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Teachers' Perceptions of Number Concepts and Arithmetic Operations in Elementary Education

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Abstract. *Many elementary school teachers face challenges in conveying the concepts of number and arithmetic operations due to limited conceptual understanding. This study is important because teachers' conceptual frameworks have a significant influence on how students develop mathematical understanding. The objective of this study is to explore and understand elementary school teachers' subjective experiences in interpreting and teaching number concepts and arithmetic operations. Employing a qualitative design with a phenomenological approach, this study involved 10 elementary school teachers selected through purposive sampling to reflect diverse teaching backgrounds. Data were collected using two instruments: a written test (questionnaire) and in-depth interviews. The written test assessed teachers' understanding of arithmetic symbols (+, −, ×, ÷), while interviews explored their teaching practices, conceptual views, and pedagogical approaches. Data were analyzed using phenomenological thematic analysis to identify themes in teachers' experiences and meaning-making processes. The results revealed a mismatch between teachers' understanding and standard mathematical conceptions, referred to as zone concept image differences (ZCID). The study concludes that teachers often construct a narrow view of number and arithmetic operations, limiting students' opportunities for deeper mathematical thinking. This study suggests the need for targeted professional development to enhance teachers' conceptual understanding and improve their instructional practices.*

Keywords: *arithmetic operations, concept image, number concept, phenomenological approach, teacher understanding*

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Introduction

Mathematics education at the elementary school level plays a crucial role in laying a strong foundation for more complex mathematical understanding in the future. Among the concepts taught at this level, a strong understanding of number concepts (Chin & Pierce, 2019; Douglas et al., 2020; Radford, 2000) and arithmetic operations (Anghileri, 2005; Onal, 2023) are key to students' success in understanding more advanced mathematical materials. Mathematics is a discipline that relies heavily on numbers and calculations, so a deep understanding of number concepts and arithmetic operations is very important. Therefore, mastering these concepts not only helps students perform calculations correctly but also develops good logical and analytical thinking skills, which in turn build higher mathematical problem-solving abilities (Ma, 2020; NCTM, 2010).

According to Der-Ching and Hung-Jin (2019), understanding numbers includes five components, namely: (a) the basic meaning of numbers (understanding the number system as a whole, including integers, fractions, decimals, percentages, and the relationships between them; (b) comparison of a number (being able to recognize and compare numbers, such as $\frac{1}{11}$ is greater than $\frac{1}{15}$); (c) representation of numbers and operations (developing and using various forms of representation to solve problems flexibly, including using pictures or symbolic notation); (d) the effect of an operation on a number (recognizing how basic operations such as addition, subtraction, multiplication, division affect computational results); and (e) assessing the reasonableness of a computational result (using estimation to estimate the results of an operation before performing detailed calculations and comparing them with the results of the calculations).

Primary school teachers have a great responsibility to teach number concepts and arithmetic operations in a constructive, engaging, and comprehensible manner. Recent studies highlight that a teacher's conceptual understanding plays a significant role in shaping how students perceive and internalize mathematical ideas (Schoenfeld, 2019). However, teachers' interpretations and pedagogical approaches to these fundamental mathematical concepts can vary widely, which in turn influences the quality and effectiveness of instruction (Wischgoll & Prediger, 2024). Therefore, this study aims to investigate primary school teachers' understanding of number concepts and arithmetic operations, as well as how these understandings are reflected in their classroom teaching practices.

Many studies have examined elementary school students' performance in mathematics over the past few decades. However, far fewer studies have specifically examined elementary school teachers' understanding of number concepts and arithmetic operations. The few studies that exist suggest that the understanding of mathematics among elementary school teachers and pre-service elementary school teachers is often inadequate. Several studies, including those by Öztürk-Tavşan and İşler-Baykal (2024), Tsao (2004, 2005), and Yang (2007), have found that many teachers exhibit weaknesses in their understanding of mathematics. These studies revealed several key findings, including (a) the misapplication of mathematical rules: many teachers and pre-service teachers tend to misapply mathematical rules due to a lack of in-depth understanding of the underlying concepts. These errors can hurt how they teach the concepts to their students; (b) not understanding the true meaning of mathematical concepts: teachers often teach mathematics procedurally without understanding the underlying meaning of the concepts being taught. This causes students to learn mathematics mechanically without understanding the true concepts; (c) unpreparedness to teach mathematics material: research results indicate that many teachers are unprepared to teach the mathematics subject matter entrusted to them. This includes a lack of understanding of how to relate mathematical concepts to each other and teach them coherently.

