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# Developing Worked Examples to Strengthen Mathematical Communication Skills in Teaching Statistics

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***Abstract.*** *Mathematical communication skills are an essential part of mathematics learning; however, numerous studies have found that students' abilities in this area remain at a low to moderate level across various mathematical topics. This study aims to design and present worked examples that support mathematical communication skills, particularly in the context of measures of central tendency and data dispersion. This study employed a Research and Development approach, utilizing the Analysis, Design, Development, Implementation, and Evaluation (ADDIE). The participants consisted of five Year 8 students who had not yet studied the topic. Data were collected through open-ended interviews, paper-based tests, and expert validation sheets. The developed worked examples were validated by three experts based on content, construction, and linguistic criteria. The validation results indicated that the worked example met both face and logical validity criteria. The ADDIE cycle was completed after the Evaluation stage, during which the result of the limited trials showed that more than 80% students achieved the minimum score in solving the paired problem. These findings indicated that worked examples designed with self-explanation prompts and contextual variations can facilitate students' mathematical communication skills, especially in understanding and expressing mathematical ideas through multiple representations.*

***Keywords****: mathematical communication skills, self-explanation, worked example*

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## Introduction

Communication skills are among the essential 21st-century skills for future work life (Taylor et al., 2022). In the current era of rapid information growth, the ability to merely distinguish between right and wrong is insufficient. An individual also needs the ability to make and convey reasonable arguments with appropriate justifications in order to defend their perspective (OECD, 2022). Therefore, the ability to speak, listen, write, and convey ideas effectively, which is referred to as communication skills, is indispensable.

In mathematics, communication skills are defined as the ability to share ideas, clarify understanding, and construct meaning in a mathematical context (NCTM, 2000), which is often termed as mathematical communication skills. These skills are an essential part of mathematics learning, which ensures continuous knowledge development throughout the learning process. Mathematical communication involves not only establishing mutual mathematical understanding during the learning process, but also applying and transferring mathematical knowledge to real-life problems (Kaya & Aydin, 2016). Therefore, mathematical communication extends beyond providing correct answers; it encompasses the ability to articulate mathematical ideas clearly, justify reasoning, and employ multiple representations such as graphs, tables, and equations.

However, mathematical communication does not naturally develop during the mathematics learning. Various studies have reported that students' mathematical communication skills across various mathematical topics remain at a low to moderate level (Aminah et al., 2018; Nurlaila et al., 2018). Rohid et al. (2019) found that only one out of three students demonstrates adequate mathematical communication skills. Many students struggle to articulate their mathematical thoughts both verbally and in writing (Teledahl, 2017). Students also face difficulties in using appropriate representations, such as graphs and tables, to support their arguments or provide sufficient justification for their solutions (Olteanu, 2015).

In educational practice, mathematical communication skills are often overlooked, as evident in assessments that generally focus on evaluating procedural abilities, such as comprehension and problem-solving (Staščik & Glavaš, 2023). These situations gradually hinder the development of students' mathematical communication skills, impeding their ability to collaborate on mathematical tasks (Baran & Kabael, 2021). According to the TIMSS report, communication between teachers and students, as well as among students themselves, remains low, despite the duration of mathematics instruction in Indonesia being longer than in other countries (Human Development Department, 2010). As a result, a substantial portion of learning time is spent on problem-solving and classroom organization, rather than on activities that support students' mathematical communication, such as reflection and discussion. Therefore, mathematical communication skills must be explicitly integrated into classroom practice.

Worked example is one instructional strategy to support the development of mathematical skills. While they are generally associated with transfer abilities such as problem-solving, recent research highlights their potential to foster communication skills. Bosshard et al. (2023) reported that worked examples can potentially enhance learners' communication skills. Similarly, research conducted by Vest et al. (2022) found that incorporating worked examples in mathematics learning fosters deeper conceptual and procedural understanding, which helps students provide faster and more accurate explanations when solving problems. Chen et al. (2023) also showed that worked examples in mathematics learning enhance students' retention abilities and working memory resources. Furthermore, greater working memory resources enhance the ability to understand and translate problems into mathematical models (Ji & Guo, 2023), a skill closely related to mathematical communication skills.

Given this background, the present study seeks to design worked examples that support mathematical communication skills. Since research specifically exploring the role of worked examples in fostering mathematical communication remains limited, this study aims to address that gap. Therefore, it examines and presents a worked example design to support students' mathematical communication skills, particularly within the topic of central tendency and measures of data dispersion.

## Method

This study was conducted using the Research and Development (R&D) method. R&D is a systematic approach that aims to build an empirical basis for the creation of educational products or instructional tools. This research was performed in five stages: Analysis, Design and Development, Implementation, and Evaluation (Table 1). The ADDIE framework is commonly used as guidelines for designing educational products. The Analysis stage focused on identifying the learning objectives related to measures of central tendency and data dispersion, along with the indicators of mathematical communication skills. The Design stage examined the principles that are considered in developing worked examples and their connection to mathematical communication skills. This principle served as the foundation for the development stage, during which worked examples were designed.

Table 1. Research stages

| Analyse | Design | Development | Implementation | Evaluation |
|---|---|---|---|---|
| 1. Analysis of learning outcomes in the topic of measures of central tendency and data dispersion<br>2. Analysis of the indicator of mathematical communication skills<br>3. Analysis of worked example principles | 1. Design worked example concepts aligned with mathematical communication skills.<br>2. Designing problem contexts related to measures of central tendency and data dispersion, aligned with the mathematical communication indicator | 1. Developing worked examples to facilitate mathematical communication skills<br>2. Validating worked examples and paired questions with experts | 1. Conducting a limited trial of worked examples and paired questions | 1. Assess the quality of the worked example and paired questions |

## Results and Discussion

The following section reports the implementation of each stage of the ADDIE framework in designing worked examples.

