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Exploratory Study on the Formation of Trigonometric Concepts by High School Students

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Abstract. *The students' formation of mathematical concepts was still relatively weak. Data from the PISA showed that procedural skills still dominate students' conceptual understanding. The purpose of this study was to describe the students' concept formation in solving trigonometric problems. This study used a qualitative approach with a case study design. The subjects were three tenth-grade students from one of the high schools in the Boyolali district, Central Java. The instruments included trigonometric tests and in-depth interviews. Then the data was analyzed through reduction, presentation, and conclusion. The results showed that students with high trigonometric ability can fulfill all indicators of concept formation and connect trigonometric concepts with others. Furthermore, the student with medium trigonometric ability, although he can fulfill all the indicators of concept formation, has errors in the calculation process and unable to connect trigonometric concepts with other mathematical concepts. A student with low trigonometric ability is only able to perform simple solutions, and difficulties in connecting trigonometric concepts with others and explaining the steps of the solution. The implications of this study are the importance of pedagogical intervention in strengthening students' conceptual understanding and providing concrete experiences in trigonometry, especially for students with medium and low abilities.*

Keywords: *APOS theory, concept formation, trigonometric*

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Introduction

Mathematics is one of the subjects that, to date, requires accuracy in both its concepts and its integration. Mastery of complex mathematical material requires a careful understanding of symbols within a concept, as well as active, critical, and objective thinking (Laurens et al., 2018). We can observe students' learning success by how they master and solve the given problems (Minarti & Wahyudin, 2018). One of the mathematics materials that needs to be mastered carefully is trigonometric (Fererde et al., 2024; Martín-fernández et al., 2019; Owusu et al., 2025). Trigonometry is a field in mathematics that examines the relationships between sides and angles in a triangle (Daulay, 2015; Musa, 2016). Trigonometry serves as a prerequisite for various advanced mathematics topics, in accordance with Adhikari and Subedi's opinion that trigonometry is an important material that students must master, as the concept of trigonometry is a prerequisite for other topics (Adhikari & Subedi, 2021). Concept formation is crucial in trigonometry because misunderstandings of concepts can directly hinder the development and understanding of interrelated concepts (Cetin, 2015). Thus, if students cannot fully grasp the fundamental concepts of trigonometry, they will struggle to comprehend material related to trigonometric concepts (Adhikari & Subedi, 2021; Dhungana et al., 2023; Tyata et al., 2021). Thus, it is necessary to form concepts for students so that they understand the concept as a whole and can sustainably grasp it (Prabawanto et al., 2023).

Concept formation is the tendency of students to approach mathematical problem situations by building actions, processes, and objects that they can organize into schemas to understand situations and solve problems (Figueroa et al., 2018). This concept formation needs to be incorporated into students' thinking, enabling them to develop a coherent schema that can be applied in solving mathematical issues (Borji et al., 2018). In other words, concept formation is not only about data to understand a concept. However, it can also develop the concept and attempt to modify the process and objects in a schema according to individual understanding (Maharaj, 2013). Mathematical concept formation is crucial for students to grasp the material, enabling them to comprehend the overall concept being studied, rather than merely memorizing the provided formulas (Hussein & Csikos, 2023). Concept formation is a knowledge that exists independently of conceptual knowledge in each student (Chirove & Ogbonnaya, 2021). Therefore, every student needs to have a good and correct concept formation so that they can demonstrate pedagogical skills, develop mental structures, analyze and construct problems, and connect with various mathematical concepts.

The indicators of concept formation possessed by students in formulating mathematical problems from real-world contexts are one of the core stages in PISA mathematical literacy (Sinaga et al., 2023). In line with PISA's assessment of mathematical reasoning skills, this approach enables students to view and manipulate concepts as a whole (Hwang & Ham, 2021). However, the level of weakness in this concept formation is evident from Indonesia's average PISA mathematics score of 366, which is far below the Organization for Economic Cooperation and Development (OECD) average of 472, indicating that most students have not yet mastered basic concepts and the ability to solve complex problems that require higher-level thinking (OECD, 2023). This data highlights the need for in-depth pedagogical interventions to encourage students to develop a strong conceptual understanding, rather than merely memorizing procedures (Djannah et al., 2024). Furthermore, the formation of concepts in accordance with their characteristics requires students to have the ability to identify relevant mathematics and formulate problems into mathematical models (Gupta & Zheng, 2020; Jupri et al., 2021; Tasarib et al., 2025). The process is influenced by students' initial ability to construct mathematical concepts (Akarsu, 2022; Hurrell, 2021).

Previous studies have explored concept formation in algebra (Nurhayati & Dasari, 2023; Rahmawati et al., 2019), geometry (Pielsticker, 2021), probability (Yohan et al., 2025), and linear programming (Frank et al., 2022), often examining it through the lens of initial ability or APOS theory. These studies confirm that initial ability affects concept formation, and many students still experience conceptual obstacles. Crucially, no specific study has investigated the formation of trigonometric concepts when reviewed in terms of initial mathematical ability (Frank et al., 2022; Nurhayati & Dasari, 2023; Pielsticker, 2021; Rahmawati et al., 2019; Yohan et al., 2025). Prior research focused on other materials or limited the scope to high-ability students. Therefore, further research is needed to fill this gap. Previous research has not identified any specific studies examining the relationship between the formation of trigonometric concepts and initial mathematical ability. Even research by Rahmawati et al. (2019), which explores the formation of algebraic factorization concepts, only focused on high-ability students. The purpose of this study is to describe the students' concept formation in solving trigonometric problems.