This study offers a fresh perspective by exploring how elementary school teachers personally understand number concepts and arithmetic operations, going beyond identifying weaknesses to examining their root causes and impact on teaching. Unlike prior research that has focused mainly on student outcomes or procedural knowledge, this study highlights the importance of teachers' deep conceptual understanding. It contributes to the literature by offering insights that can help improve elementary mathematics instruction.

This study aims to investigate elementary school teachers' understanding of number concepts and arithmetic operations, as well as how these understandings are reflected in their classroom teaching practices. To achieve these objectives, this study was guided by this main research question: How do elementary school teachers understand the concepts of number and arithmetic operations?

Method

This study employed a qualitative research design, utilizing a phenomenological approach, to explore teachers' understanding of number concepts and arithmetic operations. The focus was on capturing the teachers' subjective experiences and the meanings they construct in interpreting and teaching these mathematical ideas. A total of ten elementary school teachers in Probolinggo, East Java, were selected through purposive sampling to ensure variation in teaching experience and educational background. Data were collected through in-depth interviews, allowing participants to reflect on their personal experiences and instructional practices. The data were analyzed using phenomenological thematic analysis, which involved identifying recurring themes and patterns that emerged from the participants' narratives, thereby revealing how individual perspectives shape their mathematical understanding and pedagogical approaches.

To explore teachers' understanding of number concepts and arithmetic operations, this study used written tests and in-depth interviews. The interviews focused on their teaching experiences, conceptual understanding, and instructional approaches. A written test in the form of a questionnaire was also used to give teachers flexibility in responding at their convenience, allowing for more thoughtful and honest answers while reaching a broader sample. The written test aimed to assess teachers' understanding and skills related to the arithmetic symbols $+$, $-$, $\div$, and $\times$. It covered five key components: a) Conceptual Meaning (Kilpatrick et al., 2001); b) Understanding Symbols in Context (Pape & Tchoshanov, 2001); c) Symbol Representation (Ainsworth, 2006); d) Connections Between Concepts (Goldin & Kaput, 1996). This instrument provides a comprehensive view of how well teachers understand and can teach arithmetic concepts, ultimately supporting better mathematics instruction in elementary schools (Zazkis & Liljedahl, 2008). The final version of the instrument is shown in Table 1.

The credibility test in this study was conducted through triangulation of sources and methods. Data were collected from various teachers with different backgrounds and analyzed using in-depth interview techniques and participant observation. Data validity was strengthened by re-checking the respondents (member checking) to ensure that the researcher's interpretation was in accordance with the experiences conveyed. Data consistency was maintained by conducting an audit trail, which recorded in detail each stage of data collection and analysis, allowing for easy tracing back. In addition, dependability was tested by analyzing by several independent researchers to ensure that the results obtained were reliable and consistent. With this approach, the study is expected to provide an authentic and in-depth understanding of teachers' concepts of numbers and arithmetic operations.

The participants in this study were selected from grade 6 elementary school teachers in Probolinggo, Indonesia, who were chosen for their relevant experience and willingness to participate in in-depth, semi-structured interviews. Grade 6 teachers were selected because they typically have extensive experience teaching a broad range of mathematical concepts and can provide insights into how they approach teaching number concepts and arithmetic operations at the elementary level. Additionally, teachers with an Elementary School Teacher Education background were chosen because they possess the foundational pedagogical skills and knowledge needed to teach a wide variety of subjects, including mathematics, to young learners. Their experience in teaching general education subjects across multiple disciplines allows for a holistic perspective on how they understand and implement arithmetic concepts in a classroom setting.

The study involved ten volunteer teachers from public elementary schools in the region. The teachers' demographic information, including sex, age, years of experience, and educational background, is summarized in the Table 2, where each teacher is represented by a code name (e.g., teacher A, teacher B, etc.) to protect their privacy.

