Didactical Design with Problem Posing to Overcome Epistemological Obstacles in Problem Solving

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Abstract. Problem-solving activities allow students to apply mathematical knowledge and connect other mathematical abilities more deeply. However, based on the situation in the field, students still experience epistemological obstacles in solving problems on Straight Line Equation material, especially issues in story form. This study aims to overcome students’ epistemological obstacles in problem-solving using didactic design with a problem-posing approach. This research is qualitative research with a didactical design research method. Six students at one of the schools in West Kalimantan, Indonesia, were the subjects of this research. Tests, interviews, and observation guidelines were used as data collection instruments. The research data obtained were analyzed, reduced, and presented in narrative form. The results showed that epistemological obstacles can be overcome after implementing didactic design with problem posing. This finding has the same pattern as previous research findings, although on different materials and scaffolding. Therefore, didactical design with problem posing can be used as one of the alternative learning designs for straight-line equation material in the classroom.

Keywords: didactic design, epistemological obstacles, problem posing, problem-solving


Introduction

Problem-solving (PS) plays the most critical aspect of learning mathematics (Ersoy, 2016; Saygili, 2017; Albay, E., 2019; Klang et al., 2021; Asoma et al., 2022). Saygili (2017) revealed that the PS process is complex and requires many mathematical abilities to be used together. PS activities provide opportunities for learners to be proficient in dealing with difficulties that can be overcome using a combination of efficiently selected knowledge (Caprioara, 2015). In this case, there are steps of PS, according to Polya (1973): understanding the problem, developing a solution plan, implementing the solution plan, and checking back.

As with the explanation of PS, Problem Posing (PP) also requires problem-solving activities, but PP is done by changing a core problem into simple questions that refer to solving the core problem. This is supported by Kadir (2011), who argued that PP leads to asking questions or problems to trigger the formation of new understanding based on the available information. According to Brown and Walter (2005), in the process of doing PP, there are two critical aspects, namely accepting, which involves students being challenged by the situation given by the teacher, and challenging is an activity that involves students being challenged to propose problems based on the situation from the teacher. In addition, previous research also revealed that PP-based
mathematics learning could have a positive and significant effect on the success of learning mathematics, namely problem-solving ability, ability to propose problems, and confidence in mathematics (Kul & Çelik, 2020). Thus, PP is essential in mathematics learning because it can improve students' ability to solve mathematical problems (Rosli et al., 2014; Suarsana et al., 2019; Passarella, 2022; Calabrese et al., 2022).

When conducting questions and answers with Mathematics teachers at school, it is known that students still have difficulty solving math problems in contextual form. At the same time, contextual problems have an essential role as a starting point for students to learn and explore mathematics-related ideas based on their experiences (Widjaja, 2013). In line with this statement (Rochsun & Agustin, 2020) stated that providing contextual problems makes students fully involved in activities to connect academic learning with the real-life context they face.

After conducting a question-and-answer session with the teacher, the researcher conducted further pre-research by giving contextual math problems to find out more details about the difficulties experienced by students. It was found that students needed help understanding the information in the problem. This resulted in the emergence of student difficulties in the process of planning the solution. In addition, students are also constrained in applying mathematical formulas to contextual problems. These types of errors are indicators of epistemological obstacles (Mahyudi et al., 2023).

Epistemological obstacles can be identified by noting the tendency to generalize certain understandings to all situations (Siagian et al., 2022). This tendency occurs because there is a gap between the learning experience and the demand to connect the learning experience to various contexts beyond what has been experienced (Brousseau, 2002a). If these obstacles are not overcome, they will cause stagnation and even a decrease in knowledge (Dewi et al., 2022). So, in this case, there needs to be an effort to prevent the obstacles experienced by students in the learning process. One of the efforts is to provide a didactic design.