*Analyse*

The analysis stage examined the learning outcomes related to measures of central tendency and dispersion, indicators of mathematical communication skills, and the principles in worked examples. This stage aims to identify the essential competencies students are expected to achieve. The results of this analysis served as the foundation for designing worked examples that align with both the learning objectives and the students' mathematical communication skills.

*Learning Outcomes*

Descriptive statistics, including measures of central tendency and dispersion, are fundamental techniques used to describe the distribution of values within the data set (Lee, 2020). Measures of central tendency are used to identify the central point of a data set, typically represented by the mean, median, and mode. Measures of dispersion, on the other hand, are an extension of measures of central tendency that describe the variability of data, typically represented by range, standard deviation, and interquartile range (Vetter, 2017). When engaging with these topics, students are expected to perform various activities, such as understanding, analyzing, and interpreting. According to the NCTM (2000), when studying measures of central tendency and dispersion, students should be able to select and apply appropriate data analysis techniques relevant to the problem context, as well as understand the relationship between each type of measure of central tendency and dispersion in a given dataset.

Research by Ismail and Chan (2015) revealed that students' misconceptions about measures of central tendency are often caused by a lack of opportunities to explain their problem-solving techniques. As a result, their understanding is limited to procedural knowledge and formula memorization. In statistics, students need to be allowed to explain the strategies they use in problem-solving to support their conceptual understanding. Furthermore, Karaca and Ay (2025) found that students are frequently able to compute measures of central tendency through data organization and reduction, yet struggle to explain their meaning and interpretation in relation to the data set. Based on this, the learning objectives for measures of central tendency and dispersion are as follows: (1) To calculate measures of central tendency and dispersion of data. (2) To interpret the meaning of measures of central tendency and dispersion. (3) To compare measures of central tendency and dispersion across two or more groups of data.

*Mathematical Communication Skills*

Communication in mathematics refers to the process of conveying ideas and clarifying understanding. According to NCTM (2000), the communication skills standards for middle school students consist of 1) organizing and consolidating mathematical thinking through communication; 2) communicating mathematical thinking coherently and clearly to peers, teachers, and others; 3) analyzing and evaluating the mathematical thinking and strategies of others; 4) using mathematical language to express mathematical ideas precisely. Similarly, Lestari and Yudhanegara (2017) defined mathematical communication skills as the ability to convey mathematical ideas, both verbally and in writing, as well as the ability to understand, analyze, and critically evaluate others' ideas to strengthen understanding.

In this study, mathematical communication skills are defined as an individual's ability to understand their own and others' mathematical ideas and to express, represent, and evaluate them using mathematical language such as symbols, graphs, and diagrams. Therefore, the aspects of mathematical communication skills considered in this research are 1) understanding mathematical ideas in multiple forms, 2) expressing mathematical ideas in written form through symbols, graphs, or diagrams, and 3) interpreting the results and solutions of mathematical problems.

*Worked Example*

A worked example is an instructional materials presenting expert solutions for beginners to study. Worked examples are widely recognized as a strategy that can reduce cognitive load and support the construction of student schemas during learning (Chen et al., 2023). Students who lack a complete schema often encounter difficulties in categorizing problems, which in turn hinders their problem-solving and knowledge transfer. Thus, worked examples are considered an effective approach in mathematics learning.

The central aim of worked example-based learning is to help students understand and explain the domain principles underlying problem solving. When learning using worked examples, students are directed to focus only on problem-relevant activities, thereby giving them more capacity to explain the domain principles underlying problem solutions and build knowledge structures (Renkl, 2017; Sweller et al., 1998). Consequently, self-explanation becomes a critical principle that determines the effectiveness of worked examples. In practice, however, students tend to read worked examples passively without generating sufficient self-explanation, since worked examples often provide complete step-by-step solutions. Therefore, it is necessary to encourage students to actively explain their understanding, which can be facilitated by incorporating self-explanation prompts into worked examples.

*Design*

At this stage, the design of the Worked Example was reviewed to ensure its effectiveness as an instructional innovation for enhancing students' mathematical communication skills. The design process focused on integrating the principles identified in the prior analysis to align the worked examples with the targeted learning objectives. Specifically, the worked examples were developed within the topics of measures of central tendency and data dispersion.

*Design of Worked Examples for Mathematical Communication Skills*

In the context of mathematical communication skills, the design of a worked example should encourage students to understand and present mathematical ideas in various forms of data representation, as well as to interpret the results and solutions to mathematical problems using logical arguments. Therefore, this study focused on designing worked examples that can provide opportunities for students to construct arguments and interpret mathematical solutions. After engaging with the worked example, students are expected to explain the ideas, arguments, and reasoning underlying their approaches to mathematical problems.