Table 2. Demographics of participants in this study

Teacher Code	Sex	Age	Years of Experience	Educational Background
Teacher A	F	32	5	Bachelor's in Elementary School Teaching
Teacher B	M	38	10	Bachelor's in Elementary School Teaching
Teacher C	F	40	12	Bachelor's in Elementary School Teaching
Teacher D	M	45	18	Bachelor's in Elementary School Teaching
Teacher E	F	29	4	Bachelor's in Elementary School Teaching
Teacher F	M	50	20	Bachelor's in Elementary School Teaching
Teacher G	F	34	7	Bachelor's in Elementary School Teaching
Teacher H	M	41	15	Bachelor's in Elementary School Teaching
Teacher I	F	36	9	Bachelor's in Elementary School Teaching
Teacher J	M	33	6	Bachelor's in Elementary School Teaching

All participants hold a Bachelor's degree in Elementary School Teacher Education (commonly referred to as "Pendidikan Guru Sekolah Dasar" in Indonesia), a program focused on preparing teachers for general elementary education. While not specifically from a Mathematics Education department, all participants have completed the required coursework in mathematics as part of their training. Each teacher had a minimum of three years of teaching experience, with the majority having more, and all were actively teaching at the time of the study.

The data collection procedure in this study began with the administration of a written test to 10 elementary school teachers who had been purposively selected. The test was designed to measure their understanding of the concept of numbers and arithmetic operations, covering a variety of questions that tested both procedural and conceptual aspects. Each teacher was given 60 minutes to complete the test under controlled conditions to ensure the accuracy of the results that reflected their true understanding.

Following the written test, the data were analyzed to identify prominent patterns of understanding and errors. Based on this initial analysis, the five teachers who showed the most diverse results, both in terms of understanding and errors made, were selected to proceed to in-depth interviews. These teachers were selected because they demonstrated significant variability in their approaches to teaching mathematical concepts, which allowed for a deeper exploration of their pedagogical perspectives. The selected teachers were representative of different teaching styles and experiences, making them particularly valuable for understanding the various ways teachers perceive and approach the teaching of number concepts and arithmetic operations.

The interviews were conducted using a semi-structured approach to gain a deeper understanding of the teachers' subjective perspectives and experiences in teaching mathematical concepts. During the interviews, the researchers asked open-ended questions to explore the teachers' personal experiences with teaching mathematics, their understanding of key mathematical concepts, and the challenges they face in delivering these concepts to students. Additionally, the interviews aimed to understand how teachers' backgrounds, such as their educational training and years of experience, influenced their teaching practices and conceptions of mathematics. The interview data provided rich, qualitative insights that complemented the quantitative findings from the written test, revealing not only the teachers' strengths and weaknesses but also their attitudes and beliefs about mathematics education.

Data analysis was conducted through the thematic analysis method, where interview transcripts and written test results were examined to find recurring and important themes related to teacher understanding. To ensure the credibility of the analysis results, data triangulation was conducted by comparing findings from written tests and interviews, and by asking participants to provide feedback (member checking) on the initial interpretations made by the researcher. This aims to validate that the results of the analysis truly reflect the experiences and understandings of the teachers who participated in this study.

Results and Discussion

The results of the study showed variations in teachers' understanding of the "+" symbol in conceptual, operational, representational, and inter-conceptual contexts.

Table 3. Elementary school teachers' views on the meaning of the “+” symbol

Component	Category	Code	f
Conceptual Meaning	Addition sign	It's an additional sign	3
		It is used in addition	2
		Combining two or more numbers or objects	1
	To increase	It allows an increase in the number of objects	2
	Total sign	To calculate the total or overall amount	1
	Plus sign	It's a plus sign	1
Understanding Symbols in Different Contexts	Addition sign	Addition of two positive numbers	10
		Addition operation between two fractions	10
		Addition of positive integers to negative integers.	10
Symbol Representation	Agree	Describes the total number of fruits	7
	Disagree	Apples and bananas cannot be added	3
Connections Between Concepts	Addition sign	Shows the result of 3 plus 4×2	6
	Plus sign	Shows that 3 plus 4 times 2	2
	Total sign	Shows that 3 multiplied by the sum of 4 and 2	2

f: teacher frequency

In the context of its use, all teachers demonstrated adequate procedural understanding of the "+" symbol across various number types, including integers, fractions, and positive-negative combinations. When asked, *"How do you explain the '+' symbol when dealing with different kinds of numbers?"* all ten teachers responded consistently. For instance, teacher D stated, *“Adding two positive numbers is easy, like 3 + 5,”* while teacher F explained, *“Even when adding fractions, like 1/2 + 1/4, the '+' is still used the same way,”* indicating that their operational understanding was solid and applicable across contexts.