Suryadi (2011) developed a didactic design containing a didactic triangle describing didactic relationships, pedagogical relationships, and didactic and pedagogical anticipation relationships. Furthermore, Suryadi (2011) stated that didactic relationships are reflected in the teacher's ability to design subject matter. Pedagogical relationships are reflected in the teacher’s style, techniques, and ways in the learning process to ensure the teacher is more accessible to teach. Students feel free to explore learning materials, and pedagogical anticipation relationships can be seen from the teaching materials prepared by the teacher because, in addition to conveying knowledge, a teacher also acts as a student guide, manager of learning activities and creator of lessons that consider learning challenges.
Kusumaningsih et al. (2020) revealed that didactical learning design can help handle epistemological, didactical, and ontogeny obstacles in the learning process of congruence material. Didactic design is designed following the results of identifying student learning obstacles. This aligns with Angraini's (2021) statement that learning design must undoubtedly be designed based on didactical aspects and adjust to the anticipation of possible learning obstacles that will arise. The scaffolding used is mathematical and realistic mathematics approaches (Kusumaningsih et al., 2020).

In this study, the didactical design was made using the PP approach as scaffolding. Silver (1994) stated that there are three kinds of PP, namely, pre-solution posing type, where students are expected to be able to create problems from existing situations. Furthermore, the within-solution posing type is where students are expected to reformulate the problem into sub-questions from existing questions, and the post-solution posing type is where students are expected to modify the conditions or objectives of the issues solved to create similar problems.

Several previous studies have discussed didactical design to overcome learning obstacles. Miftah et al. (2022) analyzed the usefulness of the didactical design to overcome high school students' epistemological obstacles to mathematical reasoning in Third Dimension material. Astriani et al. (2022) examined the effective didactical design to overcome students' learning obstacles in solving flat building problems. Rahayu et al. (2021) discussed the didactical design to overcome vocational students’ didactical and epistemological obstacles in representing distance in geometry. Utami et al. (2022) analyzed didactical design to overcome students' epistemological obstacles in solving algebraic expression problems. However, research has yet to specifically examine the epistemological obstacles of students when solving straight-line equations and the use of didactical design with the PP approach to overcome these obstacles. Many students still need help understanding data in figures and tables, making mathematical models, and applying solution strategies to straight-line equation material (Triwulan et al., 2022; Putri & Hidayati, 2023). So, this study aims to overcome the epistemological obstacles experienced by junior high school students using didactic design with the PP approach in solving straight-line equation problems.

**Method**

This research is a qualitative study using the didactical design research method. Didactical design research reveals learning obstacles in the learning process to anticipate learning obstacles (Suryadi et al., 2017). There are three stages in Didactical Design Research (DDR): analysis of the didactical situation before learning in hypothetical didactical designs, including pedagogical anticipation relationships, metapedadidactic analysis, and retrospective analysis (Suryadi, 2011).
The instruments used were interview guidelines, observation guidelines, initial tests, and final tests. The instruments were validated by three validators, namely, 2 Mathematics education lecturers and 1 Mathematics teacher. The interview guidelines used more straightforward sentences so that students could understand the meaning of the questions, and the author knew the students' epistemological obstacles. The observation guideline was made by adjusting the didactical design so that the author could observe the implementation of the metapedidactic stage. The initial and final tests are equivalent tests adapted from Tan (2009), with the indicators used determining problem-solving if presented with a straight-line equation table (question number 1) and determining problem-solving if presented with a straight-line equation graph (question number 2). Tests were made to identify epistemological obstacles experienced by students. Before the test is given, validity and reliability checks are carried out. The results of testing the validity of the initial test on items 1 and 2 and the final test on items 1 and 2 were 0.885, 0.465, 0.896, and 0.948 at a significant level of 0.05, respectively. The validity test results show that both are valid because they are more than the standard r-table value of 0.361. In the reliability test results, the initial and the final tests were classified as good, with a coefficient value of 0.827. Thus, the instruments met the requirements and are suitable for collecting research data.