Self-explanation is the most appropriate principle for fostering mathematical communication skills. Before students can communicate their mathematical ideas and arguments to others, they must first be able to explain the concepts and strategies to themselves. Self-explanation facilitates the integration of new information into prior knowledge, enhances problem-solving abilities, and deepens understanding after working through examples (Vest et al., 2022). Renkl (2017) also demonstrated that the inclusion of self-explanation in worked examples promotes better knowledge acquisition in students. By applying this principle, students are expected not only to understand concepts through the provided examples but also to communicate the ideas and concepts they have constructed. In this study, worked examples were designed with embedded self-explanation prompts. Each example was accompanied by guiding questions to help students understand the concepts and articulate their reasoning. For every worked example, responses were provided in explanatory form from a student's perspective, after which students were asked to produce their own explanations for a subsequent problem.

*Design of Contextual Problem for Mathematical Communication Skills*

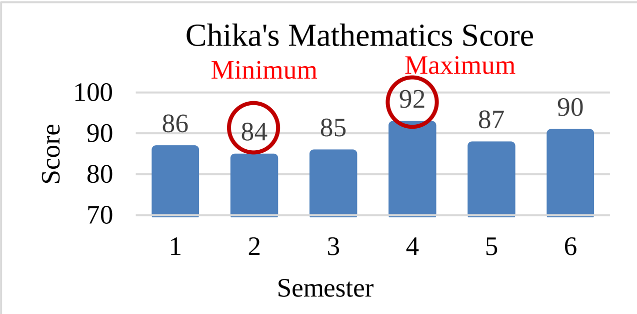
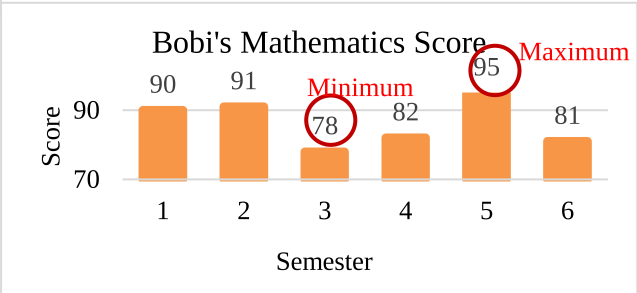
The problem contexts for measures of central tendency and data dispersion were designed as meaningful tasks that reflect real-world situations familiar to students and relevant to their reasoning (Pozas et al., 2020). Mathematical communication skills require comprehensive schemas stored in students' memory to support effective interpretation and argumentation. Therefore, the chosen contexts were those already existing in the students' working memory, such as test scores and personal financial records, so students will be enabled to apply their experiences when working with measures of central tendency and data dispersion.

The problem contexts were also designed to assess students' ability to organize and describe data systematically, as well as to construct and interpret various forms of mathematical representations and solutions. Thus, the purpose of the problems extends beyond computation, aiming to encourage students to articulate explicit arguments. Furthermore, combining contextualized problems with self-explanation prompts is expected to create opportunities for students to determine which measures are more representative within a given context and to communicate their findings effectively both in writing and visual form.

*Development*

At this stage, worked examples were developed in the domain of measures of central tendency and dispersion. The design of worked examples incorporated the principle of self-explanation and was constructed using real-life contexts, such as test scores and financial records. Each principle was aligned with communication skill indicators to ensure that the developed worked examples could facilitate and enhance students' mathematical communication skills.

Table 2. Worked example for mathematical communication skills concerning the measure of data dispersion topic

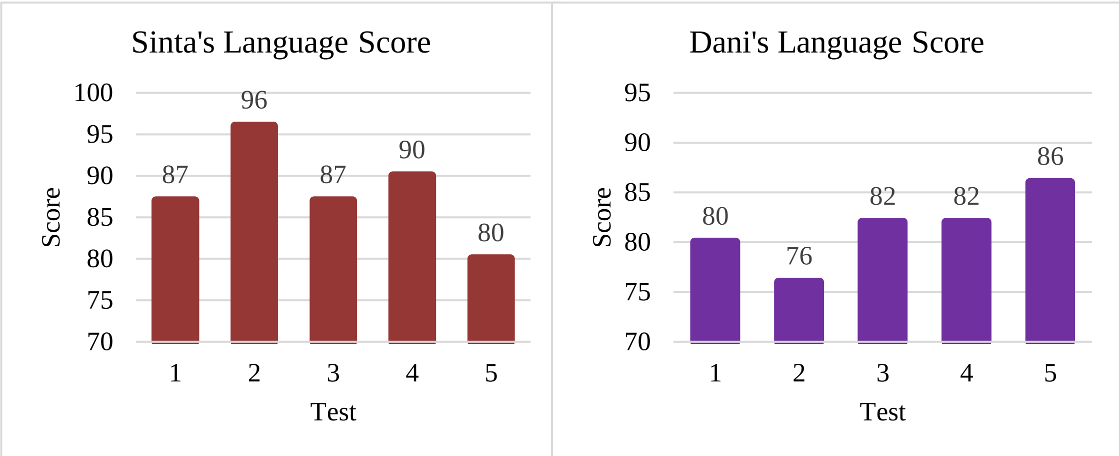
| Worked Example | |
|---|---|
|  <br>  | Chika and Bobi's math scores during their junior high school years are shown in the bar chart beside. Whose math scores are more consistent in terms of their range values? |

| Solution | | Self-explanation |
|---|---|---|
| **Step 1** | Calculate the range of Chika's Score<br>Range = 92-84<br>Range = 8<br>Chika's score range is 8 | **Why did I put special marks on the values in the diagram?**<br>I circled the highest and lowest values because the range is the range of the highest and lowest values in a data set. |
| **Step 2** | Calculate the range of Bimo's Score<br>Range = 96-78<br>Range = 18<br>Bimo's score range is 18 | **Why did I decide these values are more consistent based on the range data measure?** |
| **Step 3** | Interpretation:<br>Based on the range values of Chika and Bimo, it can be concluded that Chika's math scores tend to be more consistent than those of Bimo. | I decided Chika's value is more consistent than Bimo's because the range of Chika's value is smaller than the range of Bimo's value. This indicates that the size of Chika's values is more uniform and closely grouped compared to Bimo's values. |

