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Hypothetical Learning Trajectory on Rotation Topic Using Photography Context

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Abstract. *The concept of rotation in geometric transformations is significant because it is frequently applied in various aspects of real life. However, many students encounter difficulties in understanding the material related to rotation. This research aims to design a learning trajectory on a rotation topic using photography context for junior high school students. The method used in this study is design research, comprising three stages: the preliminary design stage, the design experiment stage, and the retrospective analysis stage. This research employed the Indonesian Realistic Mathematics Education (PMRI) learning approach. The hypothetical learning trajectory (HLT) design is created through learning activities set in a photography context. The research subjects were Grade 9 students from a junior high school in Palembang, Indonesia. Sampling was done using the purposive sampling technique. Data was collected by documentation, observation, and interviews. The study's results showed that the HLT design effectively stimulated students to explain the concept of rotation and its properties, determine the results of rotation at specific coordinates, and solve simple contextual problems related to the rotation topic. Notably, 55.6% of students obtained test results in the very good category.*

Keywords: *design research, hypothetical learning trajectory, photography, rotation*

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Introduction

Geometry is one of the important subjects in mathematics education. According to Clements (2003), learning geometry is crucial because it helps students develop spatial thinking and visualization skills, which are essential in everyday life and various fields of work in the future. Understanding shapes, utilizing visualization, and solving problems are the primary goals of learning geometry (NCTM, 2000). Geometry learning is also aimed at developing students' logical thinking skills gradually through levels of geometric thinking, as well as sharpening their spatial intuition. This ability is important because it helps students understand shapes, positions, and relationships between objects in space, which can be applied in everyday life (Van Hiele, 1986).

One of the topics in learning geometry is geometric transformation. Geometric transformation refers to the movement of a geometric shape from one position to another without altering the shape itself (Umam, 2023). Geometric transformation illustrates how shapes change their position and size according to specific rules that occur in our lives (Olivia, 2021). The topic of geometric transformations includes translation, reflection, rotation, and dilation. The concept of rotation in geometric transformations is significant because it is frequently applied in various aspects of real life. However, many students encounter difficulties in understanding the material related to rotation.

The challenges faced by students in learning geometry stem from their inability to visualize the change of an object from its initial position to its shadow position without a concrete representation (Wasi, 2022). Students often lack a visual representation of the transformation process (Taihittu et al., 2021). Additionally, based on their study, Sari and Putri (2022) demonstrated that students' reasoning abilities regarding the topic of rotation remained low. This is because instruction remains centered on the teacher without addressing students' needs for abstraction, which prevents them from thinking abstractly (Ma'rifah & Qohar, 2020). Students tend to memorize formulas when solving problems, rendering the learning process irrelevant (Evidiasari et al., 2019). Furthermore, the learning of geometric transformation material lacks connections to everyday life, and in the context of learning about rotation, students have not utilized a specific mathematical context (Sari & Putri, 2022).

To address these issues, the PMRI approach is an effective solution because it emphasizes the importance of contextual and meaningful learning. PMRI stands for Indonesian Realistic Mathematics Education. It is an adaptation of the Dutch Realistic Mathematics Education (RME) approach, tailored to the Indonesian context (Sembiring, 2010). The PMRI approach is based on RME, which originated in the Netherlands and focuses on how mathematics should be learned and taught (Kerekes, 2005). The principles of RME and PMRI are strongly influenced by Freudenthal's concept of "mathematics as a human activity" (Drijvers, 2003).

PMRI is a learning approach that begins with real problems to guide students in understanding mathematical concepts. Mathematics must be connected to reality and relevant to students so that they experience it as a human activity (Freudenthal, 1991). PMRI emphasizes meaningful learning by involving students in contextual problems related to daily life, allowing them to rediscover mathematical concepts (Zubainur et al., 2020). This aligns with Gravemeijer's perspective (1994), which states that the starting point of learning should be a problem situation that is meaningful and grounded in real-life experiences for students.

PMRI has five characteristics of mathematics learning: (1) using context, (2) vertical mathematization, (3) student contribution, (4) interactive activities, and (5) relevance to mathematics topics (Zulkardi, 2002). Thus, PMRI is an approach to mathematics learning that begins with contextual problems. It is a learning approach that starts from real-life situations for students, focusing on mathematical activities through discussion, collaboration, and argumentation, allowing them to discover and apply mathematics in solving individual and group problems (Widyastuti & Pujiastuti, 2014). Cognitive development is more meaningful when it is based on real experiences rather than solely on the language used for communication (Rifa'i & Anni, 2012). Through the PMRI approach, teachers encourage students to explore, discover, and construct their understanding of mathematical concepts. By implementing PMRI, mathematics learning can become more meaningful, engaging, and effective, thereby enhancing students' ability to apply these concepts in everyday life.