However, differences emerged in the aspect of symbolic representation. When prompted with *“If I write ‘2 apples + 3 bananas’, can they be added? How would you explain that to students?”* responses varied. Seven teachers, such as teacher B, who said, *“2 apples + 3 bananas still gives you five fruits in total,”* showed a broader interpretation, accepting the symbol as a general representation of combining quantities regardless of object type. Meanwhile, three teachers, like teacher G, disagreed, stating, *“Apples and bananas are different. You can’t add them directly like numbers.”* This indicates limitations in generalizing the "+" symbol to real-world contexts, particularly in interpreting abstract versus concrete representations.

Variations were also apparent in how teachers interpreted mathematical expressions involving multiple operations. In response to *“How would you explain the expression ‘3 + 4 × 2’ to your students?”*, six teachers, like teacher I, correctly applied the order of operations, stating, *"3 + 4 × 2 is equal to 11 because we multiply first."* However, others, like teacher E, showed misconceptions: *"It's 14 because you do 3 + 4 first, then times 2."* These differences highlight varying levels of understanding regarding the hierarchical structure of operations, which may influence how teachers convey complex mathematical reasoning in the classroom.

On a conceptual level, teachers associated the "+" symbol with various meanings. When asked, *"What does the '+' symbol mean to you? How would you define it?"* teacher A responded, *"It's the sign for addition,"* reflecting a procedural view. Teacher C noted, *"It's used to combine things, like adding apples and oranges,"* suggesting an everyday, concrete interpretation. Teacher E stated, *"It means an increase in number; something gets more,"* revealing a more intuitive, functional perspective. These varied explanations align with the earlier written data, suggesting that although teachers share similar procedural knowledge, their underlying conceptual frameworks differ.

Overall, the interview findings reveal that while the basic operational use of the "+" symbol is well understood among teachers, there remain substantial differences in how they interpret its meaning, apply it in contextual situations, and connect it to broader mathematical concepts. These variations could influence how symbolic meanings are conveyed to students, highlighting the need for reflective pedagogical development in teaching fundamental mathematical symbols.

In terms of conceptual understanding, most teachers associated the symbol "–" with subtraction, with 8 out of 10 identifying it as representing the operation of taking one number away from another. When asked, "*What does the '−' symbol mean to you?*" teacher C responded, "*It means to subtract—take away something from something else*," while Teacher F stated, "*It's the opposite of addition*." However, there were variations. One teacher interpreted the symbol as indicating negative numbers, saying, "*To me, '−' also means less than zero, like a loss or debt*," and another teacher referred to it as a difference symbol, suggesting a more relational meaning, as in "*the distance between two values*."

In the context of its use, all teachers demonstrated a consistent procedural understanding of subtraction. When asked, "*How do you explain subtraction in different situations?*" all ten teachers gave responses indicating comfort with the operation. For instance, teacher A said, "*Subtraction is taking away, whether it's 7 – 2 or 10 – 4, it's the same logic*," and teacher H added, "*Even when teaching with pictures, like 5 apples minus 3, the meaning stays the same*." Despite this, differences emerged when discussing cases where subtraction results in negative numbers. In response to "*How do you explain 5 – 7 to your students?*", most teachers recognized the possibility of a negative result. Teacher G noted, "*It gives -2, and that means you're going below zero*." However, two teachers incorrectly responded, *"You can't subtract a bigger number from a smaller one*," showing limited understanding of negative values in subtraction.