For the topic of measuring data dispersion, the worked example design is presented in Table 2, along with paired problems in Table 3. The worked example employs a bar chart representation, in which the critical information related to the range is highlighted with circles. In the solutions, the underlined sentence emphasizes the core elements of each problem-solving step. In the self-explanation column, responses are presented from students' perspectives, encouraging them to construct representative schemes and arguments mentally as they explain. The paired problems in Table 3 also include the same explanatory prompts to ensure that students have a structured framework to transfer the mathematical communication skills acquired from the worked examples.

Table 3. Task for mathematical communication skills concerning the measure of data dispersion topic

**Worked Example**




Dani and Shinta took the Indonesian test five times. The scores on each test are shown in the bar chart above. When viewed based on the range value, who has a more stable test score, according to the data above?

| Solution | Self-explanation |
|---|---|
| | **Why did I put special marks on the values in the diagram?**<br>....................................................................<br>....................................................................<br>....................................................................<br>....................................................................<br><br>**Why did I decide these values are more consistent based on the range data measure?**<br>....................................................................<br>....................................................................<br>....................................................................<br>.................................................................... |

For the topic of measures of central tendency, the worked example design is presented in Table 4. The data was represented in narrative form. However, students are still guided to identify key information through underlining, enabling them to focus on relevant aspects for determining measures of central tendency. In the self-explanation column, guiding questions and explanatory responses are provided for each measure (mean, median, and mode) to support students in developing an appropriate understanding of the role of each measure in describing data.

Table 4. Worked example for mathematical communication skills in the measure of central tendency topic

| **Worked Example** | | |
|---|---|---|
| Chika recorded her savings for seven consecutive days as follows: Monday, IDR 53,000; Tuesday, IDR 10,000; Wednesday, IDR 5,000; Thursday, IDR 7,000; Friday, IDR 8,000; Saturday, IDR 6,000; and Sunday, IDR 9,000. Based on the data, determine the most appropriate measure of central tendency (mean, median, or mode) to represent the central value of Chika's savings. | | |
| | **Solution** | **Self-explanation** |
| **Step 1** | Order of Chika's Savings<br>5000, 6000, 7000, 8000, 9000, 10000, 53000<br><br>Total savings<br>$\sum x_b = 53000+10000+5000+7000+8000+6000+9000$<br>$\sum x_b = 98000$<br><br>Mean of savings<br>$\bar{x}_b = \frac{\sum x_b}{f_b}$<br>$\bar{x}_b = \frac{98000}{7}$<br>$\bar{x}_b = 14000$<br>The mean of Chika's savings is 14.000<br><br>Median of savings<br>5000, 6000, 7000, 8000, 9000, 10000, 53000<br>The median of Chika's savings is 8.000<br><br>Mode of savings<br>None<br>Chika's saving data does not have a mode | **Why did I organize the data in this way?**<br>I organized the data to make it easier to determine the median, as the median is the middle value of the ordered data.<br><br>**Can the mean value represent the central value of the data? Why?**<br>The average is less appropriate to represent the central value of the data because it is influenced by the extreme data (outlier), which is 53.000.<br><br>**Can the median value represent the central value of the data? Why?**<br>The median is a more appropriate representation of the central value of the data because it is not influenced by outliers, thereby better reflecting the central location of the data.<br><br>**Can the mode value represent the central value of the data? Why?**<br>There is no mode in Chika's savings data set, so the mode is not an appropriate representation of the central value of the data. |
| **Step 2** | Interpretation<br>Based on the value of mean, median, and mode, the most appropriate measure of central tendency to represent the central value of Chika's savings is the median. | |

For the topic of measures of central tendency, the paired problem is presented in Table 5. In this task, students are presented with a similar problem, accompanied by a worked example and the same explanatory prompts. This design aims to help students transfer their mathematical communication skills by applying the concepts they have learned in a contextual situation.

Table 5. Task for mathematical communication skills in the measure of central tendency topic

<table>
<tr><th colspan="2">Worked Example</th></tr>
<tr><td colspan="2">Bimo is recording the money he spent while staying at his uncle's house. Bimo recorded his expenses as follows: Monday, IDR 9,000; Tuesday, IDR 6,000; Wednesday, IDR 12,000; Thursday, IDR 11,000; and Friday, IDR 7,000. Determine the type of central tendency measure (mean, median, or mode) that best represents the central value of Bimo's expense data.</td></tr>
<tr><th>Solution</th><th>Self-explanation</th></tr>
<tr><td></td><td>Why did I organize the data in this way?<br>.........................................................<br>........................................................<br>........................................................<br>........................................................<br>........................................................<br><br>Can the mean value represent the central value of the data? Why?<br>.........................................................<br>........................................................<br>........................................................<br>........................................................<br>........................................................<br><br>Can the median value represent the central value of the data? Why?<br>.........................................................<br>........................................................<br>........................................................<br>........................................................<br>........................................................<br><br>Can the mode value represent the central value of the data? Why?<br>.........................................................<br>........................................................<br>........................................................<br>........................................................<br>........................................................</td></tr>
</table>

The developed worked examples and three experts validated paired questions. Two experts are lecturers in the mathematics education study program, and one expert is a lecturer in the elementary education study program. In this study, the validation process focused on content validity, which included the validation of the instrument framework (logical validity) and the validation of individual items (face validity). This procedure was intended to ensure that the developed items adequately measured the full scope of the research variables (Allen & Yen, 1979). The validation results presented in Table 6 show that the three experts (E1, E2, and E3) provided consistent assessments of the developed worked examples and paired questions. The logical validity scores ranged from 88.89% to 100%, indicating that the instrument framework was generally valid. Meanwhile, the face validity scores ranged from 72% to 100%, suggesting that all items were appropriate for assessing students' mathematical communication skills. Based on these results, all experts concluded that the worked examples were valid, with only minor revisions required.