In the implementation of PMRI, teachers play a crucial role as learning designers to ensure that the learning process aligns with realistic principles. One tool that supports this planning is the hypothetical learning trajectory (HLT). HLT is a prediction related to how students think and understand during learning activities (Simon, 1995). HLT was first introduced by Simon, who stated that it consists of learning objectives for students, planned activities, and assumptions about learning activities in the classroom (Ilma, 2012; Simonson, 2006; Susanto et al., 2024). HLT, as part of the PMRI approach, is designed to guide the student learning process from the informal stage to the formal stage. These stages include (1) the informal stage, where students solve problems based on everyday experiences; (2) the "model of" stage, where students begin to create representations of contextual activities; (3) the "model for" stage, where they use models as more general mathematical thinking tools; and (4) the formal stage, where students achieve abstract understanding and can use formal notation and procedures (Bakker & Van Eerde, 2015; Gravemeijer, 2004; Kieran et al., 2020; Nguyen, 2022). This enables teachers to plan appropriate interventions in teaching and learning activities (Simon, 1995). By developing HLT, teachers can help students navigate from contextual experiences to formal understanding through a meaningful modeling process (Gravemeijer, 2004). Teachers should create learning plans that align with the Health and Learning Theory (HLT). Before developing a learning plan, teachers must understand the characteristics of their students, prerequisite knowledge, thinking strategies, cognitive levels, and activities that can help students enhance their thinking skills (Putra & Vebrian, 2019). In other words, teachers can use HLT to design learning processes that improve student learning outcomes (Huang et al., 2018).

According to Cárcamo Bahamonde et al. (2017), HLT is a valuable tool for developing effective mathematics teaching materials. HLT presents a structured approach to designing learning goals and facilitating gradual knowledge growth for students. It focuses on developing students' thinking and differentiating instruction based on prior knowledge (Gu et al., 2017). HLT also encourages reflection, which allows teachers to evaluate whether students truly understand the concepts about the objectives, and revision, which provides an opportunity to refine learning approaches and activities based on students' actual responses.

HLT is used as a guide to design learning activities in stages and a contextually relevant manner. One context that is relevant and close to students' lives for rotation material is photography. Photos can be a valuable tool for enhancing teaching and learning in a variety of subjects. They can provide visual aids to support learning and encourage critical thinking and analysis for students (Oksanen et al., 2024). Photography has become an integral part of everyday life for many young people, particularly in terms of taking and sharing photos using mobile phones and apps (Niemela-Nyrhinen & Seppanen, 2023). Photography, as an artistic expression in all its aspects, often utilizes mathematical principles. Composition, perspective, and lighting involve concepts of geometry, proportion, and calculation that are frequently associated with learning mathematics. Photography skills in education are beneficial for providing concrete experiences that help students build a conceptual understanding of mathematical material (Lembang & Wijiastuti, 2024). In other words, the use of photography skills influences the learning outcomes of children with special needs in mathematics. Visual language and the ability to form new ethical perspectives on reality are rooted in photography. The visual-spatial dimension in expressive photography refers to the idea of creating a form of expression of feelings in response to the visualization of spatial distance, achieved through the use of framing as a symbol of distance from oneself, others, and the environment. The development of framing elements in the embodiment of works serves as a form of visualization (Agung et al., 2017).

Research on learning using photography has been conducted by Adawi and Hilf (2023), whose research results show that the use of photographic images (photos) of mathematical objects in geometry learning is a means to increase student involvement and improve their learning experience. The integration of photography in teaching and learning mathematics was carried out by Muñoz Casado (2018), who invited his students to walk around their city while observing geometry in architecture. Students used photography and GeoGebra to create models and study the geometry of buildings (Rizzo et al., 2019). In addition, Sharp et al. (2004) proposed the use of photography to analyze the concept of slope. This research was also conducted by Lembang and Wijiastuti (2024). The results of their research suggest that utilizing photography skills as a learning tool can enhance students' mathematical abilities. Additionally, research utilizing photography has also been conducted for other subjects. The findings of Dinata's research (2016) demonstrated that the use of photographic images can enhance learning outcomes in social studies lessons related to economic activities and natural resources. However, there is no learning trajectory for rotation topics using photography context. Therefore, this study aims to produce an HLT by utilizing photography.

Method

This study employed the design research method, which aims to improve classroom learning practices through an interactive analysis of what occurs in the classroom and how to apply it (Gravemeijer, 2014). There are three main stages in design research: research preparation (preparing for the experiment), design experiments (conducting the design experiment), and retrospective analysis (Gravemeijer & Cobb, 2013; Putri & Zulkardi, 2018; Putri & Zulkardi, 2019). A key component of design research is the development of theories that connect the learning process with the factors that support learning.

The preliminary design stage aimed to develop a series of learning activities and methods for assessing the learning process. The researchers began by studying various literature on photography and several books related to rotation. Subsequently, the researchers designed the HLT, which includes learning objectives, learning activities (including learning implementation plans and student worksheets), and assumptions about students' thought processes as they progress from the informal stage to the formal stage. This serves as a guideline for each learning activity, which is flexible, can be experimentally adjusted, and has been validated through expert review. Photography is used as a starting point because it is closely related to students' daily lives.