Method

This research uses a qualitative approach with a case study design. Case studies focus on understanding phenomena within specific boundaries or within certain units of analysis. This study explores explicitly the concept formation of three high school students as they solve trigonometric problems.

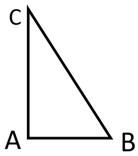
A total of 42 tenth-grade students from a public high school in Boyolali District, Central Java, participated in the trigonometric test. Students with high, medium, and low trigonometric test scores were selected to participate voluntarily in this study, without coercion, and they expressed their willingness to provide the information needed for this study. To analyze the student's concept formation, we use the APOS indicators as described and coded in Table 1.

Table 1. Concept formation indicators

Indicator	Description	Code
Action	Able to perform routine calculations with given formulas.	AC1
	Able to replicate examples of similar problem-solving.	AC2
Processes	Able to perform operations in detail and completely.	PR1
	Able to explain the sequence of steps independently.	PR2
Object	Able to identify and use concepts as input for other operations.	OB1
	Able to define concepts formally and understand their essence beyond procedures.	OB2
Schema	Able to analyze new situations and identify relevant schemas to apply.	SC1
	Able to integrate multiple concepts to solve complex problems and select trigonometric strategies.	SC2

The trigonometric test instrument consists of three problems that include the action, process, object, and schema (Table 2). The first problem used to identify the action indicators. The second one was then used to identify the process and object indicators. The last question was used to identify the schema indicator.

Table 2. Trigonometric test

Indicator	Question
Action	1. Given an acute-angled triangle ABC at A with AB = 3 cm Moreover, AC = 4 cm as shown below. Determine: a. The length of the hypotenuse b. The front side of angle B, Side on side of angle B c. The front side of corner C d. Side of the corner C 
Processes-Object	2. Given an acute-angled triangle XYZ at X with YZ = x cm and XZ = y cm, determine: a. Value $\sin Y$ and $\cos Y$ b. Value $sin^2Y + cos^2Y$ c. Using the concept of trigonometric comparison in right triangles and the Pythagorean formula, prove that $sin^2Y + cos^2Y = 1$
Schema	3. A surveyor wants to measure the width of a river. He stands at point A on the riverbank and observes point C, which is directly across the river. Then, he walks 50 meters along the riverbank to point B. From point B, he measures the angle BAC 90° and angle ABC is 60°. Explain how the concept of trigonometric comparison in right triangles can help the surveyor determine the width of the river (length AC)!

Before being used for data collection, the test instruments were first validated by two experts in the field of mathematics. The data from two experts were analyzed using the Aiken value formula, which yielded a value of 0.8. This result indicates that the questions were deemed valid, with questions 1, 2, and 3 achieving a value of more than 0.8, which categorizes them as valid. The questions have also been tested for reliability using Cronbach's Alpha, with a result of 0.87, indicating that both possess high reliability (Taber, 2018). Furthermore, data from 42 students' worksheets of the trigonometric tests were analyzed using the scoring rubric presented in Table 3.

Table 3. Rubric for assessing students' concept formation

Score	Action	Processes	Object	Schema
4	Able to perform all basic actions related to trigonometry, including identifying the front, side, or oblique, and finding the value of special angles.	Able to perform a series of procedural steps to solve fundamental trigonometric problems independently, accurately, and rarely make procedural errors.	Able to view trigonometric concepts as objects that can be manipulated to solve more complex or slightly varied problems. Understand the relationship between these concepts.	Able to integrate all trigonometric concepts into a coherent schema. Able to generalize and apply trigonometric functions in non-routine or complex situations, even connecting them with other mathematical concepts.
3	Able to perform some basic actions related to trigonometry independently, including identifying the front, side, oblique, or finding the value of special angles.	Able to perform a series of procedural steps to solve fundamental trigonometric problems, but sometimes still makes minor errors in the sequence or details of calculations.	Able to view trigonometric concepts as objects that can be manipulated to solve more complex or slightly varied problems. However, still limited to frequently used concepts.	Able to integrate all trigonometric concepts into a coherent schema, but not yet generalize and apply trigonometric concepts in non-routine or complex situations, even connecting them with other mathematical concepts.
2	Only able to perform some basic actions with significant assistance or guidance when identifying the front, side, hypotenuse, or finding the value of a special angle.	Only able to perform a series of procedural steps to solve basic trigonometric problems with significant assistance, or if the process is very similar to an example.	Less able to view trigonometric concepts as objects that can be manipulated to solve more complex or slightly varied problems. However, still limited to frequently used concepts.	Not yet able to integrate all trigonometric concepts into a coherent schema. Difficult to generalize and apply trigonometric functions in non-routine or complex situations, even when connecting them with other mathematical concepts.
1	Unable to perform any basic actions related to trigonometry, including identifying front, side, oblique, or finding the value of special angles.	Unable to perform a series of procedural steps to solve basic trigonometric problems with significant assistance, or if the process is very similar to an example.	Unable to view trigonometric concepts as objects that can be manipulated to solve more complex or slightly varied problems, and unable to understand the relationships between these concepts.	Unable to integrate all trigonometric concepts into a coherent schema. Unable to generalize and apply trigonometric functions in non-routine or complex situations, even connecting it with other mathematical concepts.