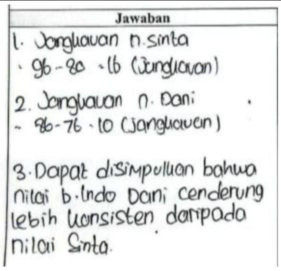
Table 6. Validation results worked example and paired question

| Experts | Percentage | | Conclusion |
|---|---|---|---|
| | Logical | Face | |
| E1 | 88.89 % | 72.00% | Valid with revision |
| E2 | 100% | 98% | Valid with revision |
| E3 | 100% | 100% | Valid with revision |

*Implementation*

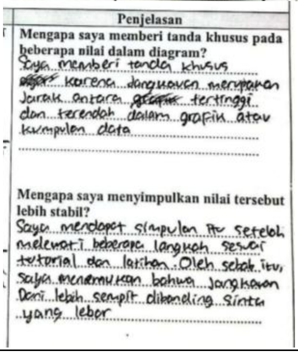
The Implementation phase is the stage where the worked examples were applied to support the learning process and facilitate students' mathematical communication skills. During this phase, a limited trial was conducted involving five Year 8 students from one junior high school in Yogyakarta, Indonesia, who had not yet been introduced to the concepts of measures of central tendency and data dispersion in their formal curriculum. These students were intentionally selected to ensure that all participants were novices in the target topic, allowing the development of their mathematical communication skills to be observed in alignment with the indicators established during the Analysis stage. The students' responses to the worked examples were subsequently analyzed and described qualitatively.

Table 7. First student's (S1) answers

| Student's Answer | Translate |
|---|---|
| <br>Jawaban<br>1. Jangkauan n. sinta<br>· 96 - 80 · 16 (Jangkauan)<br>2. Jangkauan n. Dani<br>- 86 - 76 · 10 (Jangkauan)<br>3. Dapat disimpulkan bahwa nilai b. Indo Dani cenderung lebih konsisten daripada nilai Sinta. | 1. Sinta's score range<br>$96 - 80 = 16$ (range)<br>2. Dani's score range<br>$86 - 76 = 10$ (range)<br>3. It can be concluded that Dani's language score tends to be more consistent than Sinta's score. |
| <br>Penjelasan<br>Mengapa saya memberi tanda khusus pada beberapa nilai dalam diagram?<br>karena Jangkauan adalah selisih dari nilai tertinggi dan terendah dalam suatu kumpulan data<br>Mengapa saya menyimpulkan nilai tersebut lebih stabil?<br>karena Jangkauan nilai Dani lebih kecil daripada nilai Sinta. Hal ini menunjukan bahwa rentang data nilai Sinta lebih kecil / sempit dan relatif seragam daripada nilai Dani | **Why did I put special marks on the values in the diagram?**<br>Because range is the difference between the highest and lowest score in a data set<br><br>**Why did I decide these values are more consistent based on the range data measure?**<br>Because the range of Dani's score is smaller than Sinta's range. This shows that Sinta's score is narrower than Dani's |

In the data distribution measurement task, the responses of two students were highlighted: Student 1 (S1) and Student 2 (S2). As shown in Tables 7 and 8, both students answered all the paired problems with the self-explanation prompt. The results indicate that the worked examples had a substantial impact on their ability to understand mathematical concepts in multiple forms. Both students correctly identified the concept and calculated the range after studying the worked examples. This improvement was evident in their ability to apply the formula for calculating range values. These findings are consistent with Chen et al. (2023), who reported that students learning with worked examples demonstrated stronger problem-solving skills than those who learned without them. Nevertheless, slight differences emerged in the indicators related to interpreting results and solutions of mathematical problems. Although S1 provided the correct conclusion, her self-explanation contained reasoning that was inconsistent with her conclusion. By contrast, S2's reasoning was correct, though less complete than the reasoning presented in the worked examples, suggesting that S2 understood the meaning of the range for the two data sets. Open-ended interviews with the students further revealed that S1 still lacked a clear understanding of how the range reflects data variability, leading to errors in her explanation. In contrast, S2, although relying heavily on the worked example studied, provided an explanation that more accurately reflected the concept of range. This finding aligns with Chen et al. (2019), who emphasized that for novice learners, presenting complete and explicit solutions, including initial and key steps, constitutes an effective worked example design for teachers to adopt.