At the design experiment stage, the HLT that had been created was tested on six students on November 20, 2024. The students were selected based on information from the mathematics teacher of grade 9 at SMP Bina Warga. They were then divided into two groups, each consisting of one student with high, medium, and low abilities. This arrangement aimed to investigate and predict students' strategies and ideas during the actual learning process. First, during the pilot experiment stage, one of the researchers, acting as the teacher, observed the learning process. Revisions to the HLT were made at this stage to better understand students' mindsets and strategies using the context of photography on the topic of rotation. Second, the teaching experiment stage was conducted in larger groups by the model teacher on November 23, 2024.

At the retrospective analysis stage, the researchers conducted an analysis of the learning process from the point of the teaching experiment stage. The data analysis process, which includes both observation and interview data analysis, involved comparing the results of observations during the learning process with the HLT. After the data analysis was completed, the researchers conducted an in-depth reflection to align the implementation of learning with HLT and identify aspects that needed improvement or development in the next stage.

Results and Discussion

This research yielded results from the HLT design, which included a learning trajectory for the topic of rotation for junior high school students, utilizing photography skills as a starting point in learning activities. This section presents the results of each stage of the research. The results of this study show how to effectively integrate photography skills to support conceptual understanding and student engagement in rotation topics.

Preliminary Design

The result of the preliminary design stage is the HLT design. At this stage, the researchers implemented the initial idea of using the context of photography for the topic of rotation in geometric transformation material for junior high school students. Photography skills were chosen because, according to the study by Lembang and Wijiastuti (2024), the use of photography skills as a medium for learning mathematics can enhance students' mathematical abilities. Additionally, the use of photography skills provides students with concrete experiences, helping them build an understanding of mathematical concepts. The development of the HLT is a crucial part of designing student learning activities. The design cannot be separated from the learning trajectory, which contains a lesson plan for each material to be taught. The learning trajectory represents the flow of thinking and understanding that students will experience during the learning process. In this study, the learning trajectory focuses on understanding the concept of rotation using the context of photography (See Figure 1.). This understanding is further deepened by exploring the relationship between the object's position from the starting point to the endpoint after rotation, utilizing photography skills. Once this understanding is achieved, students will discover the rotation formula for a specific angle and its application in solving everyday problems.

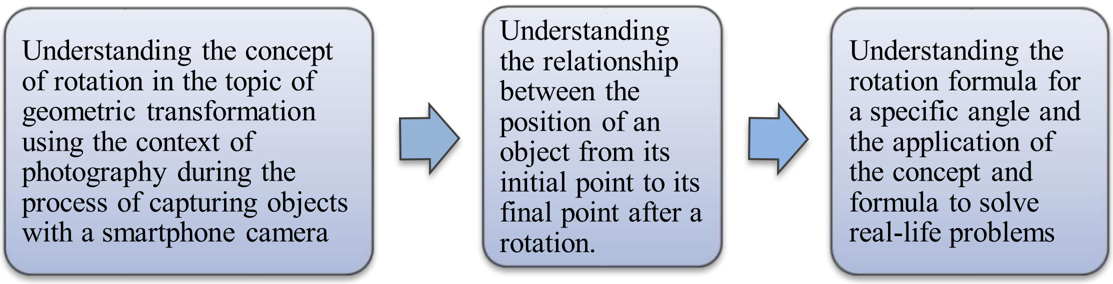


Figure 1. Learning trajectory on rotation topic for junior high school students

A set of activities on the topic of rotation is designed in the HLT based on the learning trajectory and students' cognitive outcomes. This design aims to help students achieve an understanding of the topic of rotation and apply it in everyday life. Figure 2. is an HLT design that connects learning paths, learning activities, and learning outcomes. Based on the HLT explained in Figure 2., a learning trajectory design was developed for the topic of rotation, using the context of photography for junior high school students, illustrated in the form of an iceberg (See Figure 3.).

