The Evolution of the Black-Scholes Model: A Bibliometric Analysis

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Abstract
Objective –To investigate the development of the Black-Scholes Model in financial research, identify its key contributors, and map its thematic evolution through a bibliometric analysis.

Methodology –This study employs bibliometric analysis, a research method that uses bibliographic data to analyze trends, patterns, and the impact of scholarly works in the financial research area. The data was extracted from the Scopus database, processed, and visualized using Microsoft Excel, R-Packages software, and the Biblioshiny Web Interface. The data extraction process was conducted on December 25, 2023, identifying 941 relevant documents, with 719 documents determined to be eligible for the analysis.

Results –The analysis reveals significant evolution in academic interest in the Black-Scholes Model since its introduction in 1978, with key contributors from the United States and the United Kingdom. Early research (1978–2000) focused on the theoretical basis of the model, while the period 2001–2010 marked its increasing relevance in economics. The 2011–2020 period saw the integration of the model in a wide range of economic applications. The 2021–2023 period highlights a new research focus on applications in fractional differential equations. The model has become an essential element in modern finance, driving innovation in finance education, policy, and cross-disciplinary applications.

Research limitations/implications –The study is limited by its reliance on data from the Scopus database, which may not encompass all relevant research. Future research could expand the analysis to include other databases and consider qualitative aspects of the Black-Scholes Model's applications.

Novelty/Originality –This research underscores the importance of the Black-Scholes Model in the evolution of modern finance and its diverse applications. It makes a significant contribution to the study of economics and finance by providing a comprehensive bibliometric analysis of the model's development, key contributors, and thematic evolution.

Keywords: Black-Scholes Model, Bibliometric Analysis, Thematic Evolution, Trend Topic, Research Contributors, Financial Applications.

1. Introduction

The Black-Scholes model is a widely used option pricing model that estimates the fair value of options based on mathematical theory, as well as assuming certain conditions that may not match real market conditions, leading to the development of more complicated non-linear Black-Scholes option pricing models (Meng & Wang, 2023). Revolutionary in its time, the Black-Scholes model not only opened up new avenues in the understanding of derivative instruments but also played a key role in the evolution of global financial markets (Capała & Dybiec, 2021; Protonotarios & Tassopoulos, 2021). The Black-Scholes model provides a closed-form solution for European options, allowing for analytical and numerical solutions (Christain et al., 2023). It considers factors such as stock volatility, initial stock price, and strike price to calculate option value. The model is also used to simulate and evaluate double-barrier options, analyzing the effect of barrier value on option value (Krykun, 2023). In addition, the Black-Scholes Model is used in the hedging of exotic barrier options, with analogs of the Black-Scholes formula proven for these types of options (Liao & Zhuang, 2023). Applied from pension funds to portfolio management, the model's applications are diverse, marking its significant impact on investment decisions and hedging strategies worldwide.

Modifications and extensions of the Black-Scholes model have been proposed to overcome its limitations and improve its accuracy in pricing financial derivatives. One approach is to introduce nonconstant volatility and bankruptcy risk into the model, which is more closely aligned with real financial observations (Hsu & Wu, 2020). Another approach is to consider a random environment and introduce randomness into the model coefficients, such as interest rates, drift, and volatility, leading to (B, S, X)-market securities (Swishchuk, 2023). Continuous time models are also used for theoretical research, as they allow the use of stochastic analysis tools and can provide explicit solutions or analytical expressions (Yan, 2018). Multidimensional extensions of the Black-Scholes model with stochastic volatility and correlation have been developed, offering practical advantages in pricing derivatives compared to the original model (Albevirio et al., 2006). In addition, a new type of model, called the "market model," has been developed, focusing on the implied volatility variable (Kermiche, 2014). In exotic options theory, different types of options have been considered, such as binary options, range forward contracts, and barrier options. These options have been priced using different methods, including the Black-Scholes PDE method and numerical techniques (Buchen, 2012). In addition, the constant variance elasticity (CEV) model has been generalized to the fractional ceV model to capture long memory effects in stock returns or volatility. This fractional model has been used to numerically price European double-barrier options (Rezaei et al., 2021). In credit risk assessment, a hybrid credit risk model has been proposed to price vulnerable options with stochastic volatility. The model incorporates idiosyncratic and systematic risks into asset dynamics and intensity processes, enabling comparative statistical analysis (Liang & Wang, 2021). In addition, the subdiffusive Black-Scholes model has been used to analyze financial markets, specifically to price American and barrier options. Computational methods, such as the weighted finite difference and Longstaff-Schwartz methods, have been used to value these options (Krzyżanowski & Magdziarz, 2021). A modified Black-Scholes model has been proposed to account for nonconstant volatility and bankruptcy risk. The model captures stock price trends better than Brownian geometric motion and provides a statistical calculation method for pricing European options (Hsu & Wu, 2020). Based on previous studies, it is important to analyze the evolution of the Black-Scholes model using new research approaches such as bibliometric analysis. This approach allows the identification of authors and institutions that have contributed most to the development of the model, as well as the tracking of evolving research themes, providing in-depth insights that can assist in formulating more informed policies, research strategies and decision-making in academia and research related to the Black-Scholes model (Grosseck et al., 2019; Jugović et al., 2022; Verma & Gustafsson, 2020; Zhao et al., 2023).

This study aims to analyze the development of research related to the 'Black-Scholes Model', identify the authors and institutions that are most instrumental in their contributions, and record the development of research themes using a bibliometric analysis approach. Bibliometric analysis is a method used to quantitatively classify and assess bibliographic material in the discipline of science (Lazarides et al., 2023). It involves analyzing publications, citations, authors, and institutions using computer-aided techniques (Saputro et al., 2023). It helps in charting research trends, identifying research directions, and providing insights for future work (Tatum & Russo, 2020). Thus, through bibliometric analysis, this study can help inform policy, research strategies, and decision-making in academia and research related to the 'Black-Scholes Model'.

2. Literature Review

The Black-Scholes model is a well-known mathematical model used to price European options (Abdi et al., 2022; Almeida et al., 2022; Larcher, 2023). It is widely used in financial mathematics and has been applied to various pricing techniques, such as the binomial tree model and the Crank-Nicolson method (Shao, 2023).

The Black-Scholes formula is a mathematical model used to determine the theoretical price of European options. A European option is a type of option that can only be exercised on the expiration date, not before. Here is the Black-Scholes formula for both of these types of options:

Call Option:
$$\boldsymbol{C = S0N(d1) - Xe - rTN(d2)}$$

Put Option :
$$\boldsymbol{P = Xe - rTN(-d2) - S0N(-d1)}$$

The Black-Scholes formula is derived using various methods. One approach is via the binomial tree continuum limit, which requires no knowledge of stochastic calculus or partial differential equations. Another method involves using the Black-Scholes-Merton dual equation, which has the same form as the Black-Scholes-Merton equation. This equation can be used to price different types of options and provide insight into pricing and hedging. It also allows for put-call equivalence and relates the delta and gamma of puts and calls through analytical formulas (Gianin & Sgarra, 2013).