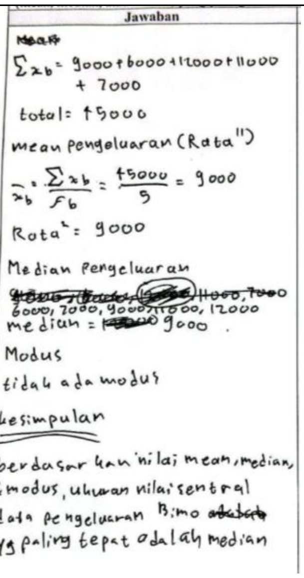
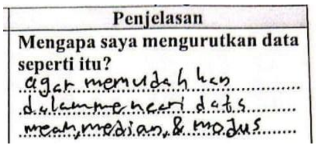
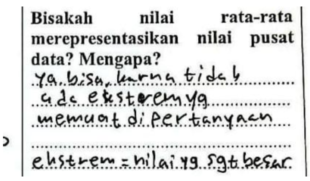
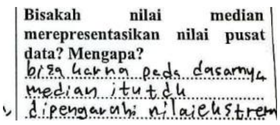
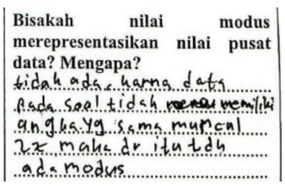
Table 8. Second student's (S2) answer

| Student's Answer | Translate |
|---|---|
| <br>Jawaban<br>Langkah 1: Hitunglah jangkauan nilai Sinta<br>Jangkauan = 96-80<br>= 16<br>Jadi jangkauan Sinta 16<br>Langkah 2: Hitunglah jangkauan nilai Dani<br>Jangkauan = 86-76<br>= 10<br>Jadi jangkauan nilai Dani 10<br>Langkah 3: Kesimpulan: Berdasarkan data yang kita ambil dari grafik atas, bahwa Dani memiliki nilai yang konsisten dan seirama dibanding Sinta | Step 1 Calculate the range of Sinta's score<br>Range = 96-80<br>= 16<br>So, Sinta's score range is 16<br>Step 2 Calculate the range of Doni's score<br>Range = 86-76<br>= 10<br>So, Doni's score range is 10<br>Step 3 Conclusion:<br>Based on the data from the graph above, Dani's scores are consistent than Sinta's. |
| <br>Penjelasan<br>**Mengapa saya memberi tanda khusus pada beberapa nilai dalam diagram?**<br>Saya memberi tanda khusus karena jangkauan merupakan jarak antara tertinggi dan terendah dalam grafik atau kumpulan data<br>**Mengapa saya menyimpulkan nilai tersebut lebih stabil?**<br>Saya mendapat simpulan itu setelah melewati beberapa langkah sesuai tutorial dan latihan. Oleh sebab itu, saya menemukan bahwa jangkauan Dani lebih sempit dibanding Sinta yang lebar | **Why did I put special marks on the values in the diagram?**<br>I give special marks because range is the distance between the highest and the lowest in a graph or a data set.<br><br>**Why did I decide these values are more consistent based on the range data measure?**<br>I arrived at that conclusion after following the tutorial's steps and example. Therefore, I found that Dani's range is narrower than Sinta's. |

Students' responses to paired questions on measures of central tendency, following engagement with the worked examples, are presented in Tables 9 and 10. In Table 9, Student 3

(S3) demonstrated proficiency in the communication skill of expressing mathematical ideas in written symbolic form. S3 used mathematical symbols appropriately when responding to the problems. However, S3 made errors in interpreting the results and solutions. In the self-explanation section, S3 acknowledged that both the mean and median can represent measures of central tendency, yet in the conclusion, only the median was mentioned. Interview data suggest that S3 was strongly influenced by the answers provided in the worked examples.

Table 9. Third student's (S3) answer

| Student's Answer | Translation |
|---|---|
|  Jawaban<br>~~Mean~~<br>$\sum x_b$ = 9000+6000+12000+11000 + 7000<br>total: 45000<br>mean pengeluaran (Rata")<br>$\bar{x}_b = \frac{\sum x_b}{f_b} = \frac{45000}{5} = 9000$<br>Rata²: 9000<br>Median pengeluaran<br>6000, 7000, 9000, 11000, 12000<br>median = 9000<br>Modus<br>tidak ada modus<br>kesimpulan<br>berdasarkan nilai mean, median, & modus, ukuran nilai sentral data pengeluaran Bimo yg paling tepat adalah median | $\sum x_b$ = 9000+6000+7000+11000+12.000<br>Total = 45000<br><br>Mean of Expense<br>$\bar{x}_b = \frac{\sum x_b}{f_b} = \frac{45000}{5} = 9000$<br>Mean = 9000<br><br>Median of expenses<br>6000, 7000, 9000, 11000,12000<br>Median = 9000<br><br>Mode<br>There is no mode<br><br>Conclusion<br>Based on mean, median, and mode, the most appropriate measure of central tendency to represent Bimo's expense is the median. |
|  **Penjelasan**<br>**Mengapa saya mengurutkan data seperti itu?**<br>agar memudahkan dalam mencari data mean, median, & modus | **Why did I organize the data in this way?**<br>To make it easier to find the mean, median, and mode |
|  **Bisakah nilai rata-rata merepresentasikan nilai pusat data? Mengapa?**<br>ya bisa, karna tidak ada eksteremya memuat di pertanyaan<br>ekstrem = nilai yg sgt besar | **Can the mean value represent the central value of the data? Why?**<br>Yes, because there is no extreme in the question<br>Extreme = very large value |
|  **Bisakah nilai median merepresentasikan nilai pusat data? Mengapa?**<br>bisa karna pada dasarnya median itu tdk dipengaruhi nilai ekstrem | **Can the median value represent the central value of the data? Why?**<br>Yes, because basically, the median is not influenced by extreme values. |
|  **Bisakah nilai modus merepresentasikan nilai pusat data? Mengapa?**<br>tidak ada, karna data pada soal tidak memiliki angka yg sama muncul 2x maka dr itu tdk ada modus | **Can the mode value represent the central value of the data? Why?**<br>No, because no data appears twice, so there is no mode. |

Meanwhile, Table 10 shows that Student 4 (S4) performed better in interpreting the results and solutions of mathematical problems. However, unlike S3, S4 avoided using mathematical symbols when solving the problems. The interviews revealed that although S4 understood the procedures and formulas for calculating the mean, he lacked a clear understanding of the symbols themselves, making him reluctant to apply them in solving problems.