Based on the results of interviews with mathematics teachers in year eight, students from one junior high school in West Kalimantan, Indonesia, consisting of 27 students, still experience challenges in solving straight-line equation problems. Before the didactic design is implemented, the didactic situation analysis stage is carried out to find out the epistemological obstacles experienced by students. The students were directed to do an initial test. After that, the initial test results were sorted and reduced to select six students. The selection of the six students used a purposive sampling technique. The purposive sampling technique selects samples based on predetermined criteria that want to be studied intensively (Kothari, 1990). In this study, the six subjects were taken based on the number of errors experienced by students in doing the initial test. The six students were interviewed to learn more about epistemological obstacles and responses that might occur when didactic design is applied.

At the metapedidactic analysis stage, the researcher developed a didactic design with problem posing that contained response predictions and didactic anticipation by overcoming epistemological obstacles structured on what students know and need, then given challenges and supports. The researcher implemented the didactical design by using the types of PP, namely presolution posing, within-solution posing, and post-solution posing to six students simultaneously. In addition, researchers also provide the PP stages individually to students who are still experiencing obstacles and need guidance on parts that still need to be understood.
After the metapedadidactic analysis stage, the researcher conducted a retrospective analysis stage by giving a final test to determine changes in epistemological obstacles after the didactic design was applied. In addition, the researcher also linked and analyzed the suitability of response predictions with the responses that occurred during didactical design. In analyzing the results of the initial and final tests, researchers used data analysis techniques consisting of data reduction, data presentation, and conclusion drawing (Sugiyono, 2018). Data reduction is used to select students who experience epistemological obstacles based on the initial test results. After that, the data is presented narratively from epistemological obstacles before and after the didactic design is implemented. Furthermore, conclusions are drawn regarding the changes in the epistemological obstacles experienced by students.

The initial and final tests in this study were description tests. The initial and final tests aim to determine the epistemological obstacles so that the test is made with the same difficulty level. The Table 1 is the initial test instrument used in this study.

Table 1. Preliminary test instrument

<table>
<thead>
<tr>
<th>Number</th>
<th>Descriptive</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Take a look at the table below! Mass (kilograms)</td>
</tr>
<tr>
<td></td>
<td>1 kg</td>
</tr>
<tr>
<td></td>
<td>2 kg</td>
</tr>
<tr>
<td></td>
<td>3 kg</td>
</tr>
<tr>
<td></td>
<td>4 kg</td>
</tr>
<tr>
<td></td>
<td>5 kg</td>
</tr>
<tr>
<td></td>
<td>6 kg</td>
</tr>
</tbody>
</table>

In a fruit shop, it is known that the price of the first 2 kg of oranges is Rp12,000 per kg. For the purchase of 3 kg of oranges, a discount will be given with a price reduction of Rp1,000.00, and if you buy more than 3 kg, it will be reduced by Rp2,000.00 from the original 1 kg price (Rp12,000.00). How can you determine the money needed to buy 10 kg of oranges?

2

It is known that Nanda goes to school by bicycle because she is afraid of being late for school due to traffic jams. Here is the graph of her journey.

![Graph of Nanda's journey](image)

What distance Nanda travels from home to school if she reaches school in 300 seconds?

Results and Discussion

Brousseau (2002) states that there are three learning obstacles: ontogeny, didactic, and epistemological. Furthermore, Brousseau revealed that ontogeny obstacles occur because students are reluctant to do learning activities, and this obstacle will exist as long as students are in the learning process. From the initial test results, students are known to experience epistemological obstacles as follows.
Table 2. Epistemological obstacles

<table>
<thead>
<tr>
<th>Epistemological Obstacles</th>
<th>Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conceptual obstacles</td>
<td>- Not yet able to identify the elements known and asked questions</td>
</tr>
<tr>
<td></td>
<td>- Not yet able to make mathematical models from contextual problems</td>
</tr>
<tr>
<td></td>
<td>- Not able to use the straight-line equation formula correctly</td>
</tr>
<tr>
<td>Procedural obstacles</td>
<td>- There are discrepancies between the solution steps done by the student and the steps instructed.</td>
</tr>
<tr>
<td>Operational and technical obstacles</td>
<td>- Not yet able to solve problems in a simple form</td>
</tr>
<tr>
<td></td>
<td>- Difficulty in performing arithmetic operations</td>
</tr>
</tbody>
</table>