This model assumes a market consisting of two assets: non-risky assets (bonds) and risky assets (stocks) (Weatherall, 2018). It is widely accepted that the assumption of constant volatility of the Black-Scholes model is violated in real-world financial markets, but it is still used by options professionals because of its simplicity and ease of implementation (Sinha et al., 2018). This model can be used as a viable risk management tool even in financial markets characterized by stochastic volatility (Ballabio, 2023). The Black-Scholes model was introduced in Stochastic Finance as part of the continuous Black-Scholes model, which is used to study European option prices (Sinha et al., 2018). Despite its limitations, the Black-Scholes model can still provide useful and tradable information, even when its core assumptions are contradictory.

The Black-Scholes model has limitations in some aspects. First, it assumes constant volatility, which is not always the case in real-world financial markets (Oh et al., 2018). Secondly, it assumes that the underlying assets follow a log normal distribution, which may not hold true in all situations. In addition, the model assumes that there are no transaction costs or taxes, which is unrealistic. Furthermore, the Black-Scholes model assumes that the market is efficient and there are no arbitrage opportunities, which may not always be the case. Finally, the model assumes that interest rates are constant and known, which may not be accurate in practice. These limitations highlight the need for more sophisticated models that can better capture the complexity of financial markets (Abdi et al., 2022; Černá & Fiňková, 2024; Chen et al., 2024; Contreras G., 2021; Edeki et al., 2023; Wang et al., 2022).

3. Methodology

This study aims to analyze the development of research related to the 'Black-Scholes Model', identify the main contributors among authors and institutions, and map the evolution of the research theme using a bibliometric analysis approach. The three main research questions examined are: 1) What is the development trend of Black-Scholes model research? 2) Who are the main researchers and institutions that contribute to the evolution of the research theme? and 3) How has the Black-Scholes model research theme evolved?

1. Trends in the Development of the Black-Scholes Model
   This section explores how academic interest in the Black-Scholes model has evolved over time. Through annual trend graphs of publications and citations, we illustrate the dynamics of attention given to this model. In addition, the annual citation analysis is presented in tabular form, highlighting the number of citations received each year, while a table of citations by country provides insight into the international influence of the model.
2. Key Contributors in Black-Scholes Research
   In this section, we identify researchers and institutions that were instrumental in the development of the Black-Scholes model. By utilizing a graph featuring the top 10 authors and institutions—selected based on the number of articles and citations—we highlight the key contributors who have advanced research in this area.
3. Evolution of Black-Scholes Research Themes
   This section examines how Black-Scholes-related research themes have emerged, evolved, and interacted over time. We use topic trend graphs to visualize how often various topics arise. Thematic maps are used to illustrate the relationships between various topics, and thematic evolution graphs show how these themes change, and theme progression quadrant graphs help position these topics in the context of their maturity and growth. Bibliometric research is a newly popular method that complements previous research as it not only allows for the identification of authors and institutions that contributed significantly to the development of the model, but also allows for the tracking of evolving research themes. As such, this analysis provides deeper insights that can support better policy formulation, research strategies and decision-making in academia and research related to the Black-Scholes model.

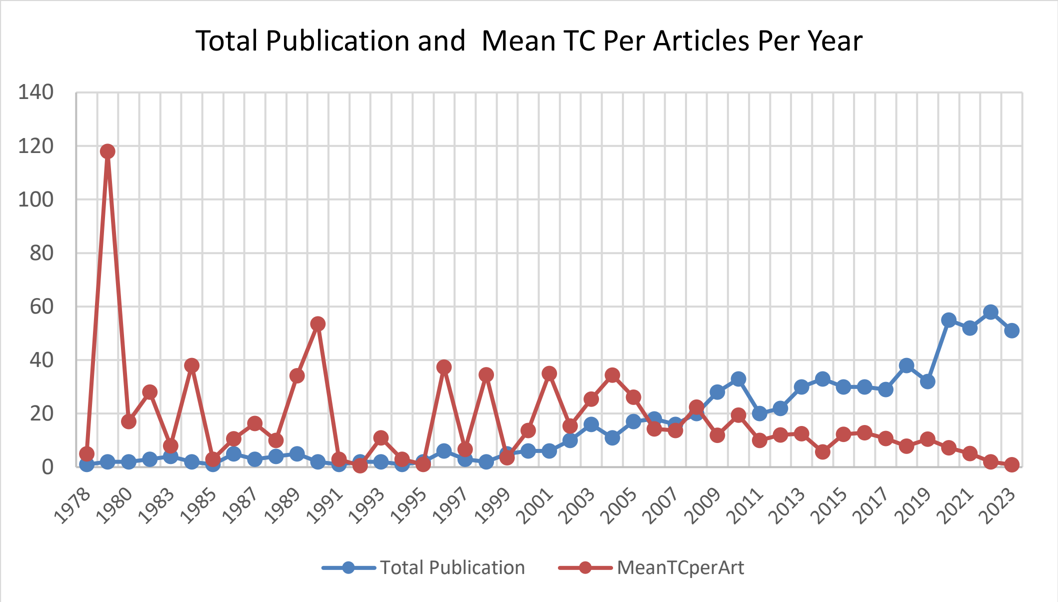
The information for this study was extracted from Scopus, and considering the relevance of Scopus in terms of scientific quality, it is considered very high. The data was then processed and visualized using Microsoft Excel, R-Packages software, and the Biblioshiny Web Interface. The data extraction process was conducted on December 25, 2023, and it successfully identified 941 relevant documents. The initial filtering stage involved the elimination of documents that did not meet the set criteria, including a restriction on the type of article only, which had to be final and published in a journal. Documents from 2024 were excluded, as that year had not yet occurred. This process resulted in a reduction of 202 documents from the total found. Of the remaining 739 documents, a further eligibility assessment was conducted with selection criteria that included a language restriction to English documents based on the use of English as the dominant international language in scientific communication. This process led to the exclusion of an additional 20 documents. After a rigorous selection process, 719 documents were determined to be eligible for inclusion in the bibliometric analysis.

3.1 Development Trends of the Black-Scholes Model

Figure 1 shows the annual distribution of total publications and the average citations per article related to the Black-Scholes Model during the research period (1978–2023).

The graph depicts the publication trend of articles related to the Black-Scholes Model as well as the dynamics of the average citations obtained by each article over time. At the beginning of the period under study, it can be seen that the number of published articles is relatively small, but each article, on average, receives a high number of citations. In contrast, in the last few years of the study period, there has been a striking shift where the number of publications has increased significantly, while the average number of citations per article has shown a sharp decline. This indicates a change in the academic impact of new articles related to the Black-Scholes Model.