Table 10. Fourth student's (S4) answer

| Student's Answer | Translate |
|---|---|
| **Jawaban**<br>urutan data pengeluaran uang:<br>6.000, 7.000, 9.000, 11.000, 12.000<br><br>total pengeluaran<br>6k+7k+9k+11k+12k = 45.000<br><br>rata.rata (mean)<br>$\frac{45.000}{5} = 9.000$<br>rata² = 9.000<br><br>Median:<br>6.000, 7.000, $\boxed{9.000}$, 11.000, 12.000<br>median: 9.000<br><br>Modus:<br>tidak ada modus dipengeluaran Bimo<br><br>kesimpulan:<br>Berdasarkan nilai mean median, & modus. Ukuran pemusatan data yg paling tepat untuk mempresentasikan nilai pusat dari data tabungan Bimo adalah mean dan Median. | Order of expense data<br>6000,7000,9000,11000,12000<br><br>Total expense<br>6000+7000+9000+11000+12000=45000<br><br>Mean<br>$\frac{45000}{5} = 9000$<br>Mean = 9000<br><br>Median of expenses<br>6000, 7000 $\boxed{9000,}$ 11000,12000<br>Median = 9000<br><br>Mode<br>There is no mode in Bimo's expense<br><br>Conclusion:<br>Based on mean, median, and mode, the most appropriate measures of central tendency to represent Bimo's expenses are the mean and the median. |
| **Penjelasan**<br>**Mengapa saya mengurutkan data seperti itu?**<br>saya mengurutkanya agar memudahkan dalam mengidentifikasi median & modus. karena median : nilai tengah, dan modus : nilai yg sering muncul. | **Why did I organize the data this way?**<br>I ordered the data to make it easier to identify the median and mode, because the median is the value in the middle, and the mode is the value that occurs most frequently. |
| **Bisakah nilai rata-rata merepresentasikan nilai pusat data? Mengapa?**<br>Ya, bisa karena hasil tersebut tidak dipengaruhi kecondongan tetapi mean dapat menyeimbangkan data tersebut | **Can the mean value represent the central value of the data? Why?**<br>Yes, because data is not affected by skew, the mean can accurately represent the balance of the data. |
| **Bisakah nilai median merepresentasikan nilai pusat data? Mengapa?**<br>Ya, tentu saja. karena median adalah nilai tengah yg membagi data jadi 2 bagian. Sehingga tidak ada yg terpengaruh dri kecondongan. | **Can the median value represent the central value of the data? Why?**<br>Yes, because the median is the value in the middle that divides the data set into two parts; therefore, the data is not affected by skew. |
| **Bisakah nilai modus merepresentasikan nilai pusat data? Mengapa?**<br>Tidak bisa. karena tidak ada nilai yg muncul berulang kali dalam data. Sedangkan modus hanya bisa digunakan jika nilai tersebut muncul berulang kali. | **Can the mode value represent the central value of the data? Why?**<br>No, because no data appears repeatedly. Whereas the mode can only be used when the data appears several times. |

Understanding mathematical concepts in various forms of data representation is a crucial indicator of mathematical communication ability. Therefore, the worked examples developed in this study incorporate various forms of data representation, specifically bar charts and narratives, as both are commonly used in problems concerning measures of central tendency and dispersion. In Table 2, a bar chart is presented alongside key information relevant to determining the range. Figure 1 shows that students were able to identify the necessary information to calculate the range, namely the maximum and minimum values in the data. This demonstrated that, after engaging with a worked example, students understood the important information that must be found in a data set, depending on the type of dispersion measure being calculated. Meanwhile, in Table 4, the data are presented in narrative form to ensure that students can identify the essential information needed to determine the most appropriate measure of central tendency.

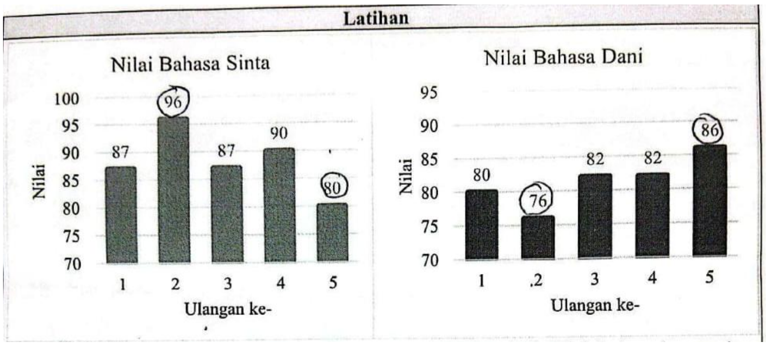



Figure 1. Fifth student's (S5) answer

*Evaluation*

The evaluation stage aims to assess the effectiveness of the developed worked examples and paired problems in facilitating students' mathematical communication skills. According to Nieveen (1999), an educational product can be considered effective if the intended learning experiences are successfully implemented and learners' outcomes demonstrate the achievement of the desired learning objectives. Similarly, Daryono et al. (2023) emphasized that the evaluation of effectiveness in educational development research is primarily based on the extent to which the designed product enables learners to achieve the predetermined performance criteria. Therefore, in this study, the developed worked examples were categorized as effective if at least 80% of students achieved a minimum average score of 75 in solving paired problems. The limited trial results are presented in Table 11.