Based on the interview results, it is known that students need more confidence in the results of their work and are not used to solving contextual problems. The teacher's statement supports this concern that students need help if given contextual form problems, so these issues are rarely given during the learning process. From the existing information, it can be seen that ontogeny obstacles and didactic obstacles cause the emergence of students' epistemological obstacles in solving contextual problems. So, this research aims to show that didactical design with PP can overcome students' epistemological obstacles in solving mathematical problems.

Didactic design with the PP approach is prepared by predicting difficulties and didactic anticipation by overcoming epistemological obstacles structured on NCTM teaching principles. NCTM (2000) states that teachers must understand that students are learners, so mathematics teaching must be based on what is known and needed, then challenge and support them to learn it well. This statement is supported by the research of Amelia et al. (2023), who found that teachers' pedagogical competence in the classroom includes models, methods, strategies, and learning techniques that can manage learning appropriately.

The didactical design contains assignments, prediction of responses, and anticipation of responses. In this case, the assignment, response prediction, and response anticipation are adjusted to the initial test instrument made by the researcher and the results of students' work on the test. In addition, the steps of PP used in this study are (1) the researcher provides known information on each condition and asks students to create questions and their solutions (pre-solution posing stage); (2) the researcher directs students to create questions and their solutions during the process of solving the core question in each condition (within-solution posing stage); (3) the researcher directs students to create questions and their solutions after solving the core question by changing the condition or purpose of the problem (post-solution posing stage). This statement is supported by Silver's (1994) opinion that there are three types of problem posing: pre-solution posing, within-solution posing, and post-solution posing. The Table 3 is a display of the designed didactical design.
Table 3. Didactical design with PP approach

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Predicted Difficulty</th>
<th>Didactic Anticipation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presented with a contextual problem of a straight-line equation using a table, students can solve the problem.</td>
<td>Students need to gain an understanding of important information from the problem.</td>
<td>- Students are directed to underline important information using different colors of ballpoint pens.</td>
</tr>
<tr>
<td></td>
<td>Students have difficulty solving problems.</td>
<td>- Students are directed to create questions to help solve the core question.</td>
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<tr>
<td></td>
<td></td>
<td>- If there are no students who do not understand, the author gives one of the sample questions.</td>
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<tr>
<td></td>
<td></td>
<td>- Students are asked to answer the questions they have made</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- If students need to remember the gradient formula to solve the problem. So, the author reminded them that the ratio of the change in the upright side to the change in the horizontal side is the gradient formula.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- After getting the gradient value, students are directed to ask further questions about the form of the equation from the existing table/graph. In this case, if no one remembers the general form of the equation of a straight line, then the author can remind them again.</td>
</tr>
<tr>
<td>Presented with a contextual problem of a straight-line equation using a graph, students can solve the problem.</td>
<td>Students have difficulty operating algebraic forms.</td>
<td>- The author guides students to try calculating addition and subtraction operations on whole numbers. After students are fluent in these operations, they continue with multiplication and division. Do the process repeatedly until students understand the concept of these operations.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- The author increases the difficulty level by determining the variable's value if the form of the equation that uses the operations of subtraction, addition, multiplication, or division is known.</td>
</tr>
<tr>
<td></td>
<td>Students have difficulty rechecking answers.</td>
<td>- The author guides the students to recheck the work using substitution and elimination. The writer can provoke students by first asking the meaning of substitution and elimination.</td>
</tr>
<tr>
<td></td>
<td>Students need help with modifying information from the problem and solving it.</td>
<td>- The author gives example questions such as what the graph would look like if you bought 3kg of sliced oranges.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- The author asks some students to mention what other questions can be asked.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- The author directs to solve the questions that have been asked.</td>
</tr>
</tbody>
</table>

When the researchers implemented the didactical design with the PP approach, they provided PP simultaneously to the six students. In addition, researchers also provide PP individually to subjects who are still experiencing obstacles and need guidance on parts that still need to be understood. In the process of implementing the didactical design, the response predictions made follow the responses given by students in the form of epistemological obstacles, and the didactical design that has been designed can overcome these obstacles. The following is an analysis of the results of several research subjects’ initial and final tests.