Based on the document analysis of the students' worksheet, researchers grouped them into three levels of trigonometric ability. The grouping provisions are presented in Table 4. The score of trigonometric ability is measured using the following formula.

$$Score = \frac{Total\ Scores}{32} x\ 100$$

Table 4. Categories of students' trigonometric ability

Trigonometric Ability	Score
High	$x \geq 75$
Medium	$60 \leq x < 75$
Low	$x < 60$

(Damayanti et al., 2024)

Based on the analysis of the students' worksheets, 20 students (48%) were placed in the high category, 18 students (43%) in the medium category, and 4 students (9%) in the low category. Furthermore, the researchers then select three students who represent each category. After discussing with the mathematics teacher, three students were selected: SH for students with high trigonometric ability, SM for those with medium ability, and SL for those in the low category. To validate the data from the students' worksheet, the researchers used in-depth interviews to reveal the students' thinking processes in solving trigonometric problems based on APOS indicators.

Results and Discussion

Action

High-ability students fully meet the concept formation indicators that enable them to successfully construct and integrate new trigonometric ratio schemes with existing knowledge, such as the Pythagorean theorem scheme from plane geometry. The results of the SH answers to the first problem of the action indicator are shown in Figure 1 below.

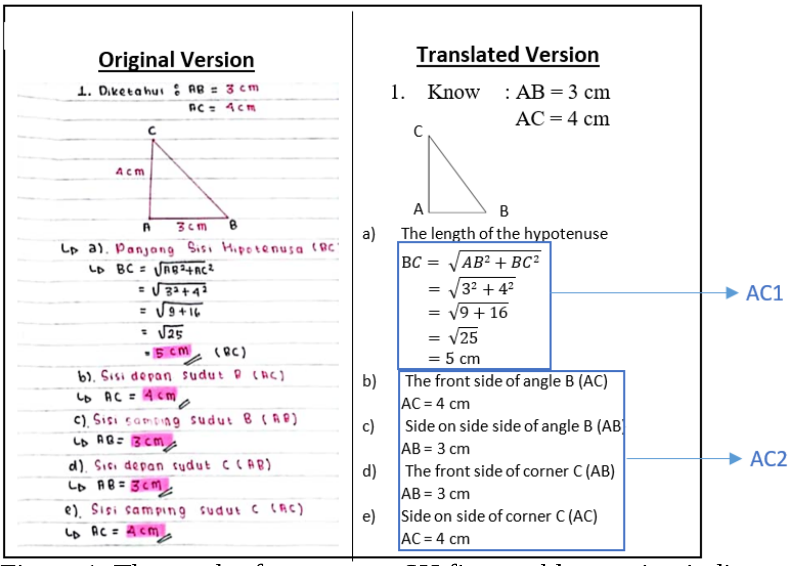


Figure 1. The result of answers to SH first problem action indicator

Figure 1 indicates that SH successfully performed detailed calculations according to the instructions, explained the steps independently, accurately identified the necessary concepts, and selected the correct solution strategy. Overall, SH fulfilled the action indicators for concept formation ability while solving the problem. Furthermore, Figure 1 shows that SH's response clearly articulated the known and requested information in the problem. Crucially, SH was able to solve the initial problem, which involved the Pythagoras theorem, demonstrating its interrelation with the subsequent problem. The following excerpt from the interview then supports this analysis.

P : *When you saw the right triangle in question number 1, what was your first thought to find the sides? (AC1)*
SH : *Find the length of the hypotenuse CB using the Pythagorean formula.*

Based on Figure 2 and interview results, SH successfully demonstrated the ability to perform all basic actions related to trigonometry independently upon explicit instruction, including the critical skill of identifying the opposite, adjacent, and hypotenuse sides of a triangle. Furthermore, students with medium ability failed to meet all concept indicators. SM has not successfully constructed a comprehensive trigonometric ratio schema and demonstrates a limited capacity to effectively integrate this concept with prior knowledge, such as the Pythagorean theorem schema from plane geometry (Figure 2).

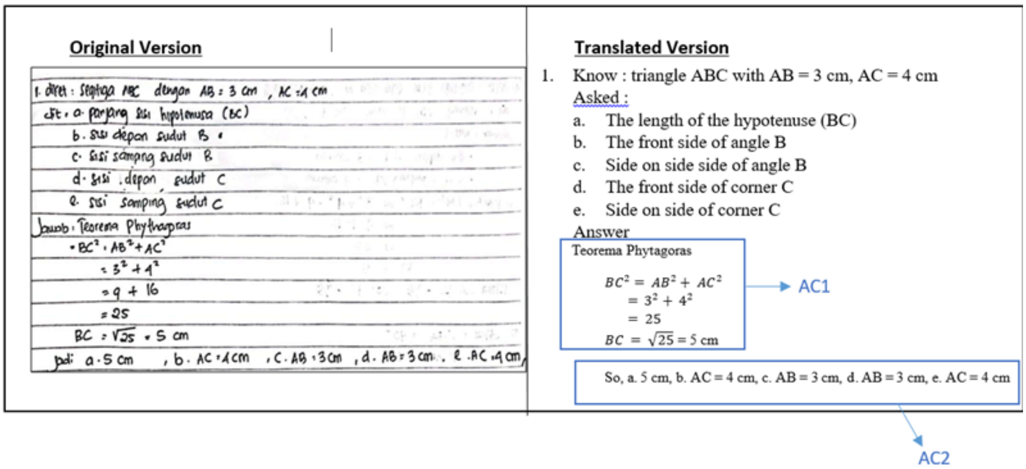


Figure 2. The result of answers to the SM first problem action indicator

Figure 2 shows that SM successfully solved the initial problem, but incorrectly redrawn the triangle. SM also applied the Pythagorean Theorem, performed detailed calculations, identified concepts, and chose the appropriate strategy. Overall, the answer given by SM is the same as SH, so SM is said to be able to solve the problem by fulfilling the action indicator in the ability to form concepts. The following interview excerpt supports SM's answer.