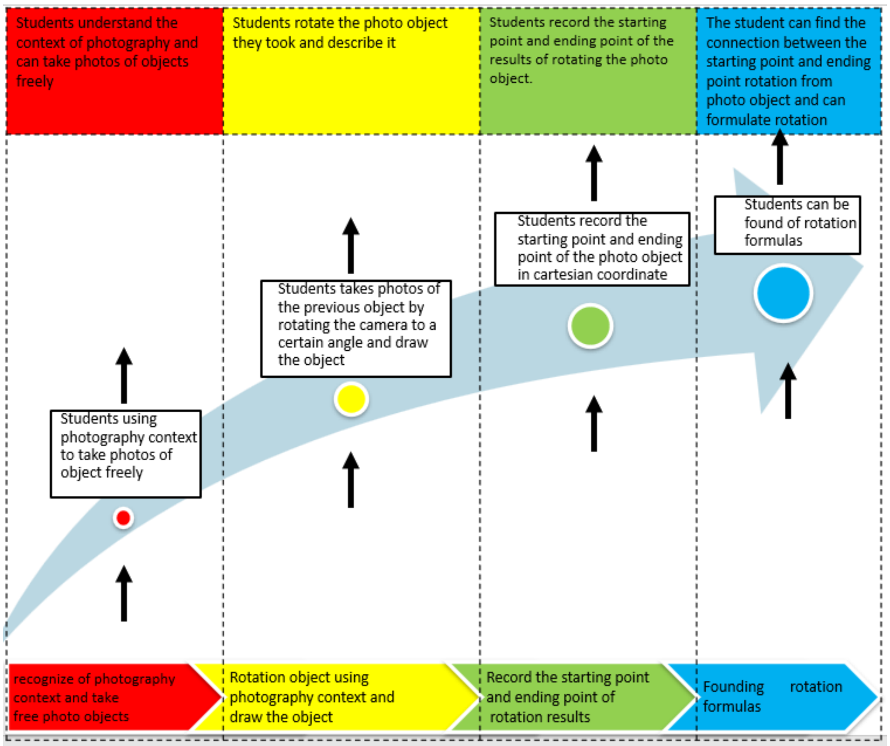


Figure 2. HLT draft on rotation topic

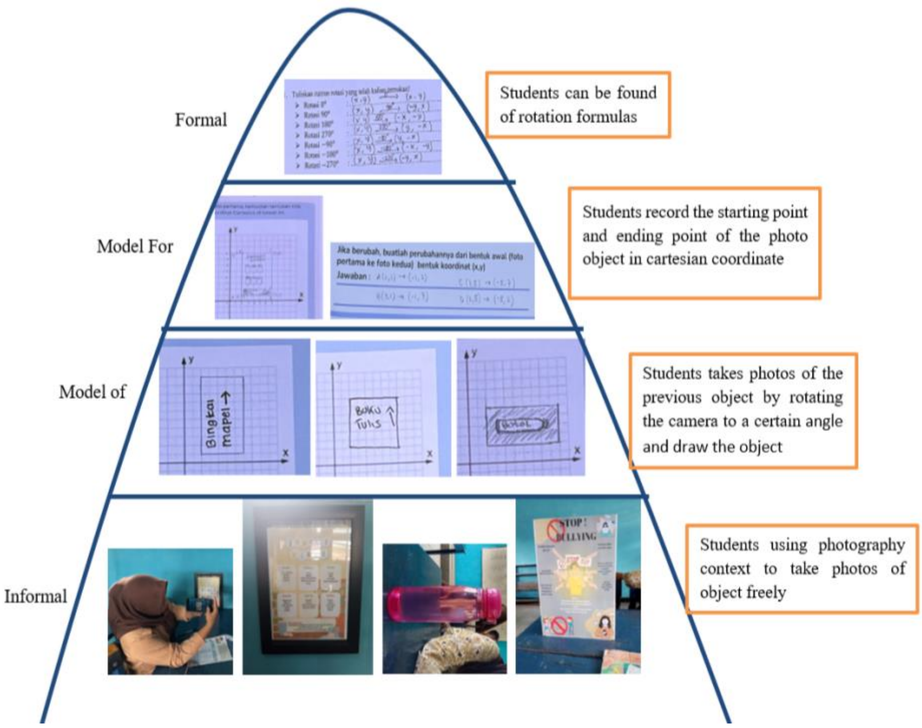


Figure 3. Iceberg on rotation topic for junior high school students

Based on the designed HLT in Figure 3., the conjecture for learning the topic of rotation was obtained, as shown in Table 1.

Table 1. Conjecture of rotation topic using photography context

Level	Activity	Activity Description	Conjecture of students' thinking
Informal	Recognize the photography context and take free photo objects	Students use photography context to take photos of objects freely.	1. Students are familiar with the context of photography and often take photos of objects they encounter in their daily lives. 2. Students are familiar with the context of photography, but they often dislike taking photos of objects they encounter in their daily lives. 3. Students often struggle to use rotation in phone applications. 4. Students are familiar with using rotation in phone applications. 5. Students take photos in the classroom 6. Students take photos outside the classroom
Model of	Rotate an object using a photography context and draw the object	Students take photos of the previous object by rotating the camera to a certain angle and drawing the object.	1. The student takes a photo of an object by rotating the phone camera to the left (counterclockwise). 2. The student takes a photo of an object by rotating the phone camera to the right (clockwise).
Model for	Record the starting point and rotate the result	Students record the starting point and rotation result of the photo object in cartesian coordinates.	1. The student can record the start point and end point by using context photography appropriately. 2. The student cannot accurately record the start and end points using context photography.
Formal	Developing rotation formulas	Students can be found in rotation formulas	1. The student can identify the connection between the starting point and ending point of rotation from the photo object and formulate a corresponding rotation. 2. The student cannot find the connection between the the starting point and the ending point of rotation from the photo object and cannot formulate the rotation.

Design Experiment and Analysis Retrospective

The results of the design experiment and retrospective analysis stages were obtained by conducting experiments with students' worksheets. At this level, the researchers tested the learning trajectory design created for grade 9 junior high school students. Furthermore, a retrospective analysis was conducted on the experimental results obtained during the design experiment stage.

In the pilot experiment stage, the designed HLT was first tested on six grade 9 junior high school students, consisting of two high-ability students, two medium-ability students, and two low-ability students. At this stage, many students were still confused about determining the direction of rotation for positive and negative angles, so the activity description model was revised. Previously, the photo object was rotated by turning the smartphone camera; we then added the use of a mica cover as a rotation aid. The model of the stage which originally contained the activity "students take photos of the previous object by rotating the camera to a certain angle and draw the object", after being revised to "students draw the objects they photograph on the student worksheet, then trace the photo object on the mica cover and rotate it".