Table 4. Elementary school teachers' views on the meaning of the “−” symbol

Component	Category	Code	f
Conceptual Meaning	Subtraction Symbol	Shows the subtraction operation between two numbers	8
	Negative Symbol	Shows negative numbers	1
	Difference symbol	Difference between two values	1
Understanding Symbols in Different Contexts	Subtraction Symbol	Shows the subtraction operation between two numbers	10
	Minus Symbol	Less or minus	10
Symbol Representation	Subtraction Symbol	Shows the subtraction operation between two numbers	10
	Minus Symbol	Less or minus	10
Connections Between Concepts	Negative Symbol	-2	9
	Minus Symbol	2	1

f: teacher frequency

Regarding symbolic representation, all teachers could accurately relate the “−” symbol to conventional mathematical notation, such as horizontal equations and number line models. However, when asked to distinguish the subtraction symbol from a negative sign, responses varied. Teacher D said, “*They look the same, but when it’s in front of a number like -5, it means the number is already below zero*,” while teacher I admitted, “*I sometimes treat them as the same, especially when I explain quickly to students*.” This indicates that although the procedural use of the “−” symbol is consistent, the distinction between subtraction and negativity may not be fully conceptualized.

On a deeper level, when asked, "*What does ‘−2’ mean in your view?*”, 9 out of 10 teachers correctly interpreted it as a negative number, reflecting an awareness of its role in expressing values less than zero. However, one teacher misinterpreted the expression, stating, “*It just means two, but with a minus, so I treat it as a small number,*” which may lead to confusion when teaching operations involving negative values. These findings align with earlier research (Ball et al., 2008; Selter et al., 2012), showing that educators may possess strong procedural fluency while still holding fragmented or intuitive conceptual models.

Overall, while most teachers demonstrated an adequate operational understanding of the "−" symbol, especially in subtraction contexts, differences in interpretation emerged regarding symbolic meaning, negative values, and the distinction between operational and relational uses. These variations may affect how students conceptualize subtraction and negativity, reinforcing the need for targeted professional development that emphasizes the multifaceted nature of mathematical symbols.

In the case of the “×” symbol, teachers demonstrated a reasonable understanding of its basic conceptual meaning, with most associating it with multiplication. The majority of teachers, specifically 3 out of 10, identified the “×” symbol as representing the multiplication operation between two or more numbers. For example, teacher A remarked, "*It's the multiplication sign, like 2 × 3 gives you 6*," while teacher D stated, "*It's used for multiplying numbers*." This aligns with the conventional understanding of the symbol as a tool for multiplying quantities.

Table 5. Elementary school teachers' views on the meaning of the "×" symbol

Component	Category	Code	f
Conceptual Meaning	Multiplication Symbol	Shows the multiplication operation between two or more numbers.	3
		Shows repeated addition	2
		Combine two numbers into one product	2
	Times symbol	Indicates that two or more values are multiplied together	3
Understanding Symbols in Different Contexts	Multiplication Symbol	Shows the multiplication operation between two or more numbers.	5
		Shows repeated addition	5
Symbol Representation	Multiplication Symbol	Shows the multiplication operation between two or more numbers.	5
		Shows repeated addition	5
Connections Between Concepts	Multiplication Symbol	Shows the multiplication operation between two or more numbers.	10

f: teacher frequency

However, some teachers showed alternative conceptualizations. Two teachers associated the "×" symbol with repeated addition, explaining it as "*adding the same number multiple times*," which is consistent with the traditional teaching of multiplication as a process of repeated addition. Teacher F stated, "*Multiplication is just repeated addition like 4 × 3 is the same as 4 added three times*," which reflects a more foundational arithmetic interpretation. Additionally, two other teachers linked the "×" symbol to the idea of combining two numbers into a single product, echoing an understanding that multiplication results in a combined outcome rather than just repeated additions.

When asked, "*How do you explain multiplication to your students in different contexts?*" responses were generally consistent. All ten teachers, for instance, were able to explain multiplication as the operation between two or more numbers, such as "5 × 3 = 15" or "2 × 4 = 8." For example, teacher E explained, "*When you multiply, you take groups of a number and combine them, like 3 × 2 means adding 3 two times*." Similarly, teacher I said, "*Multiplying is just adding the number over and over*." These responses indicate a strong procedural understanding of multiplication.

Despite this consistency, there was a notable variation in how teachers conceptualized the use of the "×" symbol in different mathematical contexts. When asked about using "×" with different types of numbers, five teachers mentioned that multiplication could be viewed as repeated addition. However, only five also correctly identified it as the multiplication operation itself. This suggests that although all teachers understand multiplication in basic terms, there may be some confusion in applying the concept to more advanced contexts, such as with fractions or larger integers (Ardiansari et al., 2024).