Figure 1.
Distribution Per Year Total Publications and Average Citations Per Article Related to Black-Scholes Model



The analysis of annual publication and citation trends can be better understood through a detailed table that presents the dynamics of the average citations obtained by articles over time. Details of this analysis are presented in "Table 1: Annual Citation Analysis of Publications."

Tabel 1.
Publication Citation Analyst Per Year

Year	TP	NCP	TC	C/P	C/CP	h-index	g-index
1978	1	1	5.00	5.00	5.00	1	1
1979	2	1	236.00	118.00	236.00	1	2
1980	2	1	34.00	17.00	34.00	1	2
1982	3	3	84.00	28.00	28.00	2	3
1983	4	2	32.00	8.00	16.00	2	4
1984	2	2	76.00	38.00	38.00	1	2
1985	1	1	3.00	3.00	3.00	1	1
1986	5	4	53.00	10.60	13.25	3	5
1987	3	2	49.00	16.33	24.50	1	3
1988	4	3	40.00	10.00	13.33	3	4
1989	5	4	171.00	34.20	42.75	3	5
1990	2	2	107.00	53.50	53.50	2	2
1991	1	1	3.00	3.00	3.00	1	1
1992	2	1	1.00	0.50	1.00	1	1
1993	2	2	22.00	11.00	11.00	2	2
1994	1	1	3.00	3.00	3.00	1	1
1995	2	1	2.00	1.00	2.00	1	1
1996	6	5	224.00	37.33	44.80	4	6
1997	3	3	20.00	6.67	6.67	3	3
1998	2	2	69.00	34.50	34.50	1	2
1999	5	3	18.00	3.60	6.00	2	4
2000	6	5	82.00	13.67	16.40	4	6
2001	6	6	210.00	35.00	35.00	4	6
2002	10	10	154.00	15.40	15.40	7	10
2003	16	15	407.00	25.44	27.13	10	16
2004	11	11	379.00	34.45	34.45	7	11
2005	17	17	444.00	26.12	26.12	10	17
2006	18	15	258.00	14.33	17.20	9	16
2007	16	15	220.00	13.75	14.67	6	14
2008	20	18	449.00	22.45	24.94	9	20
2009	28	23	333.00	11.89	14.48	10	17

Year	TP	NCP	TC	C/P	C/CP	h-index	g-index
2010	33	29	643.00	19.48	22.17	13	25
2011	20	17	199.00	9.95	11.71	7	13
2012	22	20	265.00	12.05	13.25	9	16
2013	30	26	374.00	12.47	14.38	11	19
2014	33	32	189.00	5.73	5.91	9	12
2015	30	28	368.00	12.27	13.14	9	18
2016	30	24	385.00	12.83	16.04	9	19
2017	29	23	309.00	10.66	13.43	10	17
2018	38	33	297.00	7.82	9.00	11	15
2019	32	28	336.00	10.50	12.00	10	17
2020	55	46	399.00	7.25	8.67	11	18
2021	52	42	268.00	5.15	6.38	9	13
2022	58	35	112.00	1.93	3.20	5	7
2023	51	18	49.00	0.96	2.72	4	5

Notes:
TP = Total of publication,
NCP = Number of cited publications,
TC = Total citations,
C/P = Average citations per publication,
C/CP = Average citations per cited pblication,
H = h-index,
G = g-index

To understand citation distribution and trends at a global level, "Table 2: Analysis of Publication Citations by Country" describes insights into citation averages obtained by articles in different countries, reflecting on geographical influences on research dissemination.

Tabel 2.
Publication Citation Analyst Per Country

Country	TP	NCP	TC	C/P	C/CP	h-index	g-index
United States	170	140	2,019.00	11.88	14.42	23	39
United Kingdom	151	139	2,666.00	17.66	19.18	28	46
Netherlands	112	100	1,521.00	13.58	15.21	21	34
Switzerland	51	39	324.00	6.35	8.31	11	17
Germany	32	29	379.00	11.84	13.07	10	19
Singapore	16	12	189.00	11.81	15.75	6	13
Thailand	12	2	4.00	0.33	2.00	1	2
Romania	10	7	34.00	3.40	4.86	4	5
Bulgaria	7	5	9.00	1.29	1.80	2	2
Egypt	7	5	19.00	2.71	3.80	3	4
Hong Kong	6	4	46.00	7.67	11.50	3	6
Poland	6	2	16.00	2.67	8.00	1	4
Canada	5	2	12.00	2.40	6.00	1	3
India	5	2	15.00	3.00	7.50	1	3
Iran	5	5	28.00	5.60	5.60	4	5
China	4	2	21.00	5.25	10.50	2	4
South Korea	4	4	34.00	8.50	8.50	3	4
Croatia	3	2	10.00	3.33	5.00	1	3
Japan	3	3	41.00	13.67	13.67	3	3
Serbia	3	3	16.00	5.33	5.33	2	3
Taiwan	3	2	4.00	1.33	2.00	1	2
Turkey	3	2	4.00	1.33	2.00	1	2
Azerbaijan	2	2	2.00	1.00	1.00	1	1
Greece	2	1	3.00	1.50	3.00	1	1
Kazakhstan	2	-	-	-	-	-	-
South Africa	2	1	2.00	1.00	2.00	1	1
Spain	2	-	-	-	-	-	-
Ukraine	2	-	-	-	-	-	-
Australia	1	1	1.00	1.00	1.00	1	1
Czech Republic	1	1	3.00	3.00	3.00	1	1
France	1	1	7.00	7.00	7.00	1	1
Hungary	1	-	-	-	-	-	-
Italy	1	-	-	-	-	-	-

Country	TP	NCP	TC	C/P	C/CP	h-index	g-index
Kuwait	1	1	1.00	1.00	1.00	1	1
Lithuania	1	1	1.00	1.00	1.00	1	1
Malaysia	1	1	3.00	3.00	3.00	1	1
Mexico	1	-	-	-	-	-	-
Pakistan	1	-	-	-	-	-	-
Palestine	1	1	6.00	6.00	6.00	1	1
Slovakia	1	1	9.00	9.00	9.00	1	1

3.2 Key Contributors to Black-Scholes' Research

Figure 2 describes the ten most prolific authors in the publication of articles on the Black-Scholes Model during the research period from 1978 to 2023, revealing the main contributing authors in this field.