After the HLT revision, the next stage is the teaching experiment. This stage was conducted in Grade 9 of a junior high school in Palembang with 27 students, comprising 17 male students and 10 female students. The students were divided into six groups, each with heterogeneous abilities. There are four activities at the design experiment level and are classified into the following stages.

(1) *Informal Level*

In this stage, during the first activity, students freely select objects to photograph. The teacher begins the lesson by asking students questions about photography, such as who enjoys taking photos, what types of objects they usually photograph, what tools they typically use, how long it takes to capture photos for optimal results, and how many objects they usually photograph. Students then mention the types of photo subjects they prefer. Some express a liking for taking photos of themselves, while others enjoy capturing images with people, nearby objects, buildings, scenery, and various outdoor subjects. The teacher then inquires if students are familiar with the term "photography" and whether they consider themselves proficient in photography skills. Furthermore, the teacher provides information about photography to enhance students' understanding of visualization. To establish a common perception among students, the teacher asks them to take photos freely based on their preferences. The photo subjects can be located in the classroom or outside. After the students finish taking photos, the teacher facilitates a discussion on their approaches to capturing the subjects. During this discussion, differing perceptions emerge, as some students opt for landscape photos, while others prefer portrait photos. Some students capture images in full screen, while others use a 1:1 ratio. Additionally, some students focus on their subjects while others take a broader view.

The following is an excerpt from a conversation between students and the teacher that occurred during this activity:

Students	:	*The photos are free. How do I take the photo object? Does it have to be full-screen?*
Teacher	:	*You are free to take the photo object. It can be the entire cellphone screen or, according to the ratio, 1:1, 3:4, or 9:16.*
Students	:	*Can we zoom the photo?*
Teacher	:	*Yes, please enlarge or reduce it.*
Students	:	*If you want to take a landscape position, you can.*
Teacher	:	*: Landscape or portrait positions are allowed as long as the photo object is visible and not cut off.*

The questions expressed by students in the conversation above indicate that they are free to choose any object they like and are not given any restrictions on the scale and position of the photo, whether in vertical or horizontal orientation. This freedom encourages creativity and allows students to explore different perspectives in capturing images. As a result, students become more engaged and motivated to participate actively in the learning process.

Some students took photos of drinking bottles, duty schedules, markers, subject frames, rulers, smartphones, and notebooks. The students reported that when taking photos, they began by positioning the camera as the center point of rotation. The resulting photos were then drawn with a similar shape on the available student's worksheet, and they continued to take the next photo by rotating the smartphone camera at a certain angle. Students who captured landscape photos achieved a horizontal object position, while those who took portrait photos obtained an upright object position. The results of the students' photos are shown in the following figure.

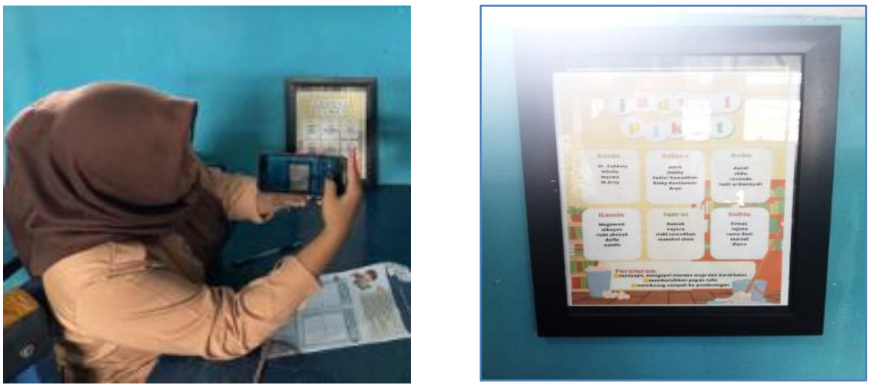


Figure 4. Student activity using photography objects

After the students took photos of freely chosen objects, the teacher and students discussed the techniques for capturing these images. They practiced again to achieve a shared understanding, making it easier to carry out activities at the next level. This activity helps clarify misunderstandings and ensures that all students can apply their photography skills. This activity also fosters a supportive learning environment where students feel comfortable experimenting and asking questions.

At this level, students were accustomed to photography activities because they often engaged in them when they encountered objects they liked. The tools for photography, such as cameras, phone cameras, and iPad cameras, were also familiar and accessible in their daily lives, making it easy for students to visualize the position of the object. However, when taking photos, students still did not realize or understand that they were applying the concept of rotation. This understanding was developed through discussions between the teacher and students regarding the concepts that can be learned from the process of capturing objects through photography activities. Ultimately, students realized that photography skills can be enhanced by rotating the photography tool, in this case, the cellphone camera, in a specific direction and angle.

The following is a discussion between teachers and students about the concepts that can be learned from the process of taking objects through photography activities.