Moreover, when discussing symbolic representation, there were consistent interpretations of the symbol across different contexts. All teachers agreed that the “×” symbol represents the multiplication of numbers. However, teacher G pointed out that students often confuse the “×” symbol with the addition symbol, especially when dealing with simple numbers. Teacher G stated, “*Young students sometimes mistake the ‘+’ and ‘×’ signs, thinking that multiplying is just like adding but with bigger numbers*.” This illustrates the challenge of teaching young learners to differentiate between these fundamental symbols.

In terms of connecting concepts, all teachers showed a unified understanding of the multiplication operation itself, agreeing that the “×” symbol represents multiplying two or more numbers together. There was no significant disagreement in their interpretation, which aligns with previous findings that show multiplication is well-understood at the basic level among educators. However, as with previous symbols, there was room for improvement in the more abstract interpretations and application of multiplication concepts in different scenarios.

Overall, while primary school teachers demonstrated a strong procedural understanding of the "×" symbol, differences in conceptualization emerged, particularly in their understanding of multiplication as repeated addition and in its application across various contexts. These differences underscore the importance of refining teachers' conceptual knowledge and the need for professional development that reinforces the deeper meanings and uses of mathematical symbols in teaching.

In terms of conceptual understanding, teachers demonstrated varied interpretations of the "÷" symbol. Four teachers identified it as a division symbol, explaining that the first number is divided by the second number to yield a result that is consistent with the standard mathematical operation of division. For example, teacher B stated, “*The ‘÷’ sign means you divide the first number by the second one*.” In contrast, two teachers associated the "÷" symbol with a ratio, describing it as representing a comparative relationship between two values or quantities. Teacher D explained, “*It’s a way to compare two things, like how many times one number fits into another.*” Additionally, four teachers interpreted the symbol as a separator, representing the division of a whole into smaller parts. Teacher G elaborated, “*It's like separating a cake into equal pieces; each piece is a part of the whole*.”

When considering the symbol’s use across different contexts, all but one teacher consistently identified the "÷" symbol as a separator in mathematical expressions. For instance, when asked, “*How would you explain the symbol ‘8 ÷ 2’ in real life?*" nine teachers responded by referring to the "÷" symbol as separating a total into smaller units. Teacher, I remarked, "*If you* *divide 8 apples between 2 people, each one gets 4 apples, so it's like separating the total into equal parts*." However, one teacher failed to apply this interpretation, stating, "*It's about how many times 2 fits into 8*," demonstrating a misunderstanding of the symbol's broader conceptual use.

Table 6. Elementary school teachers' views on the meaning of the "÷" symbol

Component	Category	Code	f
Conceptual Meaning	Division Symbol	States that the first number is divided by the second number to get the result	4
	Ratio Symbol	Implies a comparative relationship between two values or quantities	2
	Separator Symbol	Shows the separation or division of one unit into smaller parts.	4
Understanding Symbols in Different Contexts	Separator Symbol	yes, the same two things	9
	Division Symbol	No	1
Symbol Representation	Division Symbol	States that the first number is divided by the second number to get the result	10
Connections Between Concepts	Division Symbol	States that the first number is divided by the second number to get the result	10

f: teacher frequency

Regarding symbolic representation, all ten teachers correctly associated the "÷" symbol with division in numerical expressions. For example, teacher F noted, "*In 12 ÷ 3, it simply means 12 divided by 3 equals 4*," confirming that teachers possess a solid procedural understanding of the symbol in isolation. Despite this, the interpretations of the symbol's conceptual meanings varied, with some teachers emphasizing different contexts or applications, such as ratios or separations. Furthermore, when asked about the connections between different mathematical concepts, all teachers stated that the "÷" symbol represents division, reinforcing their procedural understanding. However, while some teachers could see the link between division and ratios (such as 6 ÷ 2 representing the ratio of 6 to 2), the concept of separation was also commonly applied when discussing tangible examples involving whole numbers.

Overall, while the teachers generally demonstrated a strong grasp of the division operation represented by the "÷" symbol, the diversity in their interpretations highlights the need for greater emphasis on the various contexts in which division, ratio, and separation can be applied. These differences suggest that some teachers may benefit from deeper conceptual training to better differentiate between the multiple meanings of the "÷" symbol, ultimately improving their ability to convey these distinctions to students.

