

Strengthening Mathematics Teachers' Subject Matter Knowledge for Teaching Critical Thinking through the SUPER-LS

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Abstract. Teaching critical thinking in the mathematics classroom is essential yet challenging. Mathematics teachers need a strong foundation of subject matter knowledge (SMK) to be able to design and conduct a mathematics lesson that focuses on critical thinking. However, this knowledge was found to be limited among the teachers. One of the collaborative learning platforms that can be used to facilitate teacher to strengthen their SMK is a school-university partnership mediated by the lesson study (SUPER-LS) program. Thus, this study aims to strengthen mathematics teachers' SMK for teaching critical thinking through the SUPER-LS program. The qualitative case study design with embedded units was used in this study, involving 10 in-service teachers (ISTs) and 59 students. Lesson study artefacts were employed to collect data and analyzed using transcript-based lesson analysis and performance assessment of the critical thinking scale. The study discovered that the SUPER-LS program fostered collaboration among ISTs, leading to improvement of SMK, with specialized content knowledge (SCK) emerging as the most dominant component. In addition, students' proficiency in each indicator of critical thinking ranged from level 1 to level 4, with most students at level 2. The findings call on education stakeholders to sustain the SUPER-LS program to improve students' critical thinking performance.

Keywords: school-university partnership, lesson study, critical thinking, learning community, subject matter knowledge, mathematics teachers

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Introduction

In response to global demands, international testing standards, and national priorities, the Indonesian government updated the 2013 curriculum by introducing the "Kurikulum Merdeka." This revision addresses 21st-century skills, national and global citizenship, and employability concerns. According to the 'Kurikulum Merdeka, secondary mathematics instruction aims to equip students with a range of abilities, including mathematical comprehension, reasoning, problem-solving, communication, presentation, connection-building, and positive mathematical dispositions, while also emphasizing critical thinking as one of the key Pancasila characters. These competencies are developed through project-based learning, where students collaborate to critically and creatively solve mathematical problems. Ultimately, the revamped

mathematics curriculum is designed to foster a comprehensive set of 21st-century skills (BSKAP, 2024).

Incorporating critical thinking into the mathematics curriculum necessitates teachers to have robust mathematics teaching capacities to effectively foster these skills in the classroom. Ensuring that future educators possess a solid foundation in mathematics is crucial, as a teacher's competency is directly linked to the quality of their teaching (Charalambous et al., 2012; Wilhelm, 2014) and can positively impact student performance (Koponen et al., 2017; Ng, 2012). However, teaching critical thinking in the mathematics classroom is challenging. Inadequate mathematical knowledge for teaching among teachers is a key factor contributing to the struggles of Indonesian learners in mathematics. This claim is supported by the significant impact mathematical knowledge has on teaching practices, student learning, and achievement in mathematics (Appelgate et al., 2020; Hill et al., 2005; Koponen et al., 2017). In addition, the limited mathematical knowledge among Indonesian mathematics teachers has been a concern for over two decades (Putra, 2016; Yuniarto et al., 2021; Zubainur et al., 2020).

In Indonesia, teacher education programs for pre-service teachers (PSTs) have not sufficiently equipped future educators to meet the evolving demands of teaching critical thinking (Afkar et al., 2023; Fitriati et al., 2024). Surveys conducted by Fitriati and Prayudi (2021) found that higher education institutions pay limited attention to developing 21st-century competencies such as critical thinking, creativity, communication, and collaboration (4Cs) among prospective mathematics teachers. Many teacher educators themselves lack a comprehensive understanding of these skills, resulting in inadequate preparation for PSTs (Chikiwa et al., 2019). Consequently, PSTs tend to demonstrate low to moderate proficiency in 4C skills, revealing a persistent gap between theoretical frameworks and practical application in teacher education (Fitriati et al., 2024). This shortfall in skill development also has implications for the quality of PSTs' instructional practices, particularly MKT. To bridge this gap, teacher education needs to equip PSTs with MKT and rethink strategies to better equip PSTs with the necessary skills and knowledge.

The school-university partnership mediated by lesson study (SUPER-LS) is a collaborative teacher professional development (TPD) model that combines U.S. school-university partnerships with Japanese lesson study (Fitriati et al., 2022, 2023). This model aims to improve education quality through continuous teacher development, starting from teacher preparation and continuing throughout their careers. SUPER-LS facilitates the exchange of ideas, knowledge, and experiences among teachers, enhancing their ability to support student learning. Effective teacher education and TPD programs should focus on developing teachers' mathematical knowledge for teaching to meet 21st-century educational demands (Farrell, 2021;

Thant Sin, 2021). With its demonstrated potential for improving teaching and teacher quality, lesson study inevitably raised the issue of school-university collaboration to support teacher education. Thus, many countries and cities around the world, including Indonesia (Fitriati et al., 2022; Hendriana et al., 2007), Hongkong (Tsui et al., 2009), South Africa (Letloenyane et al., 2015), Ireland (Clivaz et al., 2023), Australian (Groves et al., 2013), and England (Baldry & Foster, 2019), have established school university partnerships mediated by lesson study as a method of professional development for PST and IST. These partnerships allow PSTs to participate in lesson study cycles conducted by ISTs in schools, allowing PSTs to do so within an extremely well-established framework. As a result, the SUPER-LS has become a powerful professional development model.

Numerous research studies have demonstrated the benefits of the SUPER-LS in preparing mathematics teachers, demonstrating significant gains in professional knowledge, research development, and knowledge creation (Farrell, 2021; Fitriati et al., 2023; Green et al., 2020; Groth et al., 2020; Thant Sin, 2021). Thus, to foster knowledge creation and dissemination in mathematics teacher education, one recommendation is to place a premium on collaboration between schools and universities to improve the quality of mathematics education. Adopting the SUPER-LS program to support PSTs and ISTs with SMK for teaching critical thinking in mathematics classrooms is imperative to meeting the current demand for 21st-century skills.

In response to the challenges facing teacher education and the potential of the SUPER-LS program, this study aims to strengthen mathematics teachers' Subject Matter Knowledge (SMK) for teaching critical thinking through a structured professional development initiative. The Lesson Study (LS) model has been widely recognized as an effective form of professional development, promoting reflective practice, collaborative inquiry, and continuous improvement among educators (da Ponte et al., 2022; Lewis, 2016). By integrating university-based expertise with school-based classroom practices, the SUPER-LS program seeks to bridge the gap between theoretical knowledge and practical application, thereby improving both teaching practices and student learning outcomes (Baldry & Foster, 2019).

