translational thinking, reveals that this phenomenon occurs when students make errors while creating new representations from those previously established. These new representations are intended to aid in problem-solving. The representations representations in question include external objects such as graphs, symbols, words, and mathematical models, which act as encoding symbols, depict mathematical relationships or concepts, and support mathematical constructions. Such fragmentation is commonly observed when students attempt to solve mathematical problems (Wibawa et al., 2020). However, there are various strategies that can be employed to address fragmentation (Damayanti, 2023). Through methods such as calibration and scaffolding, students can resolve fragmentation by linking existing concepts, allowing them to develop a more thorough and accurate understanding. Establishing these connections is vital for solving problems correctly and comprehensively (Kholid et al., 2022; Mhlanga & Moloi, 2020). A student’s ability to regulate their cognitive processes significantly influences their overall understanding of concepts during problem-solving (Masduki et al., 2020; Maysaroh & Sutarni, 2023). Furthermore, limited intervention, through disequilibration and scaffolding, enabled students to activate and reorganize their cognitive schemes, which proved effective in resolving fragmentation (Wibawa et al., 2020).

The analysis of responses from S2 students, which displayed fragmentation of illogical connections, reveals that these students struggled to establish meaningful mathematical relationships and develop their conceptual understanding. This fragmentation is indicative of misconceptions held by the students. While their answers or concepts may be incorrect, they believe them to be true. This observation aligns with the findings of Seitz and Angel (2020) who argued that beliefs can form independently of truth, knowledge, or rationality. By decoupling, students can resolve fragmentation by reflecting on and comparing their previous answers with new ones. This comparison ultimately helps students recognize that their earlier answers were incorrect due to the flawed concepts they had constructed. This view is consistent with the work of Harris et al. (2008), Cho et al. (2008), and Seitz and Angel (2020), who suggested that beliefs can be modified through the validation or rejection of new evidence. Moreover, in solving mathematical problems, students can also utilize mathematical media and technologies as effective tools (Kholid et al., 2022). This finding is further supported by Palavan et al. (2022), who contended that problem-solving requires the processes of reading, comprehension, mathematical knowledge application, and the use of mathematical operations.

The efforts of defragmentation described above have proven effective in addressing conceptual fragmentation at each stage of problem-solving. In addition, the professional development of teachers plays a crucial role in supporting these efforts. Consequently, adopting a realistic and child-centered model of mathematics learning is an especially suitable approach. Setyaningsih et al. (2019) describe realistic learning as a collaborative learning strategy adapted from Realistic Mathematics Education (RME). This approach emphasizes a constructivist methodology that focuses on the learner's capabilities and progress, assessed through their responses to the learning process. Key features of RME include the use of real-world contexts, mathematical modeling, knowledge production and construction by the students themselves, an interactive teaching process, and the integration of various learning tools (Gravemeijer, 1994). This approach helps students understand abstract algebraic concepts and enables them to solve mathematical problems accurately and correctly.

Conclusion

Based on the results of research and discussion presentations, students who experience fragmentation of conceptual understanding will find it difficult to solve mathematical problems of algebra material. Reading the problem repeatedly and substituting the information obtained into formulas can be the average ability to understand concepts seen in students. Fragmentation of translational thinking and fragmentation of meaningless connections occur when students experience errors in making new representations from previously made representations whose purpose is to facilitate solving the problem at hand. So that, defrag it with limited interventions carried out, namely facilitating disequilibration and providing scaffolding assistance, making subjects indicate the occurrence of scheme knitting in understanding concepts and connecting with concepts that are already owned to minimize errors in solving mathematical problems. Further research can find other types of fragmentation and defragmentation efforts in students' implementation for solving mathematical problems based on the results obtained.

This research has several limitations, especially in describing in depth the types of fragmentation of concept understanding and defragmentation efforts that have been carried out on students. This study only covers one type of fragmentation in the context of non-routine mathematical problem-solving, so there is an opportunity for future research to explore more diverse concept fragmentation and more comprehensive defragmentation strategies.

Further research is expected to identify additional types of fragmentation that students may experience in the context of mathematics learning, as well as develop more effective and diverse methods of defragmentation. By focusing on various types of mathematics materials or topics, this advanced study is expected to broaden the insight into how the fragmentation of concept understanding impacts students' ability to solve non-routine mathematical problems. These results can significantly contribute to more adaptive teaching strategies and support the development of students' conceptual understanding.

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