To provide a visual representation of the variation in teachers' understanding of basic arithmetic symbols, Figure 1 presents a comparison of the number of teachers who demonstrated conceptual and procedural understanding of the symbols "+", "–", "×", and "÷".

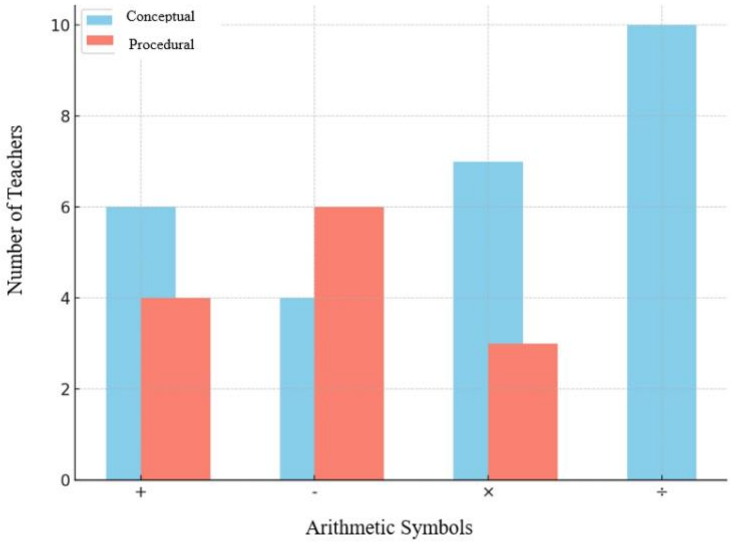

Figure 1. Variation in teachers' understanding of arithmetic symbols

The figure above shows that the symbol "÷" has the highest level of conceptual understanding (10 out of 10 teachers), while the symbol "−" tends to be understood more procedurally. The symbols "+" and "×" show a more balanced distribution between conceptual and procedural understanding. This suggests significant variation in how teachers interpret each symbol, which may impact how they teach arithmetic operations to students.

The next explanation critically analyzes the study's findings and situates them within the broader context of mathematics education and theory, responding to the research questions. The results highlight key challenges faced by elementary school teachers in their understanding and teaching of number concepts and arithmetic operations. Teachers primarily associate number concepts with basic counting and operations but struggle with abstract ideas such as fractions and irrational numbers (Aksoy & Yazlik, 2017; Sibomana et al., 2022; Unaenah et al., 2024). This finding highlights a gap between the procedural understanding that teachers often emphasize and the conceptual depth required for effective teaching. This gap aligns with the notion of the zone of concept image differences (ZCID), which highlights the discrepancy between teachers' intuitive or practiced understanding of mathematical symbols and operations and the scientifically accurate conceptions expected in formal mathematics education (Suryadi, 2019; Sulastri et al, 2022).

The study's results align with existing literature on the challenges of teaching foundational mathematics, particularly the tension between procedural and conceptual understanding (Ball et al., 2008). Many teachers focus on rote memorization and computational skills, which may stem from standardized testing pressures that prioritize accuracy over understanding (Boaler, 2016). This is supported by Skemp's (1976) distinction between instrumental and relational understanding, where many teachers remain within the former approach, focusing on methods rather than fostering deeper mathematical thinking.

The study also identifies the difficulty teachers face in explaining abstract concepts, such as fractions, which are often viewed as disconnected from whole numbers. This difficulty is consistent with Ma's (2020) findings on the limited depth of teachers' mathematical understanding. The reliance on visual aids and manipulatives reported by teachers suggests a need for tangible tools to bridge conceptual gaps, echoing Li's (2020) emphasis on the importance of hands-on learning materials in mathematics instruction.

Furthermore, teacher training emerged as a significant factor in teachers' mathematical knowledge. Variations in teacher preparedness highlight the crucial role of pre-service education in shaping teaching effectiveness. Ball et al. (2008) have emphasized the importance of specialized content knowledge for effective teaching, underscoring the need for ongoing professional development and targeted training to enhance teachers' mathematical expertise. Sulastri et al. (2022) also stated that the basic concepts of the material need to be presented in depth to minimize misconceptions.