Critical thinking is widely recognized as a core 21st-century competency and a key outcome of education, valued for its role in problem-solving, decision-making, and preparing students for complex societal and workplace demands (Evans, 2020; Facione, 2011; Lai, 2011). Rooted in Dewey's (1910) conception of reflective thinking, critical thinking is commonly defined as a multifaceted cognitive process involving the ability to analyse, interpret, evaluate, and reason logically from multiple perspectives (Alsaleh, 2020; Joma et al., 2022; Ravitz et al., 2012; Sternberg, 2016). Scholars also emphasize its metacognitive dimension, linking it to higher-order thinking and self-regulation. Beyond academic settings, critical thinking supports

informed citizenship and lifelong learning (Boss, 2013; Singh et al., 2020). Despite its acknowledged importance, students' engagement in critical thinking within mathematics classrooms remains limited, particularly in contexts lacking instructional strategies aligned with 21st-century skills (The World Bank, 2018). As such, embedding critical thinking as a focus of mathematics instruction and as an indirect yet meaningful outcome of teacher professional development is essential. Approaches such as Realistic Mathematics Education (RME) can foster critical thinking by situating problem-solving in meaningful contexts. Thus, to support the development of students' critical thinking skills, this study also incorporates the RME approach within the SUPER-LS program.

The RME approach is particularly well-suited for cultivating critical thinking, emphasizing context-rich, meaningful learning experiences that reflect real-world situations (Oldham & Price, 2020). This approach encourages students to actively construct mathematical understanding through problem-solving, exploration, and reflection, core elements of critical thinking. Previous research has consistently shown that RME supports students in mathematizing real-world contexts, enabling them to systematically analyze, interpret, and evaluate information (Mulbar & Zaki, 2018; Ndiung, 2021; Septriyana et al., 2019). Moreover, the dialogic and humanistic features of RME foster student autonomy and reasoning by promoting justification of solutions, consideration of alternative viewpoints, and peer collaboration (Fauziah et al., 2022; Putri & Zulkardi, 2019). As a result, students taught through RME demonstrate improved abilities in making inferences, drawing conclusions, and applying logical reasoning, which are key indicators of critical thinking.

Given this dual focus on teacher learning and student outcomes, the study is guided by the following research questions: (1) what aspects of Subject Matter Knowledge (SMK) are developed by mathematics teachers through their participation in the SUPER-LS program? and (2) how does students' critical thinking performance change following their teachers' involvement in the SUPER-LS program?

Method

This study is part of our ongoing project that aims to investigate the growth of mathematical knowledge for teaching (MKT) among mathematics instructors in relation to teaching 4C skills using the RME approach. To achieve the objective of the study, the SUPER-LS program was conducted to facilitate teacher learning of teaching critical thinking. The program involved in-service teachers (ISTs) from schools and pre-service teachers (PSTs) from the university. However, this study focused only on ISTs' subject matter knowledge (SMK). The research design employed in this study is a single case study approach with embedded units (Merriam & Tisdell, 2016).

These units consist of two groups of ISTs who conducted lesson study activities, including plan, do, and see in their own schools and their students. The demography of ISTs involved in the SUPER-LS is provided in [Table 1](#).

Table 1. The demographics data of ISTs

School	Pseudonym	Age Group	Year of Teaching	Current Status
School 1	IST101	41-44	25 Years	Teacher
	IST102	35-40	15 Years	Teacher
	IST103	55-60	36 Years	Teacher
	IST104	50-55	30 Years	Head of School
School 2	IST205	41-45	20 Years	Teacher
	IST206	45-50	15 Years	Teacher
	IST207	45-50	25 Years	Teacher
	IST208	45-50	30 Years	Head of School
	IST209	25-30	4 Years	Teacher
	IST210	25-30	2 Years	Teacher

The participants were purposefully selected based on their relevance to the research subject. Wannenburg and Curlewis (2023) describe purposive sampling as a method that involves deliberately selecting participants with specific characteristics that align with the research aim. In this study, 10 ISTs were selected based on specific criteria and their willingness to participate in the SUPER-LS program. They were from two secondary schools in Banda Aceh, Indonesia. The program was implemented over six months, coinciding with the start of the school's second semester. Through this initiative, a learning community was established to carry out lesson study focusing on developing RME-based instruction to enhance students' critical thinking.

Following the SUPER-LS program designed by the Indonesian government (Fitriati et al., 2023; Kemendikbudristek, 2021), the SUPER-LS program in this study was conducted in two phases: the first phase involved implementing lesson study in schools, specifically targeting the improvement of students' critical thinking skills through the RME approach. This was carried out by ISTs in collaboration with PSTs from the university. Each participating school conducted two cycles of lesson study involving a total of 59 Year 8 students. School 1 focused on the topic of "isosceles triangles", while School 2 explored "the relationship among central angles, arc lengths, and the area of sectors". The second phase took place at the university, where PSTs were supported in teaching critical thinking using the RME approach through lesson study. In this phase, PSTs enrolled in a microteaching course practiced lesson study under the guidance of their instructors, with the active involvement of ISTs from the partner schools.

The research instruments used in this study included video recordings of lesson study activities and relevant artefacts. These materials were analyzed using transcript-based lesson analysis (TBLA) to examine observation data, particularly focusing on discussions and dialogues among teachers, students, and knowledgeable others. TBLA is a research methodology used in educational studies to explore classroom interactions and instructional content by converting

spoken language, such as teacher-student dialogues and group discussions, into written transcripts (Sarkar Arani et al., 2019). This method is particularly valuable in lesson study research as it evaluates the effectiveness of learning activities and determines whether the instructional goals were achieved (Sarkar Arani et al., 2019; Winarti et al., 2021).

In this study, TBLA was employed to identify the types of SMK teachers developed during their participation in the SUPER-LS program. The analysis involved interpreting discussion transcripts and examining the application of SMK across different stages of each lesson study cycle. Various teaching and learning episodes were analyzed from multiple perspectives, including lesson structure, key features of the lesson, student interactions, and aspects of student learning. The video recordings of each lesson study cycle covering the "plan," "do," and "see" phases were analyzed using the transcript-based approach within the framework of MKT as developed by Clivaz and Shuilleabhain (2019), as illustrated in Table 1.

In addition, a critical thinking performance scale was utilized to assess students' critical thinking abilities. This scale was developed by the researcher based on the critical thinking indicators proposed by Rice (2017) which include the ability to compare information from multiple sources, analyze numerical data and factual evidence, summarize understanding of the material, evaluate conflicting viewpoints, and construct persuasive arguments using supporting data and logical reasoning to address complex problems or unresolved questions (see appendix 1). The scale demonstrated a high level of reliability as reported by Rice (2017), with a coefficient of 0.9, indicating its suitability for use in other studies.

Results and Discussion

The findings derived from this study are categorized into two primary areas to address the study problems: 1) the type of SMK developed during the SUPER-LS program and 2) the students' performance of critical thinking as their teachers participated in the SUPER-LS program.

The type of SMK developed during the SUPER-LS program

Data from video recordings of each lesson study activity across two cycles and artefacts were used to identify the type of SMK developed by ISTs during the SUPER-LS program. The discussions or dialogues were recorded, transcribed, and examined using TBLA. Following this analysis, all conversations or dialogue during the plan, do, and see phases related to SMK were coded. This study used MKT frameworks to systematically classify each step analysis, which one falls within the SMK categories, and which indicator. The process of this step is illustrated in Table 2. The number of domains of SMK that appeared in this process was calculated.