P : *When you saw the right triangle in question number 1, what was your first thought to find the sides? (AC1)*
SM : *What I thought of was the Pythagorean formula.*

Based on Figure 2 and interviews, it appears that SM successfully met the action indicators for concept formation by performing routine calculations and all basic trigonometric actions independently, including accurately identifying the sides of a triangle based on explicit instructions. In addition, students with low abilities experience difficulties at the basic level, characterized by an inability to identify opposite sides, adjacent sides, and hypotenuses consistently, or by making mistakes in choosing the correct trigonometric ratios (Figure 3). These indicators suggest that students are still at the action stage of concept formation, according to APOS theory, which requires intervention focused on strengthening the basics and providing concrete experiences before progressing to more abstract understanding.

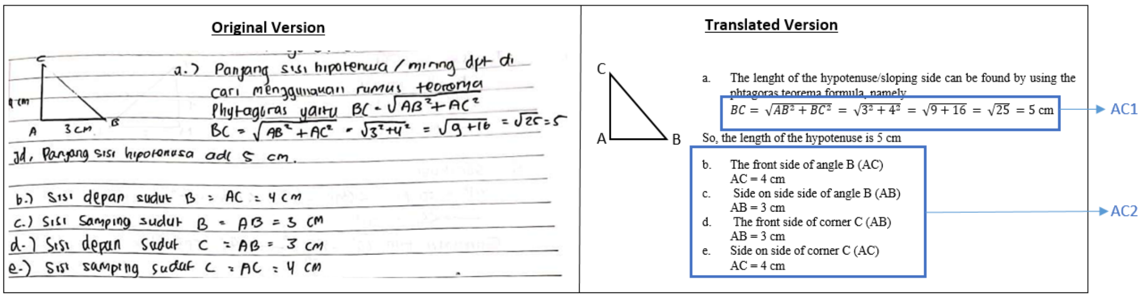


Figure 3. The result of answers to SL first problem action indicator

Figure 3 reveals that SL solved the first problem completely, accurately applying the Pythagorean theorem and performing all required calculations, steps, and concept identification. Overall, the answer given by SL is the same as SH, so SL is said to be able to solve the problem by fulfilling the action indicator on concept formation ability. The following interview excerpt supports SM's answer.

P	:	*When you saw the right triangle in question number 1, what did you think of first to find the sides (AC1)*
SL	:	*I used the Pythagorean formula.*

Thus, SL fulfills the action indicator by performing independent trigonometric calculations, but the interview revealed a reliance on guessing rather than analyzing problem sides and angles, suggesting a need for guidance on analytical habits. Based on the analysis of three different categories in action indicators, SH, SM, and SL can perform basic actions related to trigonometry, such as routine calculations using formulas (e.g., the Pythagorean theorem) and identifying the sides of triangles. The difference lies in the level of independence and accuracy in the initial analysis, with high-ability students exhibiting a more mature analysis than those of medium and low ability.

Processes

In the process indicator, SH was able to solve the problem appropriately, writing down what was known and what was asked, and providing a picture of a triangular flat shape to facilitate his analysis. After that, SH solved the problem by using the concept of the trigonometric comparison formula. Performing operations with sequential steps, as well as the calculations, is done correctly. The results of SM's answers to the second problem of process indicators are shown in Figure 4.

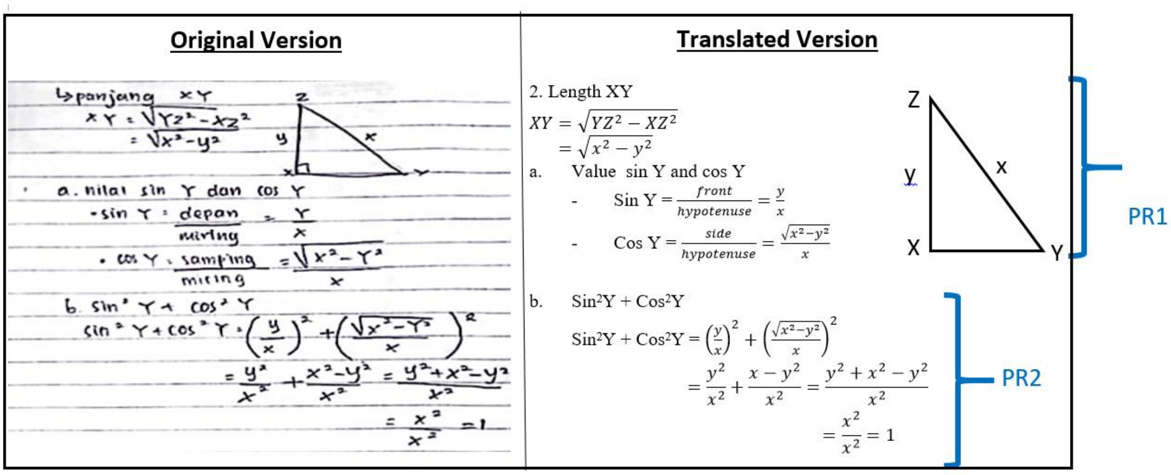


Figure 4. The result of answers to SH second problem processes indicator

Figure 4 shows that SH successfully solved the second problem for the process indicator. SH demonstrated proficiency by clearly recording known and requested information, utilizing a triangle diagram for analysis, correctly applying the trigonometric ratio formula, and executing sequential, accurate calculations. This figure is supported by the results of the SH interview as follows.