These findings align with trends observed in recent studies on arithmetic instruction. For example, research by Kusmaryono (2019) revealed that elementary school teachers often experience misconceptions in teaching mathematics, which can lead to misunderstandings in students and reinforce the cycle of inadequate mathematical knowledge. However, this study adds a new phenomenological dimension by focusing on teachers' lived experiences, providing deeper insights into the emotional and personal challenges they face in the classroom. In contrast to the general trend reported by Fitriani et al. (2023), which emphasized the importance of mathematical content knowledge in teaching, the qualitative approach employed in this study reveals how individual experiences and contextual factors influence teaching practices.

This study has certain limitations. The relatively small sample size and the limitation to a specific geographic and institutional context may affect the generalizability of the findings. Furthermore, because the data rely heavily on self-reported narratives through interviews and written responses, there is a possibility of bias, such as social desirability or selective recall. These factors may have influenced the way teachers articulated their understandings and experiences. Future research could expand the scope by including larger and more diverse samples, as well as integrating classroom observations to triangulate data and enrich findings.

Conclusion

This study examined how elementary school teachers comprehend the concepts of numbers and arithmetic operations, including addition, subtraction, multiplication, and division. The findings reveal significant variations in teachers' conceptual understanding, with many diverging from scientifically accepted meanings of mathematical symbols such as '+', '-', '×', and '÷'. These differences are not merely factual errors but reflect deeper conceptual gaps. A major finding is the emergence of the zone of concept image differences (ZCID). In this phenomenon, teachers' intuitive or experience-based interpretations of symbols conflict with their formal, scientific definitions.

The ZCID is evident in various situations, such as the incorrect symbolic translation of word problems and limited understanding of abstract mathematical operations. For instance, interpreting “3 apples and 4 bananas” as 3a + 4b or viewing the division of fractions merely as a procedural step illustrates how everyday reasoning can misguide instructional practices. Teachers also struggled to distinguish the dual roles of the minus sign, showing confusion between subtraction and negative numbers. These examples underscore the importance of bridging the gap between intuitive and scientific conceptions to facilitate accurate and meaningful mathematics instruction.

This study highlights the importance of enhancing teacher education by emphasizing conceptual understanding, rather than just procedural fluency. Professional development should focus on helping teachers critically reflect on their mathematical thinking and align it with formal principles. Future research should investigate how sustained, research-based training programs can enhance teachers' conceptual understanding and positively impact student learning. Longitudinal studies are especially recommended to evaluate how shifts in teacher understanding affect students' mathematical achievement over time.

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Table 1. The number concepts and arithmetic operations study form

Symbol	Component	Question	Symbol	Question	Symbol	Question	Symbol	Question
	Conceptual Meaning	What does the symbol mean?		What does the symbol mean?		What does the symbol mean?		What does the symbol mean?
	Understanding Symbols in Different Contexts	Explain the different contexts of using the "+" symbol in the following operations: a) 7+5 b) $\frac{3}{5}+\frac{2}{5}$ c) 5 + (-7)		Explain the meaning of the "-" symbol in each of the contexts: a) 10 - 4 b) 8 + (-3) c) 5 - (-3)		What is the meaning of the "×" symbol in each of the following operations? a) 3×4 b) -2×3 c) $\frac{1}{2}\times\frac{1}{3}$		Are 1÷ 7 and $\frac{1}{7}$ the same thing? Explain.
+	Symbol Representation	Do you agree that 3a+3b is the translation form of the sentence "3 apples" and "4 bananas"?	−	Write an arithmetic operation by using this symbol and find your result.	×	How do you represent 3×6 in different ways (array, equal group, repeated addition, or number line)?	+	Write an arithmetic operation by using this symbol and find your result.
	Connection Between Concepts	In the following mathematical statements, which is the correct use of the symbol "+" as an "addition symbol"? $3+4\times 2$ a) Shows the result of 3 plus 4×2 b) Shows that 3 plus 4 time 2 c) Shows that 3 multiplied by the sum of 4 and 2 d) Shows that 3 and 4 added together equal 2		What is the correct value to fill in …… in the following equation? 118-…..=120		Which one is correct? Explain why: $a^2\times a^3=a^5$ or $a^2\times a^3=a^6$		Is it true or false? Give reasons. $\frac{1}{2}\div\frac{3}{4}=\frac{1}{2}\times\frac{4}{3}$