Figure 2.
Top 10 Authors With The Most Articles

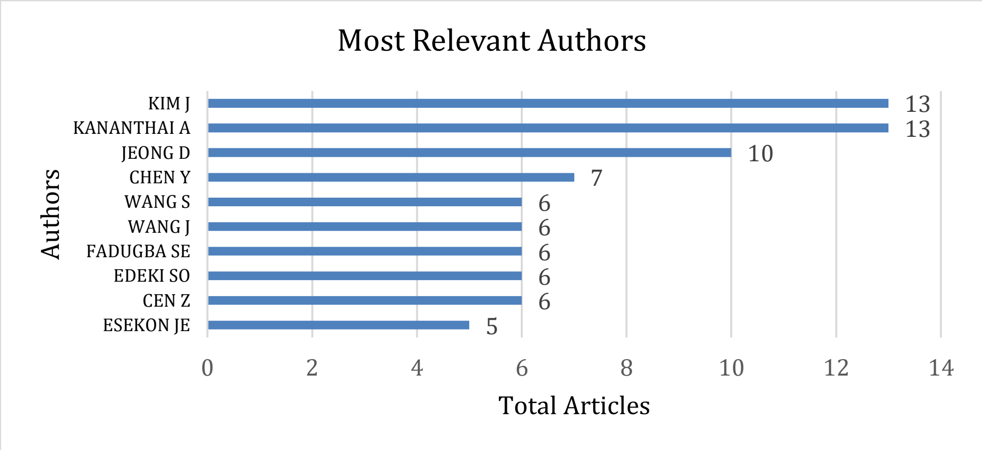
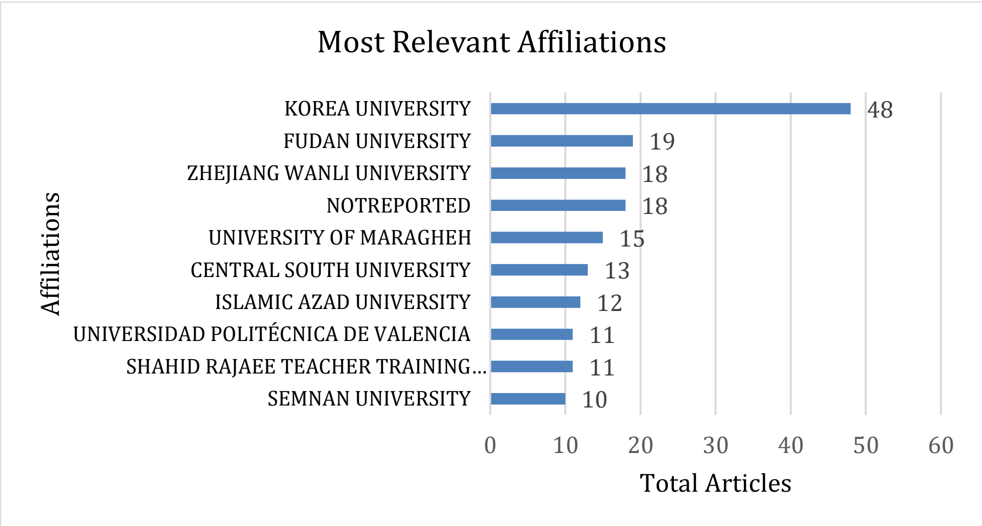


Figure 3 describes the ten most productive institutions in the publication of articles on the Black-Scholes Model during the study period, revealing the major institutional contributors in this field.

Figure 3.
Top 10 Institutions with The Most Articles



3.3 The Evolution of Black-Scholes' Research Theme

With the help of R-Packages software and WebInterface Biblioshiny, several figures can be presented, as in "Figure 4," which presents trending topics that illustrate the interrelation and development of research themes in the context of the Black-Scholes Model, showing how various topics develop.

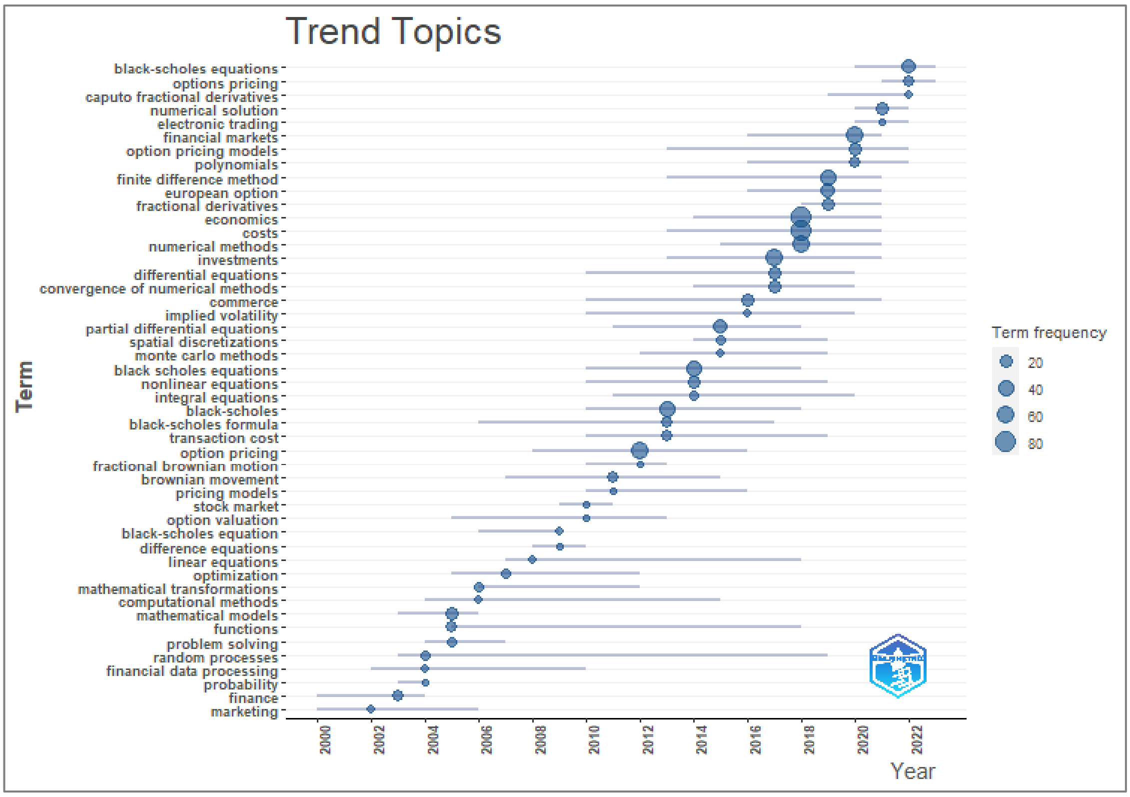


Figure 4.
Trend Topics

Figure 5 displays a quadrant diagram detailing the linkages and development of research themes related to the Black-Scholes Model. This diagram illustrates the interconnections between topics and how those themes evolve.