Table 2. Open lesson data analysis using TBLA

Code	Conversation/Dialogue	Step	Step Analyses	SMK		
				CCK	SCK	HCK
T	Okay, before we start today's lesson, let's begin with a prayer led by our friend Fathir. Fathir, please lead	4	4. Character building and giving motivation			
Fathir	The prayer begins					
SS	Students recite the prayer together.					
T	Thank you, Fathir. Alright, children, are you still excited?					
SS	Yes, still excited					
T	Keep up the spirit, even though there are many guests behind us here today observing our lesson. Alright, children, please pay attention to the display in front. Look at the picture being shown. What kind of image do you think this is?	5	5a. Technical irrelevant things; Starting the lesson by displaying a didactical situation related to today's lesson topics			
	What shape do you see? (an isosceles triangle)					
SS	Triangle		5b. Matching picture to abstract concept about triangle		SCK 8	
T	Zahratul, do you know?	6	6. Checking concept to lower achiever;		SCK 8	
Zahratul	Triangle		Matching picture to abstract concept about triangle			
T	How about this one? (<i>picture of an equilateral triangle</i>)	7	7. Change the pattern 2, Matching picture to abstract concept about triangle		SCK 8	
SS	How about this one? (<i>picture of an equilateral triangle</i>)					
T	A triangular-shaped roof. How about this one? (<i>picture of a right triangle</i>)	8	8. Change the pattern 3, Matching picture to abstract concept about triangle		SCK 8	
SS	Inaudible					
T	How about this one? (<i>picture of a traffic sign in the shape of an isosceles triangle</i>)	9	9. Change the pattern 3, Matching picture to abstract concept about triangle (unexpected answer based on attribute)		SCK 8	
SS	Traffic signs					
T	So, what can you all see in these pictures?	10	10. Reorienting picture meaning for expected result: triangle		SCK 8	
SS	Triangle					

In this study, we calculated the number of instances of transcripts related to common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK) in ISTs activities during the lesson study phases across both cycle 1 and cycle 2. Overall, the whole picture of the SMK domain demonstrated by ISTs over two cycles of LS practice is portrayed in the following [Figure 1](#).

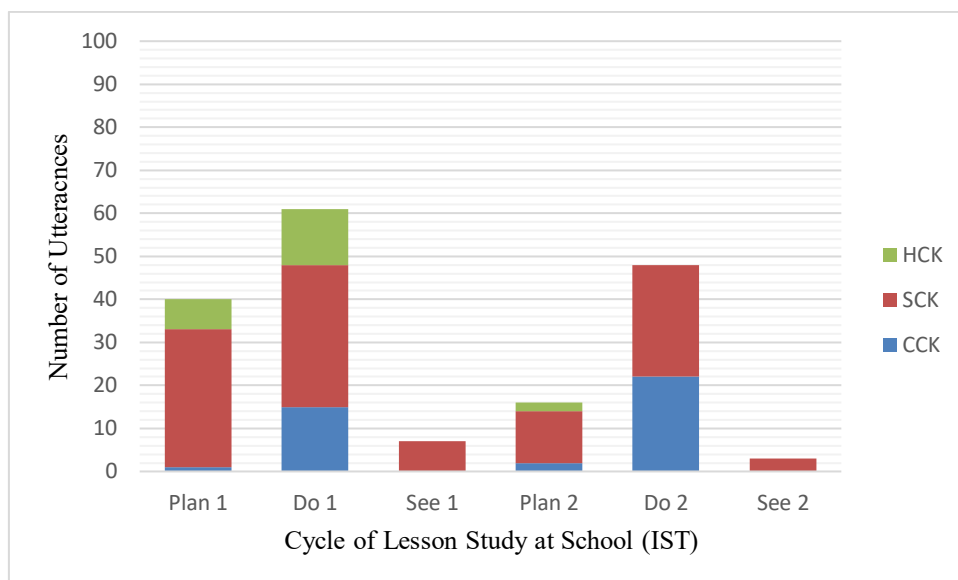


Figure 1. The type of SMK developed by ISTs over two cycles

ISTs gained a large number of SMK over two cycles of lesson study with SCK, which were the most significantly employed during designing the lesson, open lesson, and post-lesson discussion phases. This study coded 39 instances of transcripts related to CCK in ISTs activities during the lesson study phases across both cycle 1 and cycle 2. It is found that there were more instances of coding related to "determine if a solution, a definition, a representation is correct", and this feature frequently surfaced during the open lesson as open-class teachers provided feedback on students' works, whether it was correct or not. Only 3 appeared during the planning session, and none appeared during the post-lesson discussion. CCK observed in the open-lesson activity is exemplified in the following classroom dialogues.

- Teacher* : Okay, are you ready? Does anyone have something to say about the angles? What did you observe about the angles in the isosceles triangle? Which group would like to explain? Group 5? Maiza, please stand up and show us.
- Maiza* : The base angles on the left and right are coinsided.
- Teacher* : Which ones are the base angles?
- Maiza* : This one and this one (while pointing to the diagram).
- Teacher* : Alright... so when it's folded, what happens?
- Students* : They overlap.
- Teacher* : Then, which group can show what happens to the sides of the isosceles triangle when it's folded? Dina... wake up, Dina. Stand up so your friends can see. Show us which ones?
- Dina* : This one and this one (while pointing to the diagram). So, when folded, their length are equal.
- Teacher* : Same length.

This dialogue illustrates the teacher's ability to assess the correctness of students' observations, definitions, and representations such as recognizing when angles coincide or when sides are of equal length in an isosceles triangle. This evaluative skill reflects a key aspect of

CCK, which refers to the mathematical knowledge that teachers use to interpret and respond to students' mathematical thinking accurately. CCK is primarily employed during instructional practice, enabling teachers to validate students' reasoning, provide appropriate feedback, and ensure that the mathematical ideas being constructed are conceptually sound.

In addition, about 44 references related to SCK were found in the plan transcript, 59 in the open class transcript and 10 in the reflection transcript. An illustration of SCK emerging from the planning session is evident in the teacher dialogue below.

- HS : Regarding the folding part, do the students actually not know yet that the angles in an isosceles triangle coincide?*
- FH : Not yet.*
- CK : When they were folding, did they notice anything overlapping? (It seems the teacher already has an idea for a student activity, but is struggling to formulate the instructions)*
- HS : The top part coincides like that.*
- CK : If it coincides when folded, that means the angles are equal.*
- FH : So, let's just tell them to fold an isosceles triangle and measure the angles. That one is enough; we can let the students think with it. That's where their creativity comes in.*

This dialogue illustrates how one of the teachers justified the folding activity as a novel approach to introducing students to the criterion of an isosceles triangle, that if the right and left angles coincide perfectly when folded, it indicates that the angles are equal in measure. Consequently, the teachers agreed to implement the folding activity and began collaboratively formulating instructions that could also stimulate students' critical thinking. This interaction highlights that, during the planning session, teachers needed the ability to select, construct, and use mathematical representations effectively, which is an essential component of SCK (Clivaz & Shuilleabhain, 2019).