P	:	*Given the triangle in the problem to find the value of sin y and cos y, how do you determine the sides? (PR1)*
SH	:	*The first step, I marked the angle y, then determined the front, oblique, and side.*
P	:	*Do you feel that the relationship sin²y + cos²y = 1 is something that needs to be proven or something that you memorize?" (PR2)*
SH	:	*I think it is something that needs to be proven.*

Based on the analysis of the student's worksheet and interview, SH demonstrated a powerful ability to solve trigonometry problems. SH systematically identified known and requested information, used visual representations for analysis, and solved problems by applying trigonometric ratio concepts accurately through sequential and precise steps. Most importantly, the interview revealed that SH has a deep conceptual understanding rather than mere memorization; this is evidenced by SH's statement that the identity $\sin^2 y + \cos^2 y = 1$ requires proof, thus demonstrating critical thinking skills and contextual mastery of the material.

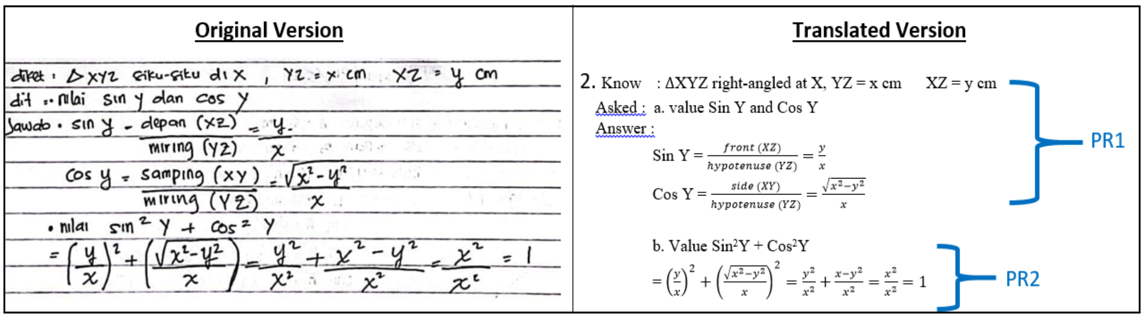


Figure 5. The result of answers to the SM second problem processes indicator

For the medium category, SM began the trigonometric process by writing down the known information first. Figure 5 shows that SM generally performed well on process indicators, accurately documenting their knowledge and using trigonometric concepts through sequential steps. However, the written solutions were incomplete, resulting in minor calculation errors. This is supported by the results of SM's interview as follows.

P : *Given the triangle in the problem to find the value of sin y and cos y, how do you determine the sides? (PR1)*
SM : *The first step is to look at the angle, then determine the front, hypotenuse, and side, and put it into the formula.*
P : *Do you feel that the relationship $\sin^2 y + \cos^2 y = 1$ is something that needs to be proven or something that you memorize? (PR2)*
SM : *I think it is something that needs to be proven.*

Thus, SM has not fully met the process indicator for concept formation ability, which is characterized by minor errors in the sequence or details of calculations, so it takes some time to recall all the steps or formulas used. Furthermore, SL did not provide a complete answer. SL immediately drew a triangle without any analysis first. Figure 6 shows that SL also did not provide a complete answer, and no appropriate solution strategy was identified. Then, the researcher interviewed SL to further explore the concept. The interview process is as follows.

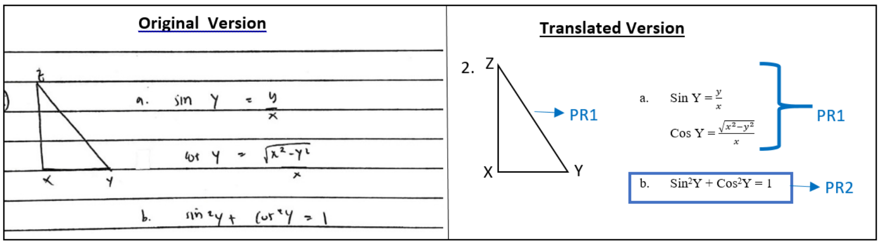


Figure 6. The result of answers to the SL second problem processes indicator

Figure 6 shows that SL solved the process indicator problem by attempting to determine sinY and cosY, successfully identifying several sides, and applying the Pythagorean Theorem. However, an error occurred in assigning the signs, resulting in an incorrect SL process, which highlights the need for improvement to ensure consistent and accurate results. This is supported by the results of SM's interview as follows.

P	:	*Given the triangle in the problem to find the value of sin y and cos y, how do you determine the sides? (PR1)*
SL	:	*Determine the front, oblique, and side.*
P	:	*Do you feel that the relationship sin²y + cos²y = 1 is something that needs to be proven or something that you memorize? (PR2)*
SL	:	*I think it is something that is memorized.*

Based on Figure 6 and the interview results, SM has not fulfilled the process indicators for forming concepts, which is characterized by numerous errors in the solution. This is indicated by SL not being able to explain the steps of the solution and not performing the operation altogether. Thus, it takes time to remember all the steps or formulas used.

In conclusion, there is a significant difference in process indicators. SH showed strong conceptual understanding and systematic trigonometric ability. SM had a basic understanding, but there were often minor flaws in detail or completeness. Then, SL showed substantial difficulty explaining steps and applying concepts, making frequent conceptual errors.