Teacher	:	*What did you take pictures of?*
Student 1	:	*Many. Our group took a photo of the duty schedule.*
Student 2	:	*We took a photo of a smartphone.*
Student 3	:	*Our group took a photo of a drinking bottle.*
Teacher	:	*Wow, it turns out you took different objects. When you took photos, what did you do to get good results?*
Student 2	:	*Sometimes, I climb a chair to get an angle from above.*
Student 3	:	*I tilted the cell phone, so it looked cooler.*
Teacher	:	*Well, that's interesting. When you tilted your cell phone, did you realize that you were rotating your camera?*
Student 3	:	*Yes, I did. But I only thought about making the photo look good.*
Teacher	:	*That's right, but actually, you are applying the concept of rotation. Do you know what rotation is in mathematics?*
Student 1	:	*Rotation is like turning an object, isn't it?*
Teacher	:	*Exactly. In photography, when you change the position of the camera—for example, from vertical to horizontal, or tilting it slightly to the left or right—you are rotating it about an axis. It could be horizontal, vertical, or even diagonal.*
Student 2	:	*So, when I take a photo from below and tilt it slightly to the right, that is also considered rotation?*
Teacher	:	*That's right. You are imagining the position of the object from a different perspective and rotating the camera to match that image. That's part of mathematics.*
Student 3	:	*Wow, I thought it was just a matter of style. Turns out there's a science to it, huh?*
Teacher	:	*That's exactly it. In the professional world, great photographers usually imagine the position of the object first, then set their cameras in certain directions and angles—by rotating—to get the results they want.*
Student 1	:	*So, when I rotate the camera to get a dramatic effect, I'm using the concept of rotation?*
Teacher	:	*Yes! To get good photos, you can experiment by rotating the camera to various angles and see the difference.*
Student 2	:	*So, learning rotation can also be a hobby, can't it?*
Teacher	:	*That's right. There are many mathematical concepts that you can find in the things you like, such as photography.*

(2) *"Model of" Level*

At this level, the students were asked to describe the results of the photo object according to specific rules outlined in the available students' worksheet. At this stage, the students had gained an understanding from the previous activity that capturing a photo object can be achieved by rotating a smartphone camera to a certain angle. In the next activity, the teacher invited the students to agree on how to draw a photo object using a worksheet provided by a student. They traced the previous photo object on the mica cover using a marker and ruler, then attached a pin to the Cartesian coordinate plane with the center point O(0,0) and a specified rotation angle. The students then rotated the mica cover counterclockwise. The same process was repeated for the rotation angles and in both counterclockwise and clockwise directions.

Figure 5. Student activity rotating photo objects

This activity was conducted to enhance students' understanding of the concept of rotation on a Cartesian plane with a specific angle of rotation leading to the resulting position. In this second activity, the students drew a photo object. The object was drawn on the Cartesian coordinates of quadrant I to establish a starting point with a positive value. At this stage, the starting points were determined based on each photo object. One of the starting points was D(2, 8). This starting point was then rotated by a specified angle to obtain a fixed rotation result point, namely the point (2, 8). Next, this starting point was rotated by another specified angle to achieve the desired rotation result, which was the point (-8, 2). Subsequently, point D was rotated by yet another specified angle to obtain the rotation result point, namely point (-2, -8). Finally, point D was rotated by a final specified angle to achieve the rotation result point, which was point (8, -2).

The object was drawn on the Cartesian coordinates of quadrant I to establish a starting point with a positive value.

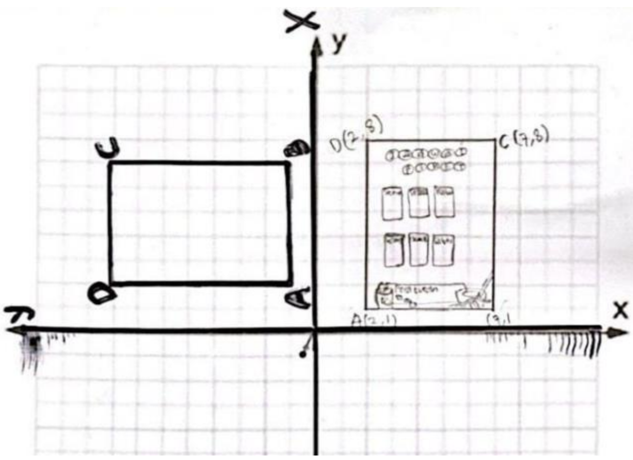


Figure 6. Students results in determining the starting point of an object and rotating it

(3) *"Model for" Level*

At this level, the students recorded the starting points and rotation results in a table provided in the student worksheet. They then replaced the numbers on the coordinate axes with the letter symbols x and y to facilitate finding the relationship between the starting points and the rotation results. Next, the teacher and students discussed the relationship between the two. After that, the teacher instructed the students to draw a photo object using the student worksheet and rotate the object to the center point O(0,0) with a rotation angle of 90^o. The same process was repeated for the rotation angles of $180^o$and 270^o. One of the starting points is A(2,1), B(7,1), C(7,8), and D(2,8). This starting point is then rotated by a specified angle to obtain the rotation result point A'(-1, 2). Next, point D is rotated by a specified angle to achieve the resulting point A"(-2, -1), and finally, point A is rotated by a specified angle to obtain the resulting point A"'(-1, -2). After recording the starting and ending points, students replace the values on the coordinate axes with the letter symbols x and y. This is done to help students easily find the relationship between the starting points and the rotation results.