1. Subject R2

![Figure 1. Illustration of subject R2's answer in the initial test number 1](image-url)
Based on the results of the work, subject R2 has yet to be able to solve the problem in Problem 2. This can be seen from the work that has yet to be able to write anything about the important information from the problem, the problem-solving plan, the problem-solving process, the recheck, and the conclusion. After giving the initial test, the researcher interviewed subject R2. The following is the interview excerpts of the researcher and subject R2.

\[ \text{R} : \text{Do you understand the meaning of question number 1?} \]
\[ \text{R2} : \text{I understand but I need clarification about how to write it, so it is incomplete.} \]
\[ \text{R} : \text{From your work to find the gradient, is it done until that point?} \]
\[ \text{R2} : \text{Yes, I think so.} \]
\[ \text{R} : \text{For the first gradient, you got } 12,000/1. \text{ Can that be simplified?} \]
\[ \text{R2} : \text{Oh yes, I got it.} \]
\[ \text{R} : \text{Can you find the simple result of } 12,000/1? \]
\[ \text{R2} : \text{Yes, I can do it. Because it is divided by one, it is quite easy} \]
\[ \text{R} : \text{For number 2, did you understand the meaning of the problem?} \]
\[ \text{R2} : \text{I was confused reading the figure.} \]
\[ \text{R} : \text{When you solved the problem, did you struggle with the calculation process?} \]
\[ \text{R2} : \text{I need help in multiplication and division operations, especially if the numbers in the problem are large.} \]

Figure 2. Illustration of subject R2's answer in the final test number 1

Based on the results of the initial tests and interviews, subject R2 experienced epistemological obstacles in solving contextual problems. The epistemological obstacles experienced are a lack of understanding of important information in the problem, especially information in the form of graphs, not being able to make a solution plan, not being able to solve problems, not understanding how to recheck answers, and difficulties in the counting process. Then, on the work results, subject R2 has yet to be able to write conclusions on each question number. Nevertheless, based on the interview, subject R2 understands how to make conclusions.
Subject R2 is just not used to making conclusions from every answer they have done. In addition, based on the interview results, the researcher got information that subject R2 experienced obstacles in the calculation process. Subject R2 was directed to work on the final test questions after applying the didactic design. The results of subject R2's work are listed in Figure 2 and Figure 3.

Figure 3. Illustration of subject R2's answer in the final test number 2

2. **Subject R5**

Based on the answer to question number 1, subject R5 needs to be more precise in solving the problem in the problem. It appears that subject R5 has not been able to identify the known and questioned elements correctly, has not been able to plan the solution, has not been able to make problem-solving, has not been able to do a back check, and has not been able to conclude. Based on the work results, subject R5 has yet to be able to solve the problem in Problem 2. This can be

Illustration of subject R2's answer in the final test number 1
Stage of understanding the problem. Subject R2 can write the information asked in the problem. Although R2 cannot write the known information, R2 can make a solution with the right answer. In the stage of planning the solution, R2 can make the problem permissive even though it needs to be more precise.

Here is the stage of implementing the solution plan. R2 has determined the gradient and equation of the straight line by paying attention to the interval. After that, he substituted the value of x based on the information asked.

This stage is the stage of rechecking the results obtained. Based on the work results, R2 substituted the value of y in the equation made at the planning stage and obtained a result from the information from the problem.

Problem understanding stage. Subject R5 incompletely wrote the known information in the problem.