This data indicates that ISTs need SCK primarily during designing and implementing the lesson, compared to the post-lesson discussion. Moreover, 22 instances of transcripts related to HCK in ISTs' activities during the lesson study phases across both cycle 1 and cycle 2. This domain of SMK was frequently appeared in the open lesson as open-class teachers demonstrated how a concept of mathematics related to other concepts in mathematics or other subjects. About 9 references related to HCK were found in the plan transcript, 13 in the open class transcript, and no instance was found within the reflection transcript. The following teacher dialogue provides an example of HCK identified during the planning activity.

- CK : So back to the paper-folding activity, in this iceberg, there is only one folding. Then the question is to measure the angle. If we don't fold it, can we still measure the angles?*
- AM : We can, using a tool.*
- CS : With a tool. So, should we change the question? If we want to ask students to fold the paper, because if we ask them to measure, they can do that without*

folding, for example by using a protractor. But here, our goal is for students to see that the two base angles are equal. That's why, by folding, they can observe that the angles coincide, no more, no less, just right, which means the angles are equal. They've also already learned about congruence. So, the question here may need to be revised, maybe into something like: "What can you say about the size of the angles?"

This discussion demonstrates how the teachers explored alternative ways to apply mathematical knowledge in identifying the criteria of an isosceles triangle. They also emphasized the importance of ensuring that students had already learned prerequisite concepts, such as triangle congruence, and the practice of consistently providing justifications for their answers. This activity highlights the need for teachers to draw on their HCK, as it involves connecting current learning goals with prior knowledge and broader mathematical ideas.

Figure 1 also shows that the planning and do session aided teachers in establishing more SMK domains than in another phase of lesson study. Teachers engage in collaborative planning and in-depth study of specific mathematics content, which is then organised according to the iceberg form (the learning trajectory of the RME approach). This approach facilitates the enhancement of their comprehension of mathematical concepts, procedures, and the interrelationships among them, which is crucial for proficient instruction (Ni Shuilleabhain & Clivaz, 2017; Schoen et al., 2024).

In addition, the occurrence of SCK in teachers' discussions was predominant throughout all the phases. As an example of a less frequently occurring knowledge from the data, in their planning phase, teachers had the opportunity to utilize their CCK (indicator: performing mathematical task) while testing various tasks to potentially include within the research lesson. Additionally, participating in the lesson study cycle required teachers to rely on various kinds of SMK in their discussion and decisions around preparation, open class or open lesson, and reflecting on the lesson (da Ponte et al., 2022; Dudley, 2015; Lewis et al., 2019)

It is also interesting to note the lack of evidence of horizon content knowledge (HCK) in teachers' LS conversations in this study (Figure 2). This may be due to the fact that within this cycle of lesson study, there is a lack of content knowledge among the ISTs. In addition, all the findings above exhibit that planning the lesson with the RME approach during the lesson study practice in school could train ISTs to develop their SMK, especially SCK, which appeared to be dominantly used. This finding confirms what Clivaz and Shulaibhin (2019) have found that teachers need all domains of SMK during the planning sessions. Thus, practicing lesson study might help teachers to develop that knowledge.

The SUPER-LS program enabled ISTs to acquire three categories of knowledge in the context of SMK: CCK, SCK, and HCK. Teachers engage in collaborative planning and in-depth study of specific mathematics content, which is then organised according to the iceberg form (the

learning trajectory of the RME approach). This approach facilitates the enhancement of their comprehension of mathematical concepts, procedures, and the interrelationships among them, which is crucial for proficient instruction. ISTs exhibited various elements of CCK during the lesson study activities. The open lesson emphasised the importance of employing many representations, such as visual, symbolic, and numerical, to convey mathematical concepts. ISTs acquired the skill of seamlessly transitioning between many forms of representation and comprehending the unique ways in which each can be employed to elucidate and strengthen their understanding of subject matter (da Ponte et al., 2022; Dudley et al., 2019). By establishing a solid base of shared content knowledge and supporting ISTs in improving their comprehension, they can strengthen their ability to teach mathematics with greater effectiveness, ultimately leading to improved learning experiences for their students. These findings confirm the previous research conducted by Clivaz and Shuilleabhain (2019) and Nguyen and Tran (2023) that discovered the contribution of lesson study to help teachers develop their MKT.

The students' performance of critical thinking as their teachers participated in the SUPER-LS program

Cultivating students' critical thinking skills is essential in today's mathematics classroom. Teachers must provide students with the capacity to think critically when solving problems, enabling them to participate actively in society and preparing them for future educational obstacles. Within the scope of this study, teachers from the schools involved created a set of mathematics lessons using the RME technique to help their students improve critical thinking skills. Regarding the students' critical thinking development, the findings are illustrated in the following Figure 2.

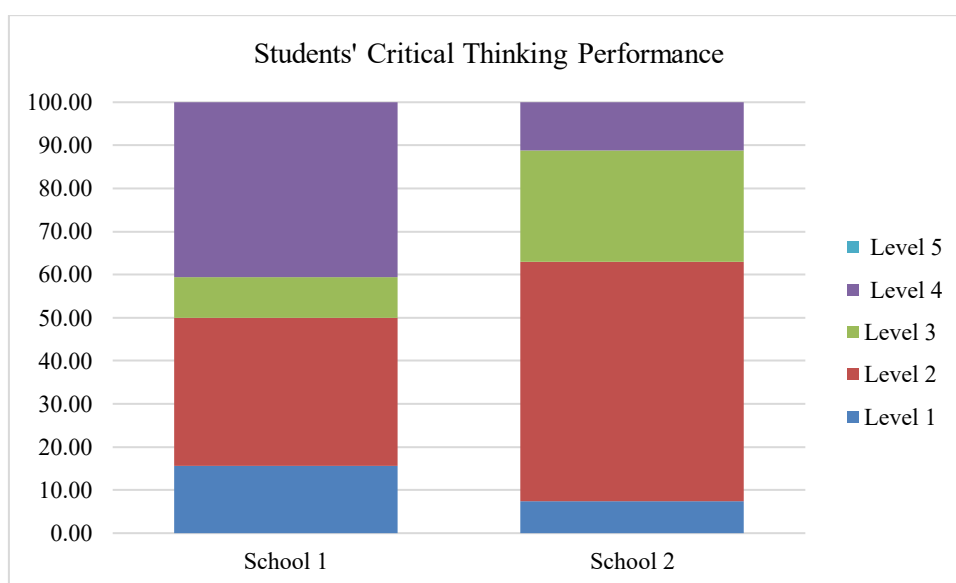


Figure 2. Students' critical thinking performance (%)

Figure 2 illustrates the range of students' critical thinking skills spanning from level 1 to level 4. The performance assessment revealed that students from both schools demonstrated critical thinking skills during the lesson. This included their ability to compare information from multiple sources, analyse numerical data and factual evidence, summarise their understanding of the material, evaluate conflicting viewpoints, and construct persuasive arguments using supporting data and logical reasoning to address complex problems or unanswered questions. Nevertheless, the critical thinking abilities of students from both schools ranged from level 1 to level 5, with most of them in the middle of the level (level 2 and level 3).