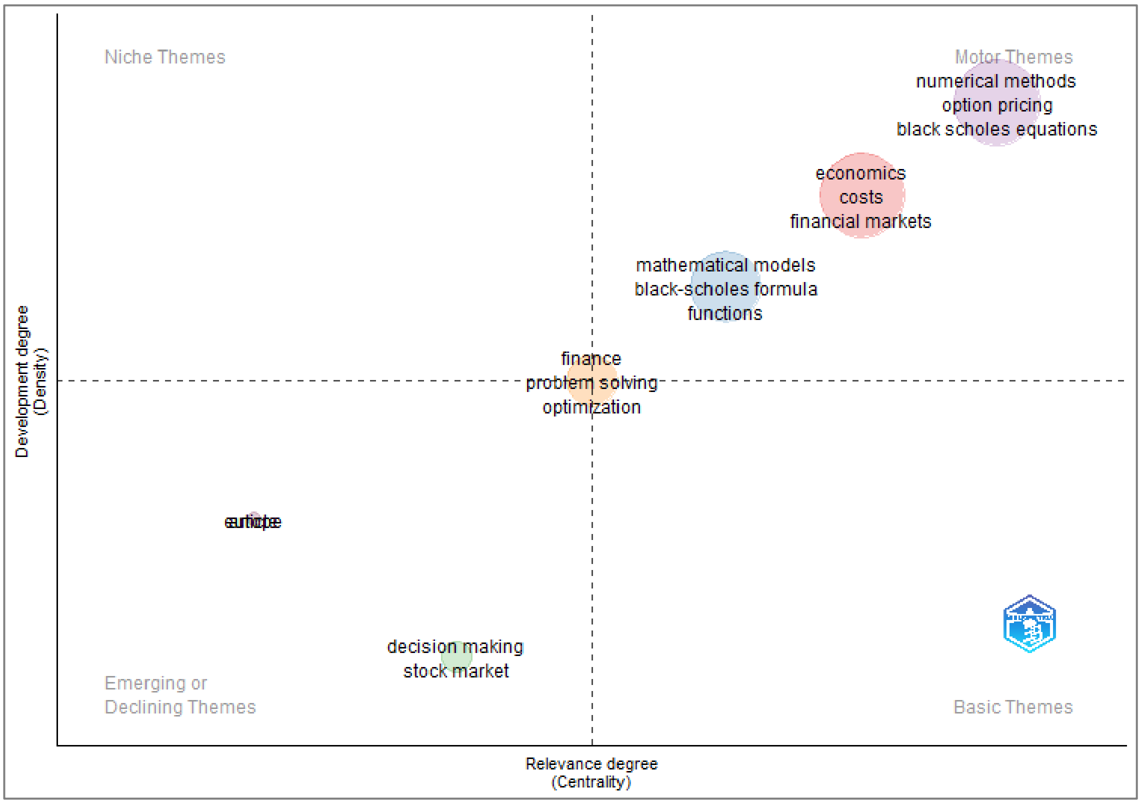


Figure 5.
Thematic Map

Figure 6 makes use of Sankey diagrams to illustrate the evolution and transition of research themes related to the Black-Scholes Model. These diagrams provide an intuitive visualization of how research topics change and evolve over time.

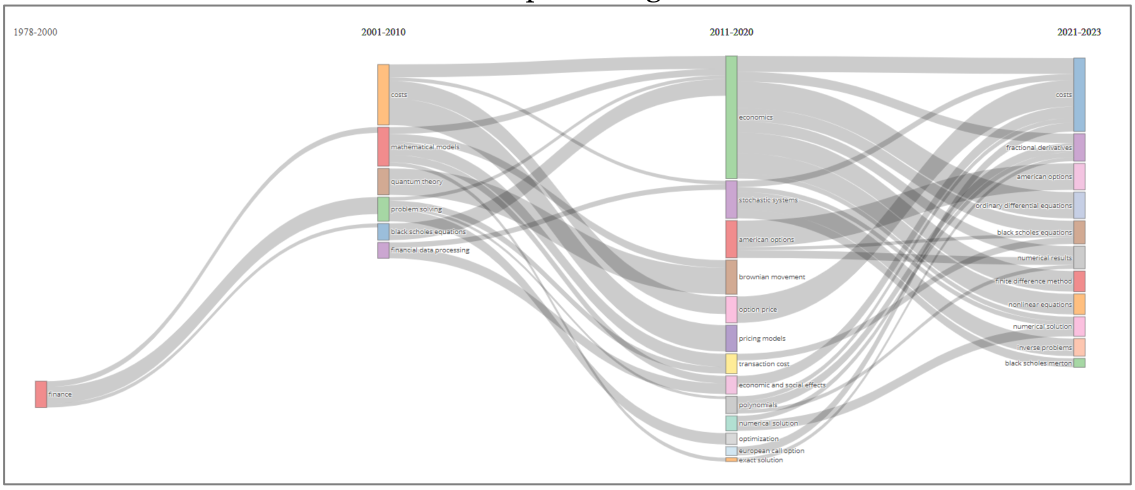


Figure 6.
Sankey Diagram of Thematic Evolution

Sankey diagrams are used in bibliometry to present the results of the dynamic evolution of themes visually and intuitively. These diagrams are commonly used to map value flows in systems at the operational level or along global value chains; they are also used to analyze the thermal efficiency of steam engines and to illustrate the energy and material balances of complex systems (Bornmann & de Moya-Anegón, 2019; Chong et al., 2021; Otto et al., 2022; Rambau et al., 2024).

To further examine the evolution of the Black-Scholes Model research theme, "Table 3: Thematic Evolution Metrics" provides a detailed analysis. This table records thematic changes in research over time, allowing for a deeper understanding of the dynamics of research within this field.

Tabel 3.
Thematic Evolution Metric

From	To	Words	WII	II	Occurrences	SI
finance--1978-2000	black scholes equations--2001-2010	differential equations	0.14	0.20	2	0.05
finance--1978-2000	mathematical models--2001-2010	option pricing	0.21	0.20	3	0.03
finance--1978-2000	problem solving--2001-2010	finance;marketing;random processes	0.64	0.20	4	0.03
black scholes equations--2001-2010	economics--2011-2020	black scholes equations;finite difference method;nonlinear equations;partial differential equations;numerical methods;differential equations;convergence of numerical methods	0.63	0.06	12	0.01
costs--2001-2010	economics--2011-2020	costs;economics;black-scholes;investments;stock price	0.49	0.04	15	0.01
costs--2001-2010	option price--2011-2020	option price	0.67	1.00	2	0.04