Table 11. Limited trial results

| Student | Score Average | Indicator's Average Score | | |
|---|---|---|---|---|
| | | Indicator 1 | Indicator 2 | Indicator 3 |
| S1 | 79.17 | 100.00 | 75.00 | 62.50 |
| S2 | 91.67 | 100.00 | 87.50 | 87.50 |
| S3 | 91.67 | 100.00 | 100.00 | 75.00 |
| S4 | 83.33 | 87.50 | 75.00 | 87.50 |
| S5 | 87.50 | 100.00 | 87.50 | 75.00 |
| **Average** | **84.17** | **97.50** | **85.00** | **77.50** |

Based on the limited trial results in Table 11, all students achieved the minimum average score of 75, indicating that the worked examples effectively facilitated students' acquisition of mathematical communication skills. Moreover, the mean scores across all indicators exceeded the minimum threshold (except indicator 3 in S1), suggesting consistent improvement across the indicators of mathematical communication. As shown in Tables 2 and 4, worked examples consist of problems, solution steps, explanatory arguments, and final solutions. Therefore, worked examples are highly considered for beginners. During the early stages of skill development, including mathematical communication, students often lack a full understanding of the principles and their application in learning. As a result, students tend to rely on superficial strategies such as memorization, copying, and seeking similarities between tasks (Renkl, 2017). Conversely, the use of worked examples can enhance students' working memory resources, enabling them to learn the structure and principles of a new domain more effectively (Chen et al., 2023). Thus, learning through worked examples promotes the construction of more complex schemas in students' working memory, which positively influences their understanding and reasoning.

Mathematical communication emphasizes the importance of argumentation and the interpretation of solutions to mathematical problems. Therefore, the objective of the developed worked example is not only to arrive at the correct answer but also to promote better and broader knowledge transfer (Purnama & Retnowati, 2020). The worked examples in this study were designed with open-ended objectives that encourage students to interpret and argue. This approach enables students to explore the principles of each measure of central tendency and data dispersion in describing data conditions. The self-explanation principle further supports the development of argumentation and interpretation skills. The self-explanation section in each worked example provided opportunities for students to articulate their understanding by solving the slightly modified but equivalent guiding questions. The questions provided, which align with the problem context, encourage students to think critically and facilitate the transfer of knowledge. As Renkl (2002) emphasized, the self-explanation effect is crucial in avoiding excessive scaffolding, thereby ensuring that worked examples yield effective learning outcomes.

The inclusion of self-explanation in the worked examples is a key factor in supporting the development of students' mathematical communication skills. McGinn et al. (2024) demonstrated that incorporating self-explanation prompts into problem-solving practices can enhance students' mathematical skills. Self-explaining significantly influences schema construction in memory. Miller-Cotto et al. (2022) also found that self-explanation has a greater impact on problem-solving abilities than sketching a problem. This finding suggests that students with well-developed schemas are providing more explanations, even without explicit prompts. They also formulate better mathematical arguments consistent with the problem. Similarly, Bichler et al. (2022) demonstrated that combining worked examples with self-explanation prompts in statistics learning is highly effective, as it conveys important problem-solving strategies beyond mere algorithmic procedures. These findings align with mathematical communication skills, which are generally not limited to algorithmic procedures but instead focus on the ability to construct arguments and interpretations of mathematical problems within appropriate schemas.

Overall, this research highlights that the key aspect of mathematical communication skills can be effectively developed through the use of worked examples. Therefore, teachers should carefully design and select examples, recognizing that communication skills are not limited to the sequence of answers but also encompass students' ability to articulate their ideas. These findings are consistent with those of Cai et al. (1995) and Silver and Cai (1996), who proposed the use of mathematical tasks that enable students to demonstrate their mathematical thinking and reasoning. As teachers can use the explanations provided by students through these tasks to evaluate students' mathematical communication skills, this approach is also supported. Hence, the worked example structure developed in this study can be used to facilitate students' mathematical communication skills across all dimensions of written expression.

## Conclusion

Worked examples can be designed to facilitate various transfer skills and mathematical abilities, including mathematical communication skills. However, several principles must be carefully considered to ensure that worked examples are relevant and effective in supporting the targeted skills. In this study, the principle of self-explanation emerged as the key principle that worked examples must be incorporated to support students' mathematical communication skills. The limited trials' result show that students not only perform calculations but also understand mathematical ideas through multiple representations and interpret solutions in line with the indicator of mathematical communication skills.

In this study, real-life contexts were chosen as they draw on students' prior experience, which strengthens understanding and enriches schemas in working memory. Contexts that are familiar to students, such as daily activities, also encourage more logical and relevant reasoning, thereby better supporting the development of mathematical communication skills. The implementation stage, tested in a limited trial, indicated that worked examples can facilitate students' communication skills across the identified indicator. However, as not all students demonstrated improvement in every aspect of mathematical communication, further research is necessary to evaluate the quality of the worked examples developed in large samples.

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