Furthermore, Figure 2 shows that School 1 outperformed School 2 in this regard. None of the students in any of the schools have achieved the most significant degree of critical thinking competence (level 5); on average, only 11.52% have reached level 1 competency. Furthermore, both schools have the potential to attain a level 4. In order to provide a more meaningful understanding of students' performance, the scale is segmented into five distinct competency levels. The levels, ranging from 1 to 5, are determined by the skills required to execute the tasks inside them properly. Level 1 is defined based on the absence of these skills. A student who has achieved a level 4 proficiency on the critical thinking scale can demonstrate six indicators of critical thinking skills. However, there may be some instances of errors or inappropriate applications.

Across the schools, 40.63% of students from School 1 and 11.11% from School 2 demonstrate proficiency at level 4. Despite School 1 outperforming on this level, the percentage of pupils performing at Level 1 is also significant (15.63%) compared to School 2, which only has 7.41%. Furthermore, School 2 demonstrates a proficiency level 2 for over 50% of its students, but School 1 only achieves this level for 34.38%. This proficiency level is the most frequently observed in both schools, with an average of 44.97%. In addition, 9.38% of students from School 1 and 25.95% of students from School 2 have demonstrated proficiency at level 3. Level 3 was the third most prevalent competence level in both schools, with an average of 17.65%. These findings suggest that more measures are necessary to improve students' critical thinking skills. It also indicates that students still need more training on developing critical thinking skills. This discovery validates the findings of Elisanti et al. (2018), who also observed a decrease in critical thinking abilities among senior high school students.

In order to address low students' critical thinking, it is important to engage students in the process of inquiry-based learning, provide them with training to apply the existing knowledge to solve problems, and consistently encourage them to engage in mathematical modelling. By doing so, students can enhance their critical thinking skills and become more empowered in their learning (Fitriati et al., 2020; Raslan, 2023). Furthermore, the emphasis on memory learning

should not take precedence in the teaching of mathematics. Train students to consistently present logical justifications for their answers, and teachers should exemplify the correct way to do so. Jaelani et al. (2023) suggested employing metacognitive scaffolding as a means to support students in solving mathematical issues by encouraging them to thoroughly analyse the problem situation, thus facilitating the discovery of optimal solutions. In addition, Hagemann and Kluge (2017) proposed the utilisation of a cooperation and collaboration approach in the learning process to assist students in resolving challenging situations. Furthermore, suggested that there should be a transition in learning from emphasising lower-order cognitive abilities to giving priority to higher-order thinking capabilities.

Furthermore, the introduction of this particular lesson study program in Indonesia was aimed at facilitating the implementation of the new national curriculum, known as *Kurikulum Merdeka*. These curricula necessitated a fundamental change in teaching methods, shifting towards a student-centered approach to learning and reducing the emphasis on teachers simply transmitting passive knowledge (Akhbar et al., 2023; Bangkara et al., 2022). In addition, the curriculum emphasizes the implementation of a learning community to facilitate teacher learning. The government demands all schools in Indonesia to build a "*komunitas belajar*" or learning community to help teachers learning to implement the curriculum as stated in the regulation number 4236/B/HK.04.01/2023 (Kemendikbudristek, 2021). The SUPER-LS program could be employed to realize this government regulation.

Establishing a school as a learning community has the potential to maintain the longevity of the SUPER-LS program in the mathematics education context. A potential framework exists to facilitate the establishment of a community for mathematics educators, which would include in-service teachers from schools, pre-service teachers from universities, and other individuals who possess expertise in teaching. The lesson study within the SUPER program serves as a professional development model that enables all community members to collaborate in solving school problems through the process of preparing, teaching, observing, and reflecting on a research lesson as part of a lesson study cycle (Fitriati et al., 2024; Nurwidodo et al., 2018; Saito et al., 2015). The objective is to enhance teaching methods in order to have a beneficial effect on student learning and to develop communities of professional teachers. By engaging in collaborative learning with the school community, mathematics instructors (including both experienced teachers and new teachers) can identify research subjects by analysing and discussing various aspects of teaching (Yu, 2021). This process allows them to receive feedback on their teaching practices, creating a mutually beneficial relationship between research and teaching.

This study contributes to mathematics education by providing empirical evidence on the effectiveness of the SUPER-LS model in enhancing teachers' SMK, highlighting lesson study as

a tool for professional growth and critical thinking in instruction, and offering valuable insights for policymakers and educators on effective strategies for teacher development and curriculum innovation. Despite this contribution, several limitations should be acknowledged. Since the organization of the lesson study was conducted in groups of ISTs across two lesson study cycles, it was not possible to track the development of SMK at the individual level. As a result, the study does not provide detailed insights into how each IST developed their SMK throughout the program. Future research could address this limitation by designing a study that specifically follows the SMK development of individual participants over time.

Additionally, the short time frame and the use of a single research site limited the scope of this investigation. Constraints in scheduling collaborative sessions and allocating adequate time for deep reflection and iterative redesign during each lesson study cycle may have hindered the full development of SMK. Therefore, future studies should consider conducting lesson study programs over a longer duration and across multiple sites to gain more comprehensive and generalizable findings. Expanding the period of observation and increasing data collection efforts would likely yield deeper insights and allow for more robust conclusions regarding the effectiveness of lesson study in developing SMK and supporting critical thinking integration.

Conclusion

ISTs acquired proficiency in many SMK domains, such as CCK, SCK, and HCK, through the SUPER-LS program. All the domains that were presented were essential for instructing critical thinking. Every domain of SMK that is developed during the program contributes to the effective instruction and acquisition of mathematical knowledge for teaching in the 21st century. Nevertheless, it is necessary to augment teachers' depth of knowledge in the subject matter in order to empower them to create mathematical tasks that cater to the abilities of their slower learners, facilitating the development of critical thinking at a speed suitable for each individual. Educators must assess their students' comprehension, critical thinking abilities, and challenges in order to gauge the magnitude of those obstacles, so that they can devise tailored instructional strategies that cater to the individual requirements of their students. Therefore, it is recommended to direct attention towards the cognitive processes of students, analyse the reasons behind their occurrence, and identify techniques for enhancing them. By possessing this information and these skills, teachers enhance students' learning and the quality of mathematics instruction.

Regarding the students' critical thinking performance, the study revealed that all indicators of critical thinking were observed during the lesson. Further analysis revealed that students demonstrated varying levels of proficiency across the critical thinking indicators during the implementation of the SUPER-LS program, ranging from level 1 to level 5. Most students were

observed to be performing at level 2 across these indicators. These findings suggest a need for continued support and professional development through follow-up SUPER-LS activities, with the aim of further enhancing opportunities for students to engage in critical thinking. While this study does not claim a direct causal impact, the patterns observed provide valuable insights into the ways in which students' critical thinking is expressed within the instructional context shaped by the SUPER-LS program.