From	To	Words	WII	II	Occurrences	SI
costs--2001-2010	pricing models--2011-2020	pricing models	1.00	1.00	2	0.04
costs--2001-2010	stochastic systems--2011-2020	commerce;option pricing models	0.14	0.05	6	0.02
financial data processing--2001-2010	economic and social effects--2011-2020	stock market;risk assessment	0.40	0.25	3	0.13
financial data processing--2001-2010	stochastic systems--2011-2020	financial data processing	0.20	0.25	3	0.05
mathematical models--2001-2010	american options--2011-2020	error analysis	0.04	0.06	2	0.02
mathematical models--2001-2010	brownian movement--2011-2020	brownian movement;fractional brownian motion	0.29	0.13	6	0.03
mathematical models--2001-2010	economic and social effects--2011-2020	economic and social effects	0.29	0.20	4	0.03
mathematical models--2001-2010	economics--2011-2020	option pricing;black-scholes model;financial markets;european option	0.25	0.04	18	0.01
mathematical models--2001-2010	exact solution--2011-2020	mathematical transformations	0.14	0.17	5	0.03
mathematical models--2001-2010	numerical solution--2011-2020	functions	0.07	0.09	9	0.03
mathematical models--2001-2010	stochastic systems--2011-2020	stochastic systems;stochastic models	0.10	0.05	5	0.02
mathematical models--2001-2010	transaction cost--2011-2020	transaction cost	0.50	0.50	4	0.04
problem solving--2001-2010	brownian movement--2011-2020	option valuation	0.08	0.13	3	0.03
problem solving--2001-2010	economics--2011-2020	black-scholes formula;boundary conditions;computer simulation;implied volatility	0.13	0.04	5	0.01
problem solving--2001-2010	optimization--2011-2020	optimization	0.43	0.33	3	0.04
problem solving--2001-2010	polynomials--2011-2020	black-scholes equations	0.10	0.08	5	0.03
problem solving--2001-2010	stochastic systems--2011-2020	parameter estimation	0.03	0.05	2	0.02
problem solving--2001-2010	transaction cost--2011-2020	algorithms	0.25	0.50	2	0.04
quantum theory--2001-2010	brownian movement--2011-2020	quantum theory	1.00	1.00	2	0.13

From	To	Words	WII	II	Occurrences	SI
american options--2011-2020	american options--2021-2023	american options	1.00	1.00	11	0.06
american options--2011-2020	black scholes equations--2021-2023	mathematical operators	0.13	0.14	2	0.04
american options--2011-2020	fractional derivatives--2021-2023	fractional differential equations;second-order accuracy	0.09	0.06	4	0.02
american options--2011-2020	numerical results--2021-2023	mathematical techniques;numerical scheme	0.29	0.17	4	0.05
brownian movement--2011-2020	costs--2021-2023	quantum theory	0.08	0.13	3	0.03
economic and social effects--2011-2020	costs--2021-2023	economic and social effects;risk assessment	0.43	0.20	4	0.03
economics--2011-2020	black scholes equations--2021-2023	black scholes equations;integral equations;fractional partial differential equations	0.50	0.14	29	0.01
economics--2011-2020	costs--2021-2023	economics;costs;investments;financial markets;numerical methods;black-scholes model;black-scholes;european option	0.60	0.04	46	0.01
economics--2011-2020	finite difference method--2021-2023	option pricing;finite difference method;iterative methods;numerical experiments;finite difference scheme	0.78	0.13	27	0.01
economics--2011-2020	fractional derivatives--2021-2023	convergence of numerical methods;fractional derivatives;differential equations;convergence;stability and convergence;efficiency;fourier analysis;unconditionally stable	0.36	0.04	14	0.01
economics--2011-2020	nonlinear equations--2021-2023	nonlinear equations;black-scholes formula;perturbation techniques	0.78	0.25	12	0.02
economics--2011-2020	numerical results--2021-2023	numerical results;time discretization	0.43	0.17	6	0.02
economics--2011-2020	numerical solution--2021-2023	partial differential equations;boundary conditions;black-scholes partial differential equations	0.19	0.06	19	0.01

From	To	Words	WII	II	Occurrences	SI
economics--2011-2020	ordinary differential equations--2021-2023	ordinary differential equations	1.00	1.00	2	0.02
european call option--2011-2020	costs--2021-2023	european call option	0.33	0.50	3	0.04
exact solution--2011-2020	fractional derivatives--2021-2023	linear transformations	0.14	0.17	2	0.03
numerical solution--2011-2020	fractional derivatives--2021-2023	differentiation (calculus);caputo fractional derivatives	0.20	0.09	4	0.03
numerical solution--2011-2020	numerical solution--2021-2023	numerical solution;radial basis function networks;functions	0.37	0.09	6	0.04
option price--2011-2020	costs--2021-2023	option price	1.00	1.00	3	0.04
polynomials--2011-2020	costs--2021-2023	black-scholes equations;generalized black-scholes equations	0.18	0.08	4	0.03
polynomials--2011-2020	fractional derivatives--2021-2023	polynomials;interpolation;convergence analysis	0.33	0.08	6	0.03
polynomials--2011-2020	numerical results--2021-2023	spatial discretizations	0.14	0.17	4	0.06
stochastic systems--2011-2020	black scholes merton--2021-2023	black scholes merton	0.33	0.33	2	0.05
stochastic systems--2011-2020	costs--2021-2023	commerce;option pricing models;electronic trading	0.23	0.05	10	0.02
stochastic systems--2011-2020	fractional derivatives--2021-2023	error estimates	0.03	0.05	2	0.02
stochastic systems--2011-2020	inverse problems--2021-2023	inverse problems	0.67	0.50	4	0.05
stochastic systems--2011-2020	numerical solution--2021-2023	stochastic systems;stochastic models	0.19	0.06	13	0.03
transaction cost--2011-2020	black scholes equations--2021-2023	transaction cost	0.25	0.50	5	0.13

Notes:

WII = Weighted Inclusion Index:

A weighted inclusion index is a composite index that measures financial inclusion by assigning different weights to different dimensions based on their importance. It aims to overcome the limitations of the previous index, which assumed the same weighting criteria for all dimensions (Figlewski, 2002; Hanivan & Nasrudin, 2019).

II = Inclusion Index:

Measurement of the proportion of occurrences of a topic compared to the total occurrences of all topics in the same dataset. II provides perspective on how dominant or often discussed a topic is compared to other topics in the same context (Mäntylä et al., 2018).

Occurrences:

The absolute number of times a topic appears in the dataset. It is a direct quantitative measure that does not consider the context or relevance of the occurrence (De Vita & Rosa, 2018).

SI = Stability Index:

The Stability Index measures how consistently a topic appears over time or across different segments of a dataset. A low value indicates that the topic appears to have relatively the same consistency, while a high value indicates that there is more variability in the frequency with which the topic appears (Jensen et al., 2019).