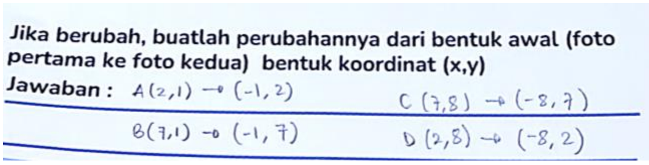

Jika berubah, buatlah perubahannya dari bentuk awal (foto pertama ke foto kedua) bentuk koordinat (x,y)
Jawaban : A(2,1) → (-1,2) C(7,8) → (-8,7)
B(7,1) → (-1,7) D(2,8) → (-8,2)

Figure 7. The result of 90^o rotation

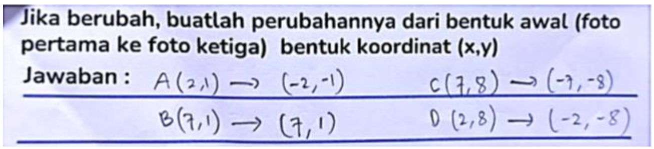

Jika berubah, buatlah perubahannya dari bentuk awal (foto pertama ke foto ketiga) bentuk koordinat (x,y)
Jawaban : A(2,1) → (-2,-1) C(7,8) → (-7,-8)
B(7,1) → (7,1) D(2,8) → (-2,-8)

Figure 8. The result of 180^o rotation

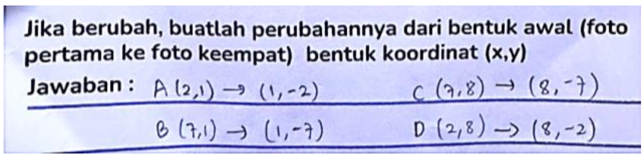

Jika berubah, buatlah perubahannya dari bentuk awal (foto pertama ke foto keempat) bentuk koordinat (x,y)
Jawaban : A(2,1) → (1,-2) C(7,8) → (8,-7)
B(7,1) → (1,-7) D(2,8) → (8,-2)

Figure 9. The result of 270^o rotation

(4) *Formal Level*

At this level, students connected their findings to the student worksheet that had been provided. Following this, a mathematical model was derived based on the established relationship. The teacher then invited students to discuss the mathematical model identified from this relationship. Furthermore, the teacher guided the students to rotate a certain point about point O(0,0) with a rotation angle of 0^o, 90^o, 180^o, and 270^o without the help of photo objects and Cartesian coordinates. The results were compared by rotating the point about point O(0,0) with a rotation angle of 90^o, 180^o and 270^o without the help of photo objects and cartesian coordinates. The same results were obtained for both methods. At the end of the lesson, the teacher encouraged students to formulate mathematical conclusions based on the process of capturing photo objects that had been conducted. The conclusion obtained was the rotation formula centered at point O(0,0) with a rotation angle of 90^o, 180^o and 270^o. The students successfully derived the rotation formulas through the activity of photographing objects, as illustrated in the following figure.

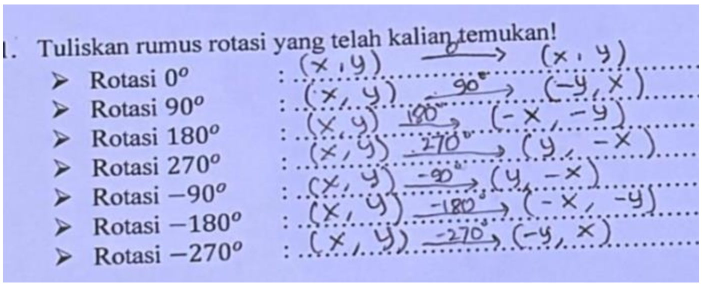

1. Tuliskan rumus rotasi yang telah kalian temukan!
- Rotasi 0^o : (x, y) → (x, y)
- Rotasi 90^o : (x, y) —90°→ (−y, x)
- Rotasi 180^o : (x, y) —180°→ (−x, −y)
- Rotasi 270^o : (x, y) —270°→ (y, −x)
- Rotasi -90^o : (x, y) —−90°→ (y, −x)
- Rotasi -180^o : (x, y) —−180°→ (−x, −y)
- Rotasi -270^o : (x, y) —−270°→ (−y, x)

Figure 10. The results indicate that students successfully found rotation formulas through photography activities

After reviewing the answer to question no. 1, the students answered question no. 2. This question encourages students to deepen their understanding of the geometric transformation concepts involved in rotation. In this question, students identified the similarity of the formula for the results of the rotation of the angle, as shown in the following figure.