Object

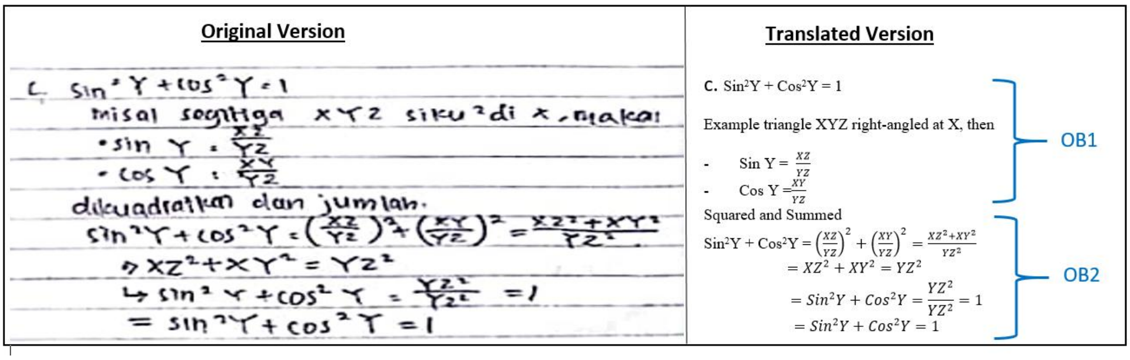


Figure 7. The result of answers to the SH second problem object indicator

In the object indicator, SH was able to identify the concept the solution used, the concept it used to prove other concepts, and distinguish the concept of trigonometry from other concepts. Figure 7 showed that SH was able to perform operations with sequential steps and calculate correctly. The initial steps involve defining sine and cosine, followed by squaring and adding, and recognizing trigonometric identities as intact and functional mathematical facts. This is supported by the results of the SH interview as follows.

P	:	*Can you explain your thought process when proving that sin²y + cos²y = 1 using the Pythagorean theorem, which one did you use to prove that? (OB2)*
SH	:	*I used sin y =* $\frac{a}{c}$ *so sin²y =* $\frac{a^2}{c^2}$ *and cos y =* $\frac{b}{c}$ *so, cos²y =* $\frac{b^2}{c^2}$. *For sin²y + cos²y =* $\frac{a^2}{c^2}+\frac{b^2}{c^2}=\frac{a^2+b^2}{c^2}=\frac{c^2}{c^2}=1$. *So, it is true that the value is 1.*

Thus, based on Figure 7 and the interview results, SH fulfills all the objective indicators. It is suggested that students view trigonometric concepts as objects that can be manipulated to solve more complex or slightly varied problems, thereby gaining a deeper understanding of the relationships between these concepts. Meanwhile, SM was able to identify the concept as a solution; however, he did not write down the trigonometric operations but provided an explanation related to the concept used to solve the problem. Thus, SM was not fully able to fulfill the object indicator.

Original Version	Translated Version
• $\sin^2 Y + \cos^2 Y = 1$ Karena definisi sin dan cos menggunakan sisi dalam segitiga siku-siku yang memenuhi teorema phythagoras.	c. $\text{Sin}^2Y + \text{Cos}^2Y = 1$ Because the definitions of sin and cos use the inner side of a right triangle that satisfies the phytagoras theorema

OB2

Figure 8. The result of answers to the SM second problem object indicator

Figure 9 illustrates that SM only displays the identity for $\sin^2 y + \cos^2 y = 1$, along with a brief explanation in a right triangle that satisfies the Pythagorean theorem, without providing any proof steps. This indicates that SM demonstrated a conceptual understanding of the identity's relationship with Pythagoras but had not yet reached the Object stage because there was no visual evidence of the proof steps being transformed into an entity that could be formally manipulated. This is supported by the results of SM's interview as follows.

P	:	*Can you explain your thought process when proving that $\sin^2 y + \cos^2 y = 1$ using the Pythagorean theorem? (OB2)*
SM	:	*I did not write the proof using trigonometric, but I explained the concepts used.*
P	:	*Are you sure the concept you wrote down is correct?*
SM	:	*Yes, I am sure.*

Based on Figure 8 and the interview results, SM has not been able to fulfill the object indicator on the concept of trigonometry. It can be seen that SM did not write the solution steps to identify the trigonometric concept, but only provided an explanation related to the concept used. Furthermore, SL has not been able to fulfill the object indicator on the concept of trigonometry. Figure 9 shows that SL did not write the solution steps to identify trigonometric concepts, as they were unable to identify and distinguish the concepts used.

Original Version	Translated Version
c. $\sin^2 y + \cos^2 y = 1$ $\frac{y}{x} + \frac{\sqrt{x^2-y^2}}{x} = 1$	c. $\text{Sin}^2Y + \text{Cos}^2Y = 1$ $\frac{y}{x} + \frac{\sqrt{x^2-y^2}}{x} = 1$

OB2

Figure 9. The result of answers to SL second problem object indicator

Based on Figure 9, it can be seen that SL tries to identify the problem. However, there are no squaring and addition steps shown to produce trigonometric identities, as well as potential conceptual errors in determining the values of sinY and cosY. Thus, SL has not reached the Object stage for this identity. The proof process has not been written completely and accurately. In this case, the researcher interviewed SL to further explore the concept. The interview process is as follows.

P	:	*Can you explain your thought process when proving that $sin^2y + cos^2y = 1$ using the Pythagorean theorem?" (OB2)*
SL	:	*I forgot how to prove it, all I know is that the value $sin^2y + cos^2y = 1$.*
P	:	*What made you forget?*
SL	:	*I memorized the proof, and when I was given the problem, I forgot.*

Based on SL's answer and the interview, SL explicitly stated, "I forgot how to prove it," and admitted that he just "memorized the proof." It shows that he has not been able to describe the steps of the proof and has not understood the identity. Thus, SL has not been able to fulfill the object indicator.

Based on the analysis of three subjects, SH shows the ability to manipulate concepts as objects and understand the relationship between concepts. SM has a conceptual understanding but struggles with formalization and proof. While SL tends to memorize without a deep understanding, making it challenging to identify and distinguish concepts.