4. Results and Discussions

An analysis of publication trends related to the Black-Scholes model since 1978 shows a significant increase in interest, with a peak of activity in 2022 recording 58 articles. This trend confirms the relevance and importance of this model in financial studies. The biggest spike in publications occurred in 2020, with an additional 23 articles, indicating an important event in this field. Fluctuations in the number of publications from year to year reflect changes in research priorities and global economic conditions. For example, the sharp decline in 2011 indicates a change in priorities or a response to economic conditions. Since the beginning of the research period in 1978, there have been fluctuations in the number of publications and citations. However, the 2000s saw a significant increase, reflecting a wider recognition of the importance of these models in dealing with the complexity of financial markets. The year 2020 recorded the highest number of publications, but with a decline in citations per paper (C/P), indicating that research is becoming more specific and niche. Recent years have shown a decline in the number of publications and citations, possibly due to theme saturation or changes in research methodology.

Geographically, the United States and the United Kingdom dominate publications, followed by European countries such as the Netherlands, Switzerland, and Germany. Asia and Eastern Europe also made significant contributions, with participation from countries such as Singapore, Thailand, Romania, and Bulgaria. Egypt represented Africa, albeit with fewer numbers.

Kim J. and Kananthai A. are the two leading authors in Black-Scholes model-related publications, with 13 articles each. Jeong D follows with a contribution of 10 articles, placing him as a prolific author. In terms of h-index, Kim J. and Jeong D. lead with a score of 6, signifying their great influence with at least six publications cited at least six times. Other authors with significant h-indexes include Wang S, Wang J, Roul P, Ren F-Y, Qiu W-Y, Magdziarz M, and Cen Z, each with an h-index of 5. Kim J. and Jeong D. show consistency and significant influence in this area, with their work frequently cited by other researchers. From an institutional perspective, Korea University stands out as the most productive institution, with a total of 48 publications, indicating a strong focus on research in this area.

Fudan University, Zhejiang Wanli University, Central South University, and Islamic Azad University also made significant contributions, with 19, 18, and other significant numbers of publications. The strong presence of these institutions in Asia signifies the region's role in the development of research related to the Black-Scholes model. Universitat Politècnica de València in Spain and Shahid Rajaee Teacher Training University in Iran, with 11 articles each, show a wide geographical spread of research. In addition, the 'Not Reported' category with 18 articles highlights the existence of relevant research that may not be attributed to a specific institution.

To show how the Black-Scholes Model idea has changed over the last 40 years, from 1978 to 2023, this study uses the Black-Scholes Theme Evolution Sankey Diagram to show how the research theme has changed over time and through different subtopics.This evolutionary period is divided into four important phases: 1) the period 1978-2000, which marks the foundation and early application of the model; 2) the period 2001-2010, characterized by the expansion to mathematical solutions and computational problems; 3) the period 2011-2020, which focuses on the integration of socio-economic effects and stochastic systems; and 4) the period 2021-2023, which shows the movement towards newer methodologies and the complexity of financial products.

4.1 1978-2000: Foundation and Early Application

During this early period, the focus of research on the Black-Scholes Model mainly revolved around understanding the underlying mathematical fundamentals, such as 'differential equations', and its application in 'option pricing'. Indications from the data suggest that during this period, there was a significant effort to integrate the Black-Scholes Model into the existing financial framework by exploring the mathematics of finance and introducing the Black-Scholes Model as a tool to assist in the pricing of derivative instruments.

To explore the relationship between the 'Relevance Degree' and 'Co-occurrence Score' of research themes related to the Black-Scholes model during this early period (1978–2000), we can examine figure 7 below.

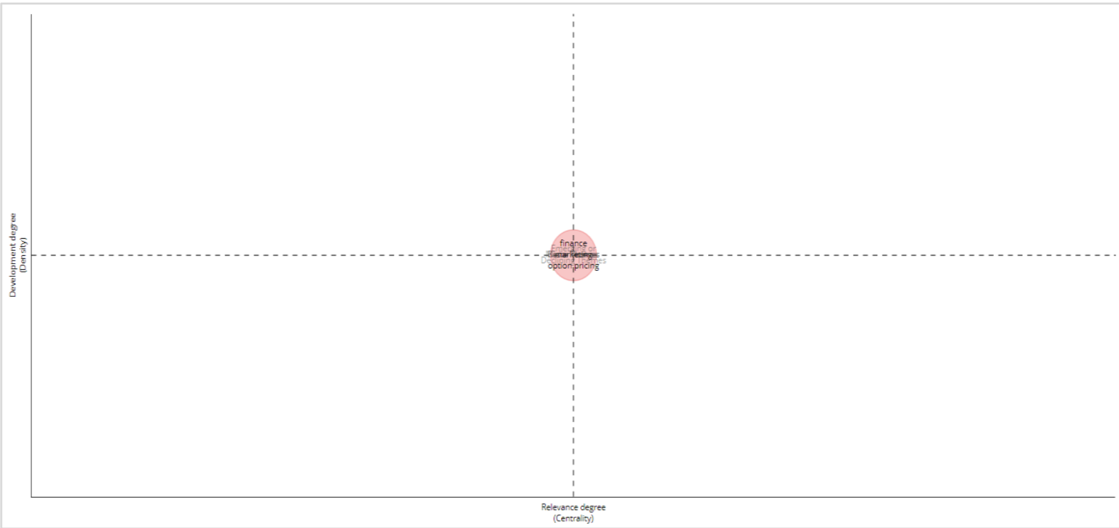


Figure 7. Evolution of Black-Scholes Related Research Themes 1978-2000

The figure shows that the dots representing the research themes are clustered close to the horizontal axis in the center, indicating that the Black-Scholes model had a moderate level of relevance over the period. The dots are not widely spread up or down, indicating that the 'Co-occurrence Score' is generally low, meaning that the model rarely co-occurs with other research themes in the same corpus of literature.

The positioning of these points in the lower quadrant, closer to the center of the horizontal axis than the vertical, implies that the Black-Scholes model, while recognized in finance, has not been at the center of much research or academic discussion. The low co-occurrence score illustrates that the model has not been widely integrated with other sub-topics or has not been frequently cited along with other concepts in economic and financial research during this period.