Figure 4. Illustration of subject R5's answer in the initial test number 1
seen from the work of subject R5, who has yet to be able to write anything related to important information from the problem, planning for solving problems, solving problems, checking back and conclusions. After giving the initial test, the researcher interviewed the subject. Interview excerpts of researcher and subject R5.

R : *In Problem 1, do you understand the known and asked information from the problem?*

R5 : *I do not understand. I am also confused about working on story form problems and applying formulas to this material.*

R : *From your worksheet, can you explain why you did not write anything in number 2?*

R5 : *Yes, I do not know how to read the graph and only know the question's meaning.*

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The stage of understanding the problem by writing down important information from the problem and making a solution plan by writing the equation according to the number of variables used.

The stage of implementing the solution plan starts with writing the gradient and equation using the interval.

The stage of re-examining the solution by substituting the answers obtained in the straight-line equation.

Figure 5. Illustration of subject R5’s answer in the final test number 1
Based on the results of the initial test of subject R5 and interview excerpts, it is known that subject R5 experiences epistemological obstacles in solving math problems, especially problems in contextual form. The teacher concerned also explained that the subject was still not used to and still had difficulty if asked to solve contextual problems, even though the problem was classified as easy. After the didactical design with problem posing was implemented in the learning process, the researcher gave directions to subject R5 to work on the final test questions. Figure 5 and Figure 6 are the results of the work of subject R5.

Based on the initial test results, the subject experienced conceptual, procedural, and operational technical obstacles in working on contextual problems of straight-line equations. Meaningful learning that needs to be delivered appropriately in the classroom is one of the factors causing epistemological obstacles to appear. During the learning process in class, teachers have yet to be able to provide learning starting from what students know. The researchers designed a didactical design with a PP approach by analyzing the location of student errors and interviews. Guiding students from the simplest stage of completion that they have understood before providing triggering questions that can help students link each known information with the
learning objectives to be achieved are the stages in the didactical design that has been designed. Raising questions can attract students' attention and trigger mathematical reactions (Kontorovich, 2020).

The stages of the didactic design were designed by considering the standard and efficient teaching principles to be used (NCTM, 2000). Then, the researcher directs students to fill out the final test, intending to see changes in students' problem-solving skills. Based on the results of the final test analysis, the epistemological obstacles experienced by the subject can be overcome by using a didactical design with the PP approach. The results of this study are supported by previous research that didactical design is effectively used to overcome student learning obstacles and improve student abilities in mathematics (Fitrianna et al., 2019; Lestari, 2020; Jannah et al., 2016; Supriadi, 2019; Nopriana et al., 2022). In addition, Chua and Toh (2022) revealed that using PP can lead to connecting problems to real-life contexts. Learners are more involved, add insight into mathematical understanding, and increase their awareness of problem-solving strategies. While implementing the didactical design using PP, the subjects were directed to develop problems based on situations commonly experienced in daily life so that the didactical design could overcome the epistemological obstacles experienced by students.

**Conclusion**

From the results of the initial test work and interviews, the research subjects needed help understanding the problem and planning the solution to the problem. This causes students to have difficulty solving problems and checking the correctness of the work results. These difficulties are part of epistemological obstacles in the conceptual, procedural, and operational technique obstacles. Epistemological obstacles arise due to limited knowledge, and students assume their knowledge is fully understood.

Didactical design with the PP approach is used to overcome these obstacles. After the didactical design was implemented, it was found that the subject's epistemological obstacles could be overcome. It can be seen from the results of the final test that the subject can identify important elements in the problem, make a solution plan, apply mathematical formulas correctly to answer problems, suitability the steps taken with the steps ordered, solve problems up to a simple form, able to check back and able to use arithmetic operations properly. The results of this study have similar patterns to previous research but are used on different materials and scaffolding.

The researcher expects to make the didactical design that has been organized into one of the alternative learning designs. In addition, the researcher expects future researchers to include didactical design with the PP approach in learning devices other than evaluation tools, such as lesson plans, students' worksheets, and media.
References