Schema

SH correctly solved the third problem using trigonometric comparison but displayed a weakness in the solution schema. While the final answer was correct and supported by a picture, SH failed to document key steps, including listing the knowns, the question, the formula, and the complete solution steps. This result indicates strong conceptual ability but poor formal communication of the process.

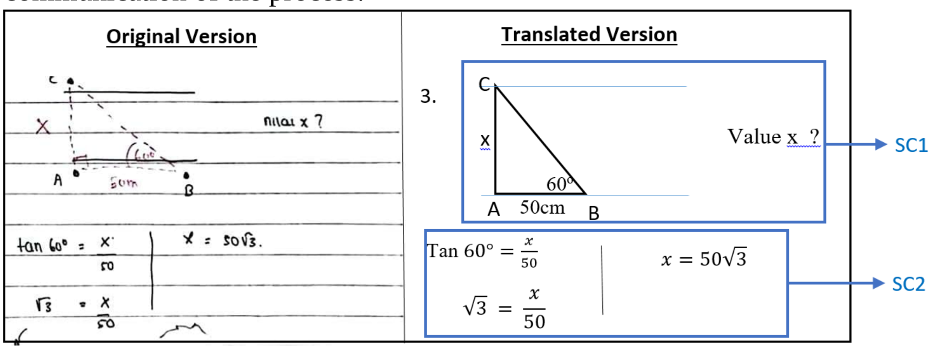


Figure 10. The result of answers to SH second problem schema indicator

Based on Figure 10, SH achieved the correct answer and showed an understanding of trigonometric concepts that led to the Schema stage. However, the steps were still incompletely presented. There is no explicit identification of variables, indicating that although students can apply solutions in a coordinated manner to solve problems, the presentation of their thoughts and assumptions is not yet fully formal or comprehensive. This is supported by the interview results with SH, as follows.

P	:	*Given the contextual problem, what immediately comes to your mind? (SC1)*
SH	:	*Immediately to the picture of a right triangle with its ABC position.*
P	:	*Did you write the information from the problem in your answer? (SC1)*
SH	:	*Oh yes, I forgot to give the information, but I wrote it on the picture.*
P	:	*If there is another context, can you apply a similar method? (SC2)*
SH	:	*Yes, I use that method if I find a problem with a similar context.*

SH failed the schema indicator because he could not generalize and apply trigonometry in non-routine situations or explain their thinking coherently. This failure was confirmed during the interview, where SH admitted to forgetting to document the complete strategy and initial information. Furthermore, SM has also not fulfilled the schema indicator in integrating concepts to solve problems by using a solution strategy that is in accordance with trigonometric comparison. The result obtained from the solution is correct; however, some concepts and solving strategies remain incomplete, providing insufficient clarity for contextual problems.

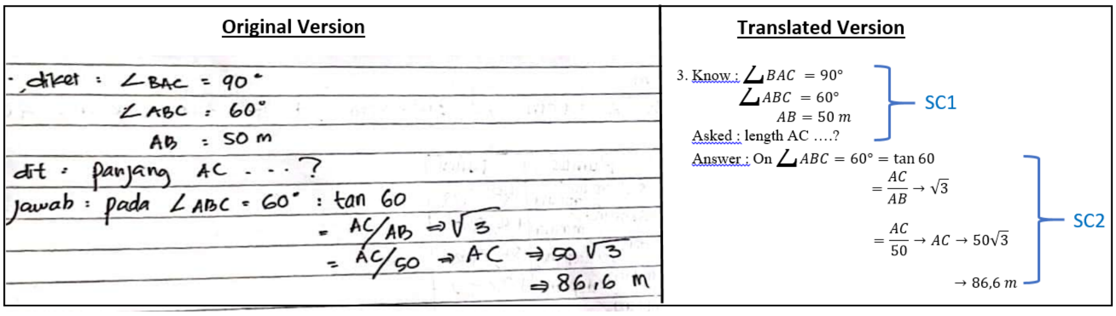


Figure 11. The result of answers to SM second problem schema indicator

Figure 11 shows that SM has a strong functional scheme in solving application problems, successfully identifying information, selecting the appropriate trigonometric function (tan), and calculating accurately. However, the solution has shortcomings in its presentation, including the triangle diagram, justification for the function selection, and explanation of variables. This result indicates the need to improve the comprehensive communication of mathematical thinking processes. This is supported by the interview results with SM, as follows.

P	:	*How did you decide to use the concept of trigonometry there? (SC1)*
SM	:	*Because it is a right triangle.*
P	:	*In your solution, what do you mean by* $\sqrt{3}$ *on the side? (SC2)*
SM	:	*I think* $\sqrt{3}$ *is the value of.*
P	:	*Next step? (SC2)*
SM	:	*I forgot, sir.*
P	:	*I see, if there is another context, can you apply a similar method? (SC2)*
SM	:	*If the problem is a right triangle, maybe I can.*

Thus, SM has not fulfilled the schema indicator, characterized by an inability to integrate all trigonometric concepts coherently. The interview confirmed this limitation, as SM was unable to explain how to complete an unfinished solution and showed hesitation when linking trigonometry to other concepts. Furthermore, SL also fails to fulfill the schema indicator in integrating concepts to solve problems by using a solution strategy that is inconsistent with trigonometric comparison. In the first step, SL took was to draw triangle ABC, but the triangle drawn was inappropriate. Thus, the final solution by SL is incorrect. The problem arose because SL could not analyze the concepts in the contextual problem.