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References

- Afkar, R., Bêteille, T., Breeding, M. E., Linden, T., Mason, A. D., Mattoo, A., Pfitze, T., Sondergaard, L. M., & Yarrow, N. (2023). *Fixing the foundation: Teachers and basic education in East Asia and Pacific*. World Bank Publications. <https://openknowledge.worldbank.org/bitstreams/5035a402-5f95-4a638c82-863271eac3e/download%0A%0A>
- Akhbar, A. F., Sobri, A. Y., Triwiyanto, T., Kusumaningrum, D. E., Maitreepun, W., Sari, L. A., & Maulina, S. (2023). Superior characteristics and future of the independent curriculum (kurikulum merdeka) management system in indonesia elementary schools in the era of digital technology. *ICEMT*, 191–197. https://doi.org/10.2991/978-2-38476-156-2_20
- Alsaleh, N. J. (2020). Teaching critical thinking skills: Literature review. *TOJET: The Turkish Online Journal of Educational Technology*, 19(1), 21–39. <http://www.tojet.net/articles/v19i1/1913.pdf>, retrieved by 25 April 2021%0Ahttps://eric.ed.gov/?id=EJ1239945
- Appelgate, M. H., Dick, L., Soto, M., & Gupta, D. (2020). Growing a greater understanding of multiplication through lesson study: mathematics teacher educators' professional development. *Mathematics Enthusiast*, 17(2–3), 583–613. <https://doi.org/10.54870/1551-3440.1498>
- Baldry, F., & Foster, C. (2019). Lesson study partnerships in initial teacher education. *Lesson study in initial teacher education: Principles and practices*, Emerald Publishing Limited, pp 147–160. <https://doi.org/10.1108/978-1-78756-797-920191011>
- Bangkara, B. M. A. S. A., Manullang, S. O., Pratiwi, E. Y. R., Husen, N., & Sabtohadhi, J. (2022). Rethinking the “kurikulum merdeka for learning.” *EDUTECH: Journal of Education And Technology*, 6(2), 201–216. <https://doi.org/10.29062/edu.v6i2.424>
- Boss, S. (2013). *PBL for 21st Century success: Teaching critical thinking, collaboration, communication, and creativity* (J. Larmer (ed.)). Novato, CA: Buck Institute for Education.

- BSKAP. (2024). *Capaian pembelajaran pada pendidikan anak usia dini, jenjang pendidikan dasar, dan jenjang pendidikan menengah pada kurikulum merdeka* (Issue 021). Kementerian pendidikan, kebudayaan, riset, dan teknologi.
- Charalambous, C. Y., Hill, H. C., & Mitchell, R. N. (2012). Two negatives don't always make a positive: Exploring how limitations in teacher knowledge and the curriculum contribute to instructional quality. *Journal of Curriculum Studies*, 44(4), 489–513. <https://doi.org/10.1080/00220272.2012.716974>
- Chikiwa, S., Westaway, L., & Graven, M. (2019). What mathematics knowledge for teaching is used by a grade 2 teacher when teaching counting. *South African Journal of Childhood Education*, 9(1), 1–9. <https://doi.org/10.4102/sajce.v9i1.567>
- Clivaz, S., Daina, A., Batteau, V., Presutti, S., & Bünzli, L. O. (2023). How do dialogic interactions contribute to the construction of teachers' mathematical problem-solving knowledge? Construction of a conceptual framework. *International Journal for Lesson and Learning Studies*, 12(1), 21–37. <https://doi.org/10.1108/IJLLS-03-2022-0031>
- Clivaz, S., & Shuilleabhain, A. N. (2019). What knowledge do teachers use in lesson study? A focus on mathematical knowledge for teaching and levels of teacher activity. In R Huang, A. Takahashi, & J. da Ponte (Eds.), *Theory and Practice of Lesson Study in Mathematics. Advances in Mathematics Education*, 419–440. https://doi.org/10.1007/978-3-030-04031-4_20
- da Ponte, J. P., Quaresma, M., & Mata-Pereira, J. (2022). Teachers' learning in lesson study: insights provided by a modified version of the interconnected model of teacher professional growth. *ZDM - Mathematics Education*, 54(2), 373–386. <https://doi.org/10.1007/s11858-022-01367-1>
- Dewey, J. (1910). How we think. *the problem of training thought*, 14. <https://doi.org/10.1037/10903-000>
- Dudley, P. (2015). *Lesson study: Professional learning for our time* (Ed). Routledge.
- Dudley, P., Xu, H., Vermunt, J. D., & Lang, J. (2019). Empirical evidence of the impact of lesson study on students' achievement, teachers' professional learning and on institutional and system evolution. *European Journal of Education*, 54(2), 202–217. <https://doi.org/10.1111/ejed.12337>
- Elisanti, E., Sajidan, S., & Prayitno, B. A. (2018). The profile of critical thinking skill students in XI grade of senior high school. *Proceedings of the 1st Annual International Conference on Mathematics, Science, and Education (ICoMSE 2017)*, 205–209. <https://doi.org/10.2991/icomse-17.2018.36>
- Evans, C. M. (2020). Measuring student success skills: A review of the literature on critical thinking. *National Center for the Improvement of Educational Assessment*. <http://www.nciea.org>
- Facione, P. A. (2011). Critical thinking: What it is and why it counts. *Insight Assessment*, 1(1), 1–28. <https://www.insightassessment.com/CT-Resources/Teaching-For-and-About-Critical-Thinking/Critical-Thinking-What-It-Is-and-Why-It-Counts/Critical-Thinking-What-It-Is-and-Why-It-Counts-PDF>
- Farrell, R. (2021). The school–university nexus and degrees of partnership in initial teacher education. *Irish Educational Studies*, 1–18. <https://doi.org/10.1080/03323315.2021.1899031>