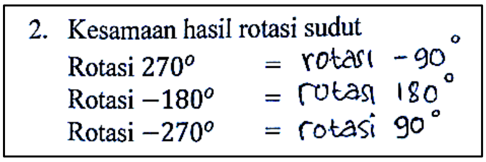
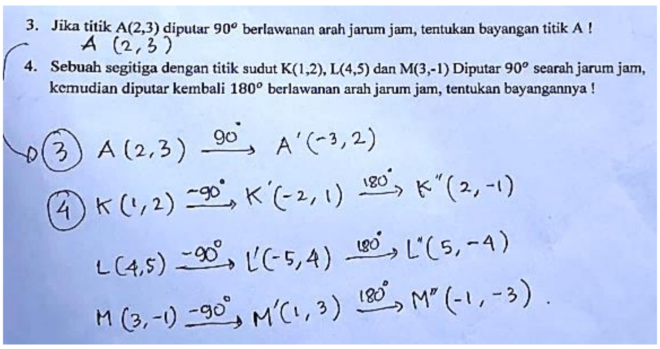

2. Kesamaan hasil rotasi sudut
Rotasi 270^o = rotasi −90°
Rotasi -180^o = rotasi 180°
Rotasi -270^o = rotasi 90°

Figure 11. The results demonstrate that students successfully found the equation formula for the results of angular rotation

The teachers and students solved other problems related to the use of rotation concepts and formulas, as seen in questions 3 and 4 in Figure 12.


3. Jika titik A(2,3) diputar 90° berlawanan arah jarum jam, tentukan bayangan titik A !
4. Sebuah segitiga dengan titik sudut K(1,2), L(4,5) dan M(3,-1) Diputar 90° searah jarum jam, kemudian diputar kembali 180° berlawanan arah jarum jam, tentukan bayangannya !

(3) A (2,3) —90°→ A′(−3,2)
(4) K (1,2) —−90°→ K′(−2,1) —180°→ K″(2,−1)
L(4,5) —−90°→ L′(−5,4) —180°→ L″(5,−4)
M (3,−1) —−90°→ M′(1,3) —180°→ M″(−1,−3).

Figure 12. The result shows that students successfully solved problems related to the concept of rotation

In this final stage, the students understood the concepts of rotation, rotation formulas, rotation angles, and directions of rotation. They were able to apply the concepts and rotation formulas to solve everyday problems. After that, the students were given a test. The test results are presented in Table 2.

Table 2. Student learning outcomes

Score	Category	Amount	Percentage
86 – 100	Very Good	15	55.6%
71 – 85	Good	8	29.6%
56 – 70	Moderate	4	14.8%
40 -55	Poor	0	0%
0 – 40	Very Poor	0	0%
Total		**27**	**100%**

The average score of student learning outcomes was 89.48, which fell into the very good category. The results of the student tests showed that 55.6% (15 students) achieved a very good category, 29.6% (8 students) achieved a good category, and 14.8% (4 students) achieved an adequate category. These results indicate that most students have successfully mastered the material, which reflects the effectiveness of learning using the context of photography.

Based on the description above, it can be concluded that the use of HLT helped determine the learning trajectory of students in understanding the concept of rotation. This finding aligns with various studies that employed the design research method, including rotation learning designs using batik and Ferris wheel motifs (Ismail et al., 2021), rotation learning designs using kawung batik motifs (Risdiyanti & Prahmana, 2018), rotation learning designs using Palembang tanjak (Hasanah et al., 2024), and geometric transformation learning designs using batik cloth motifs (Mufti & Aziz, 2024). From the results of this study, it can be concluded that the use of HLT effectively determined the learning trajectory of students in understanding the concept of rotation.

Through this HLT, students explored, discovered, and constructed their understanding of the concept of rotation, guided by the teacher. The selection of a photography context was a student activity that was very close to everyday life. The use of photographic images (photos) of mathematical objects in geometry learning is a means to increase student engagement and enhance their learning experience (Adawi & Hilf, 2023). The photography context also improved students' mathematical skills (Lembang & Wijiastuti, 2024). Furthermore, according to Dinata (2016), the photographic image method improved student learning outcomes. Learning within this context was a characteristic of PMRI, which enabled students to understand mathematical concepts in depth through real-world experiences and connect them with other mathematical concepts (Rangkuti et al., 2024; Zulkardi, 2002). This finding aligned with research conducted by PMRI, which indicated that one approach to the learning stages included various activities related to the real world (Yulia et al., 2022). Mathematics learning was presented in real-life contexts or within the students' environment (Diva et al., 2022). In PMRI learning, activities centered on students involved the process of searching, finding, and forming new knowledge that began through real conditions in the students' environment (Ramadhan et al., 2022).

Conclusion

The results of this research indicate that the photography context can be used in learning geometric transformations, particularly regarding the topic of rotation. The use of photography provides concrete experiences for students, who no longer have difficulty imagining the change of an object from its initial position to its shadow position, as they already have a clear visual image. This makes understanding the concept of rotation easier. The activities conducted are related to daily life, which helps students solve everyday problems.

This research is limited to the development of HLT rotation material. Further studies are needed on the development of HLT with a photography context for other mathematical topics, such as other geometric transformations (translation, reflection, and dilation), similarity, flat and spatial shapes, and three-dimensional concepts. These future explorations are expected to broaden the scope and effectiveness of photography-based learning in enhancing students’ mathematical understanding.

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