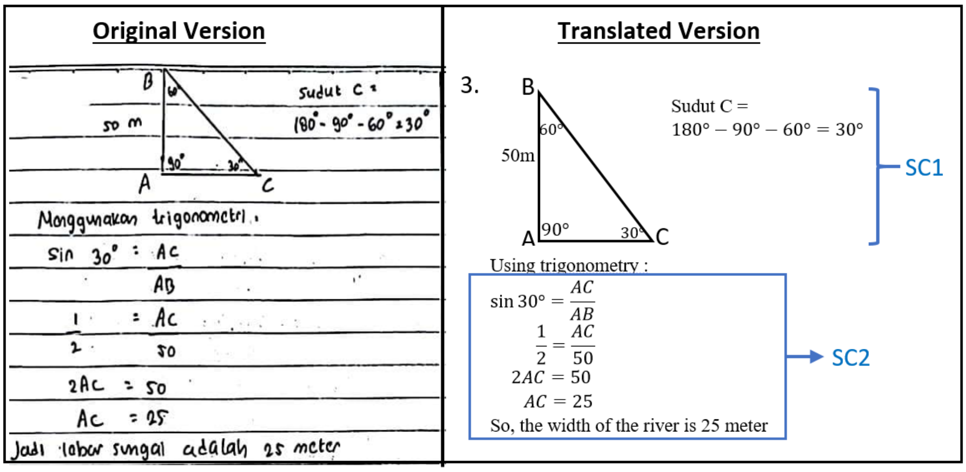


Figure 12. The result of answers to SL second problem schema indicator

Figure 12 illustrates that SL enables the visualization of problems, identification of information, and calculation of results. However, this schema is not entirely accurate because it contains significant conceptual errors in trigonometric identities, indicating a misconception in the overall schema. The results of the interview with SL support this.

P	:	*How did you decide to use the concept of trigonometry there? (SC1)*
SL	:	*From the information in the problem.*
P	:	*How are all the concepts you learned connected to solve this problem? (SC2)*
SL	:	*I understand from the formula.*
P	:	*If there is another context, can you apply a similar method? (SC2)*
SL	:	*Maybe I can or maybe not.*

SL students have not met the scheme indicators due to their inability to coherently integrate trigonometric concepts, generalize applications, or explain their thinking logically, despite having sufficient verbal understanding. Based on the document and interview analysis. The schema formation varies significantly for the three subjects. SH integrates well but needs better presentation. SM only explains partial integration and lacks confidence during interviews, while SL has deep misunderstandings and fails to integrate concepts for complex problems coherently.

Highly trigonometrically able students demonstrate a comprehensive and structured mastery of concepts, completing all stages of the APOS framework. They can perform Actions accurately, internalize them as flexible Processes by understanding that trigonometric identities must be proven, manipulate concepts as Objects, and reach the Schema stage. This finding aligns with research conducted by (Mulbar et al., 2017), which found that students with high mathematical ability and a perspective rooted in APOS Theory can effectively connect all the stages they have gone through. Furthermore, research results by Kusuma and Masduki (2016) show that students in the high category tend to have a better level of concept formation than those in the medium and low categories. In line with Syaiful et al. (2025) research, on the process indicator, students with high mathematical abilities can apply concepts algorithmically well in solving problems, students with medium mathematical abilities are less able to develop the concepts learned, and students with low mathematical abilities must dig deeper to develop their concepts.

Students with medium abilities have a sufficient foundation at the Action and Processes stages, but they are hindered in execution, often making small mistakes and struggling with formalization. This failure prevents them from fully reaching the solid Object and Schema stages. On the other hand, low-ability students fail from the start; their concepts are fragmented due to reliance on memorization, performing Actions based on guesswork, and only seeing identities as something to be memorized. As a result, they fail to manipulate concepts as Objects and are unable to form Schemas at all. In line with the research conducted by Mahfudhoh and Kurniasari (2025), the results of their study indicate that all three levels (high, medium, and low) have reached the action stage, which involves correctly calculating the central measure of data. According to research conducted by Yerizon et al. (2024), students with medium mathematical abilities do not fully grasp the concept, while students with low mathematical abilities are still unable to understand the concept's identity relationship. In line with research conducted by Afgani et al. (2017) students with low mathematical concept comprehension abilities have only reached the action stage.

Conclusion

The results of this study provide an overview of how students' concept formation in solving trigonometry problems. Concept formation involves building actions, processes, and objects into a schema to solve mathematical problems. Students with high concept formation ability can achieve all APOS indicators (action, process, object, schema) and integrate trigonometric concepts with other materials, such as the Pythagorean theorem. In contrast, students with medium ability tend not to fully achieve the indicators of concept formation, as they still exhibit minor errors in calculations, and they are less able to connect trigonometric schemas with other concepts fully. Meanwhile, students with low ability experienced significant difficulties in the process, object, and schema stages, characterized by the inability to identify the sides of the triangle, errors in choosing trigonometric ratios, and a tendency to memorize without deep conceptual understanding. This study highlights the importance of teaching approaches tailored to students' trigonometric abilities, facilitating more effective concept formation.

Although this study provides a general insight of how students' concept formation based in their trigonometry ability, several limitations are noteworthy. First, the study subjects were limited to three students at a single school in Boyolali District. This prevents generalization to a broader population or to different educational contexts. Second, this study only analyzed students' concept formation in Trigonometry. Therefore, the findings cannot yet represent students' concept formation in other topics. Third, the data collection technique was limited to tests and interviews. The use of other data collection techniques, such as think-aloud protocols and observations, would provide a more comprehensive picture. Based on the limitations of the study described, further research can expand the research subjects to different educational contexts, analyze concept formation in other mathematical topics, and use more diverse data collection techniques. This will allow for more comprehensive research findings related to students' concept formation.

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