- Fauziah, A., Ilma, R., Putri, I., Info, A., Learning, C., & Study, L. (2022). Collaborative learning through lesson study in pmri training for primary school pre-service teachers: The simulation of poligon matter. *Infinity Journal of Mathematics Education*, 11(1), 1–16.
- Fitriati, F., Novita, R., & Johar, R. (2020). Exploring the usefulness of rich mathematical tasks to enhance students' reflective thinking. *Cakrawala Pendidikan*, 39(2), 346–358. <https://doi.org/10.21831/cp.v39i2.24047>
- Fitriati, F., & Prayudi, S. (2021). 21st century teaching and learning: Student teacher's perception and challenges in higher education institutions. *Al Khawarijmi: Jurnal Pendidikan Dan Pembelajaran Matematika*, 5(1), 12–23. <https://doi.org/http://dx.doi.org/10.22373/jppm.v5i1.9342>
- Fitriati, F., Rosli, R., & Iksan, Z. H. (2023). Enhancing prospective mathematics teachers' lesson planning skills through lesson study within school university partnership program. *Journal on Mathematics Education*, 14(1), 69–84. <https://doi.org/10.22342/jme.v14i1.pp69-84>
- Fitriati, F., Rosli, R., & Iksan, Z. H. (2022). Lesson observation practice of mathematics teachers through school university partnership mediated by lesson study. *Creative Education*, 13(2), 675–690. <https://doi.org/10.4236/ce.2022.132042>
- Fitriati, F., Rosli, R., Iksan, Z., & Hidayat, A. (2024). Exploring challenges in preparing prospective teachers for teaching 4C skills in mathematics classroom: A school-university partnership perspectives. *Cogent Education*, 11(1), 2286812. <https://doi.org/10.1080/2331186X.2023.2286812>
- Green, C. A., Tindall-Ford, S. K., & Eady, M. J. (2020). School-university partnerships in Australia: A systematic literature review. *Asia-Pacific Journal of Teacher Education*, 48(4), 403–435. <https://doi.org/10.1080/1359866X.2019.1651822>
- Groth, R. E., Bergner, J. A., Weaver, S. D., & Welsh, G. S. (2020). Using japanese lesson study yo merge inservice professional development and pre-service clinical experience. *Clearing House: A Journal of Educational Strategies*, 93(2), 93–99. <https://doi.org/https://doi.org/10.1080/00098655.2020.1729082>
- Groves, S., Doig, B., Widjaja, W., Garner, D., & Palmer, K. (2013). Implementing japanese lesson study: An example of teacher-researcher collaboration. *Australian Mathematics Teacher*, 69(3), 10–17. <http://hdl.handle.net/10536/DRO/DU:30056167> Reproduced
- Hagemann, V., & Kluge, A. (2017). Complex problem solving in teams: The impact of collective orientation on team process demands. *Frontiers in Psychology*, 8, 1–17. <https://doi.org/10.3389/fpsyg.2017.01730>
- Hendriana, D., Nugraha, I., Hendayana, S., & Supriatna, A. (2007). *A School-university partnership mediated by lesson study to promote professional learning in the TCTP program*. Universitas Pendidikan.
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372–400. <https://doi.org/10.5951/jresmetheduc.39.4.0372>
- Jaelani, A. K., Hasbi, M., & Baharullah, B. (2023). A critical thinking profile of mathematics education students in solving ill-structured problem based on mathematical ability. *JTAM (Jurnal Teori dan Aplikasi Matematika)*, 7(2), 545–559. <https://doi.org/10.31764/jtam.v7i2.13378>

- Joma, A., Audrey, G., Arvisais, O., & Allawati, E. (2022). The effectiveness of educational practices for developing critical thinking skills in basic school students: A Systematic Review. *The Researchers' International Research Journal*, 8(1), 67-86. <https://doi.org/10.21276/tr.2022.8.1.AN5>
- Kemendikbudristek. (2021). *Panduan kemitraan dosen LPTK dengan guru di sekolah*.
- Koponen, M., Asikainen, M. A., Viholainen, A., & Hirvonen, P. E. (2017). How education affects mathematics teachers' knowledge: Unpacking selected aspects of teacher knowledge. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(6), 1943–1980. <https://doi.org/10.12973/eurasia.2017.01209a>
- Lai, E. R. (2011). Critical thinking: A literature review research report. *Critical Thinking*, 1–49.
- Letloenyane, M. D., Jita, L. C., & Chair, S. (2015). School-university partnerships for professional development of teachers: A case of lesson study intervention in mathematics. *Journal of higher education in Africa/Revue de l'enseignement supérieur en Afrique*, 13(1-2), 147-167.
- Lewis, C. (2016). How does lesson study improve mathematics instruction ? *ZDM*, 48, 571-580. <https://doi.org/10.1007/s11858-016-0792-x>
- Lewis, C., Friedkin, S., Emerson, K., Henn, L., & Goldsmith, L. (2019). How does lesson study work? Toward a theory of lesson study process and impact. *Theory and practice of lesson study in mathematics: An international perspective*, 13–37. https://doi.org/10.1007/978-3-030-04031-4_20
- Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research a guide to design and implementation*. Jossey-Bass Inc.
- Mulbar, U., & Zaki, A. (2018). Design of realistic mathematics education on elementary school students. *Journal of physics: Conference Series*, 1028(1), 012155. <https://doi.org/10.1088/1742-6596/1028/1/012155>
- Ndiung, S. (2021). Using the RME Principles to Support Students Problem Solving be HOTS Oriented. In *Proceedings of the 1st International Conference on Education, Humanities, Health and Agriculture, ICEHHA* (pp. 1-9). <https://doi.org/10.4108/eai.3-6-2021.2310654>
- Ng, D. (2012). Using the MKT measures to reveal Indonesian teachers' mathematical knowledge: Challenges and potentials. *ZDM - Mathematics Education*, 44(3), 401–413. <https://doi.org/10.1007/s11858-011-0375-9>
- Nguyen, D. T., & Tran, D. (2023). High school mathematics teachers' changes in beliefs and knowledge during lesson study. *Journal of Mathematics Teacher Education*, 26(6), 809–834. <https://doi.org/10.1007/s10857-022-09547-2>
- Ni Shuilleabhain, A., & Clivaz, S. (2017). Analyzing teacher learning in lesson study: Mathematical knowledge for teaching and levels of teacher activity. *Quadrante: Revista de Investigacao em Educacao Matematica*, 26(2), 99–125.
- Nurwidodo, N., Hendayana, S., Hindun, I., & Sarimanah, E. (2018). Strategies for establishing networking with partner schools for implementing lesson study in Indonesia. *Jurnal Pendidikan Biologi Indonesia*, 4(1), 11-12. <https://doi.org/10.22219/jpbi.v4i1.5489>
- Oldham, E., & Price, E. (2020). Teaching and learning science and mathematics in the 21st century: Challenges and aspirations. In L. Leite, E. Oldham, A. S. Afonso, & F. Viseu (Eds.), *Science and Mathematics Education for 21st Century Citizen Challenges and Ways Forward* (pp. xv–xxvii). Nova Science New York.

- Putra, Z. H. (2016). Evaluation of elementary teachers' knowledge on fraction using anthropological theory. *13th International Congress on Mathematical Education Hamburg, 24-31*, 1–4.
- Putri, R. I. I., & Zulkardi, Z. (2019). Designing jumping task on percent using PMRI and collaborative learning. *International Journal on Emerging Mathematics Education*, 3(1), 105–116. <https://doi.org/10.12928/ijeme.v3i1.12208>
- Raslan, G. (2023). Critical thinking skills profile of high school students in ap chemistry learning. In K. Al Marri, F. Mir, S. David, & A. Aljuboori (Eds.), *BUID Doctoral Research Conference 2022 Multidisciplinary Studies*, 320, 79–96). Springer. https://doi.org/10.1007/978-3-031-27462-6_5
- Ravitz, J., Hixson, N., English, M., & Mergendoller, J. (2012). Using project based learning to teach 21 st century skills: Findings from a statewide initiative. *Annual Meetings of the American Educational Research Association.*, 16, 1–9.
- Rice, C. L. (2017). *21st century teaching and learning: Teachers' perceptions and practices in four high schools of one district*. Arkansas Tech University.
- Saito, E., Murase, M., Tsukui, A., & Yeo, J. (2015). *Lesson study for learning community: A guide to sustainable school reform*. Routledge.
- Sarkar Arani, M. R., Lander, B., Shibata, Y., Kim-Eng Lee, C., Kuno, H., & Lau, A. (2019). From “chalk and talk” to “guide on the side”: A cross-cultural analysis of pedagogy that drives customised teaching for personalised learning. *European Journal of Education*, 54(2), 233–249. <https://doi.org/10.1111/ejed.12340>
- Schoen, R. C., Lewis, C. C., Rhoads, C., Lai, K., & Riddell, C. M. (2024). Impact of lesson study and fractions resources on instruction and student learning. *Journal of Experimental Education*, 92(2), 225–246. <https://doi.org/10.1080/00220973.2023.2183374>
- Septriyana, Y., Fauzan, A., & Ahmad, R. (2019). The influence of realistic mathematics education (RME) approach on students' mathematical problem solving ability. In *1st International Conference on Innovation in Education (ICoIE 2018)*, 165–169. <https://doi.org/10.2991/icoie-18.2019.38>
- Singh, C. K. S., Ong, E. T., Mohtar, T. M. T., Sing, R. S. M., & Mostafa, N. A. (2020). Quality teachers of the 21st century: An overview of theories and practice. *International Journal of Innovation, Creativity and Change*, 13(1), 1481–1494. <http://www.ijicc.net>
- Sternberg. (2016). Critical thinking: Its nature, measurement, and improvement. *National Institution of Education*, 11(1), 217–232. <https://pdfs.semanticscholar.org/80a7/c7d4a98987590751df4b1bd9adf747fd7aaa.pdf>
- The World Bank. (2018). *Improving teaching and learning in Indonesia*. <http://www.worldbank.org/end/country/indonesia/brief/improving-teaching-and-learning-in-indonesia>
- Thant Sin, K. K. (2021). School-university partnership: Perceptions and experiences of school teachers and student teachers on current collaboration in teacher education in Myanmar. *Journal of Adult Learning, Knowledge and Innovation*, 4(2), 62–75. <https://doi.org/10.1556/2059.2021.00050>
- Tsui, A. B. M., Edwards, G., Real, F. L., Kwan, T., & Law, D. (2009). *Learning in school-university*. 3.
- Wannenburg, E., & Curlewis, L. (2023). Exploring the need for numeracy skills in legal practice. *Cogent Education*, 10(1), 0–15. <https://doi.org/10.1080/2331186X.2023.2190307>

- Wilhelm, A. G. (2014). Mathematics teachers' enactment of cognitively demanding tasks: Investigating links to teachers' knowledge and conceptions. *Journal for Research in Mathematics Education*, 45(5), 636–674. <https://doi.org/10.5951/jresmetheduc.45.5.0636>
- Winarti, A., Saadi, P., & Rajiani, I. (2021). Applying transcript based lesson analysis in enhancing communication pattern between teacher and students in chemistry classroom. *European Journal of Educational Research*, 10(2), 975–987. <https://doi.org/10.12973/EU-JER.10.2.975>
- Yu, L. (2021). *The impact of lesson study on teacher professional development* (Dissertation). University of Canterbury. <https://ir.canterbury.ac.nz/server/api/core/bitstreams/fdfc7df6-a6e0-461b-97dd-f819f79a5382/content>
- Yunianto, W., Indra Prahmana, R. C., & Crisan, C. (2021). Indonesian mathematics teachers' knowledge of content and students of area and perimeter of rectangle. *Journal on Mathematics Education*, 12(2), 223–238. <https://doi.org/10.22342/JME.12.2.13537.223-238>
- Zubainur, C. M., Johar, R., Hayati, R., & Ikhsan, M. (2020). Teachers' understanding about the characteristics of realistic mathematics education. *Journal of Education and Learning (EduLearn)*, 14(3), 456–462. <https://doi.org/10.11591/edulearn.v14i3.8458>

APPENDIX

4Cs for 21st Century Skills

Students' name: Group or personal

Category: Critical Thinking

Definition	Indicators	Likert Scale for Performance Assessment / Portfolio				
		1	2	3	4	5
A multifaceted mental cognitive process that entails a variety of abilities and attitudes needed to solve a problem (Rice:2012)	1. Compare information from different sources before completing a task or assignment. Keywords: - compare information - different sources - completing task	Not Compare information from same or different sources, and not completing a task or assignment	Compare information from same sources and not completing a task or assignment	Compare information from same sources but completing a task or assignment	Compare information from different sources but not completing a task or assignment	Compare information from different sources before completing a task or assignment
	2. Draw their conclusion based on analysis of numbers, facts, or relevant information. Keywords: - draw conclusion - analyses - numbers, facts, relevant info	Could not draw conclusion based on analysis of numbers, facts, or relevant	Draw conclusion not based on proper numbers, facts, relevant information	Draw conclusion based on analysis of numbers, facts, but irrelevant information	Draw conclusion based on reading of numbers, facts, or relevant information	Draw conclusion based on analysis of numbers, facts, or relevant information
	3. Summarise or create their interpretation of what they have read or been taught. Keywords: - Summarise / Create Interpretation - reading experience - learning experience	Could not Summarise or create their interpretation and could not remember what have read or been taught	Could not Summarise or create their interpretation but remember what have read or been taught	Summarise or create their interpretation but not from of what they have read or been taught	Summarise or create their interpretation of what they have read only	Summarise or create their interpretation of what they have read or been taught

Definition	Indicators	Likert Scale for Performance Assessment / Portfolio				
		1	2	3	4	5
4. Analyse competing arguments, perspectives, or solutions to a problem. Keywords - Analyses - competing arguments, perspectives or solution - problem centred		Could not Read competing arguments, perspectives, or solutions to a problem.	Reading non competing arguments, perspectives, or solutions to a problem.	Understand competing arguments, perspectives, or solutions to a problem.	Analyse noncompeting arguments, perspectives, or solutions to a problem.	Analyse competing arguments, perspectives, or solutions to a problem.
5. Develop persuasive arguments based on supporting evidence or reasoning. Keywords - Develop persuasive argument - supporting evidence / reasoning		Could not Develop arguments based on supporting evidence or reasoning	Develop non-persuasive arguments based on supporting evidence or reasoning	Develop persuasive arguments but based on less-supporting evidence or reasoning and the answer is incorrect	Develop persuasive arguments but based on less-supporting evidence with correct answers	Develop persuasive arguments based on supporting evidence or reasoning
6. Try to solve a complex problem or answer questions with no single correct solution or answer. Keywords - Trying to solve - complex problems / answer Q - no single correct solution/answer		Do/does not try to solve a complex problem or answer questions with no single correct solution or answer	Try to solve a simple problem or answer questions with single correct solution or answer	Try to solve a simple problem or answer questions with no single correct solution or answer	Try to solve a complex problem or answer questions with single correct solution or answer	Try to solve a complex problem or answer questions with no single correct solution or answer