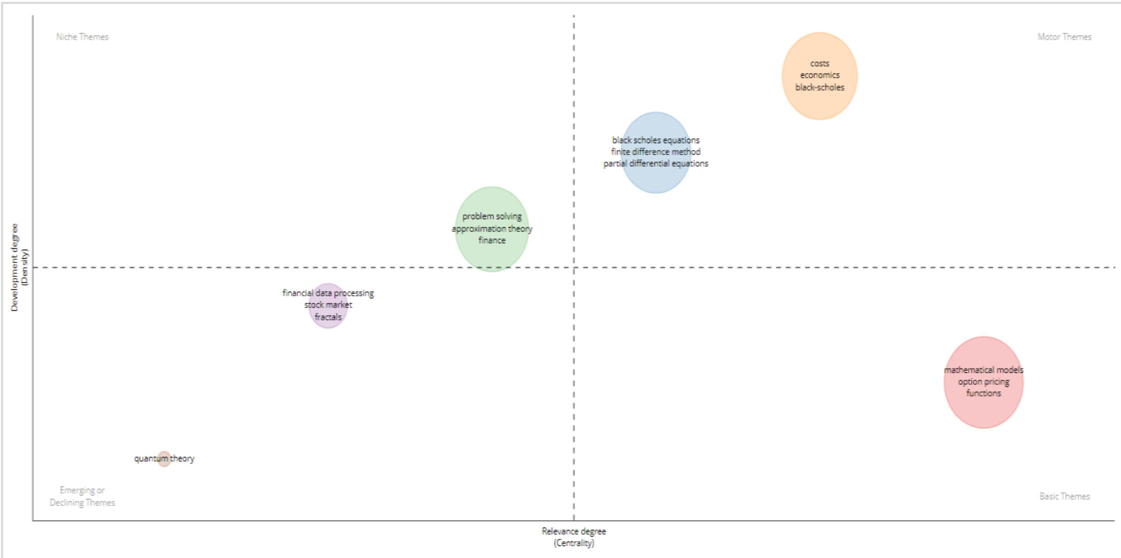
It can be concluded that the position of the dots in Figure 7 supports the view that the Black-Scholes model, during the period 1978–2000, was in the early stages of acceptance and integration in the finance literature. The model had indeed started its journey of becoming an important tool in option valuation but had not yet reached the level of penetration and importance that would be demonstrated in the following decades.

4.2 2001-2010: Expansion into Solutions of Mathematics and Computational Problems

The transition to the 21st century saw research expand towards more complex problem-solving methods and the development of mathematical models. Research began to explore "stochastic systems," reflecting attempts to accommodate randomness and uncertainty in financial markets. In addition, 'pricing models' and 'transaction costs' also became important areas of study, underscoring the quest to understand transaction costs and their implications for option pricing.

To explore the relationship between the 'Relevance Degree' and 'Co-occurrence Score' of research themes related to the Black-Scholes model during the period of expansion into mathematical solutions and computational problems (2001–2010), we can analyze figure 8 below.

Figure 8.
Evolution of Black-Scholes Related Research Themes for the 2001-2010 Period



The themes related to 'Black Scholes Equations', 'Finite Difference Method', and 'Partial Differential Equations' are located in the upper-central quadrant. The bubble is located more towards the center than in previous periods, signaling increased centrality or relevance in academic discussions and research. The topics are becoming more important, but the collaboration between them and other topics may not be as strong as it used to be, as shown by the lower co-occurrence scores that are closer to the horizontal axis. This is in line with the observed decrease in II.

During this period, an increase in the centrality of Black-Scholes-related themes becomes evident, as indicated by the position closer to the center on the 'Relevance Degree' axis. This suggests that these themes are starting to play a more significant role in academic discussions and research. The relatively higher position on the 'Development Degree' axis also indicates that research in these themes is developing and becoming more advanced.

Overall, the placement of these themes in the diagram confirms that the Black-Scholes model has gained a firmer foundation in the finance and economics literature over the past decade, cementing its role as an important framework in financial market modeling and analysis.

4.3 2011-2020: Integration of Socio-Economic Effects and Stochastic Systems

During this decade, research has focused not only on the mathematical aspects but also on the 'economic and social effects'. This signaled a greater recognition of the broad influence of financial markets and pricing models on the economy as a whole. Research on'stochastic systems' and 'brownian movement' shows a trend towards a more in-depth understanding of the underlying market dynamics and their influence on option pricing behavior. In addition, topics such as 'European option' and 'fractional Brownian motion' highlight the adaptation and application of models in various financial products.

To explore the relationship between the 'Relevance Degree' and 'Co-occurrence Score' of research themes related to the Black-Scholes model during the integration period of socio-economic and stochastic systematic effects (2011–2020), we can analyze figure 9 below.

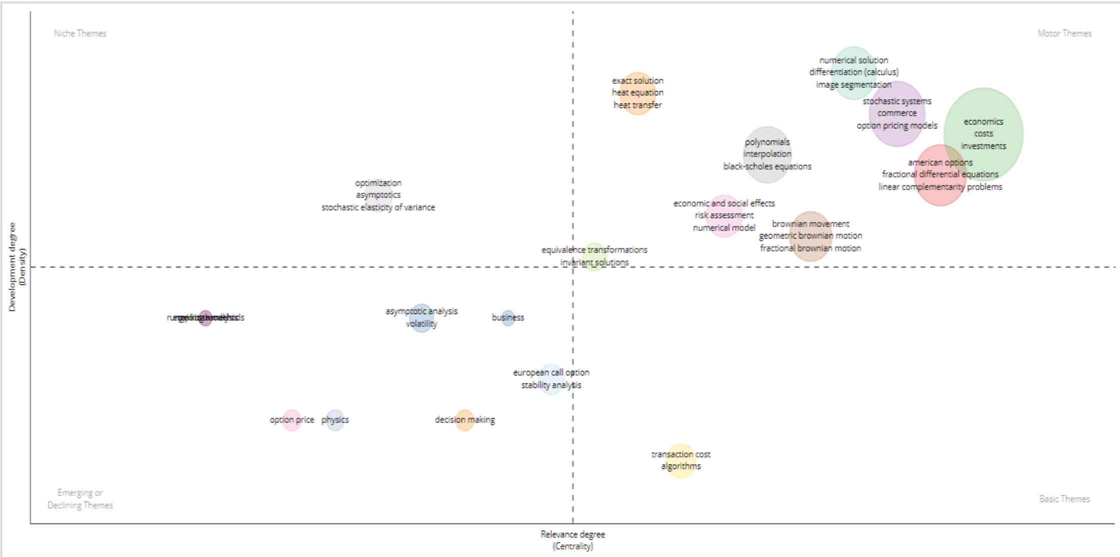


Figure 9.
Evolution of Black-Scholes Related Research Themes for the 2011-2020 Period

There are several bubbles that indicate research topics related to economics, mathematics, and other fields. These themes are located in various quadrants, indicating the degree of centrality and development of the themes in research related to the Black-Scholes model.

During this period, the Black-Scholes model was increasingly integrated into economic research. This is reflected in Figure 4, where bubbles relating to 'Black-Scholes Equations', 'Polynomials', 'Interpolation', and other economics-related topics such as 'Risk Assessment', 'Economic and Social Effects', and 'Numerical Models' are located in the top-right quadrants, indicating these topics as motor themes. These bubbles show a high degree of centrality (judging by the position close to the horizontal 'Relevance degree' axis) and a high degree of development (judging by the position close to the vertical 'Development degree' axis).

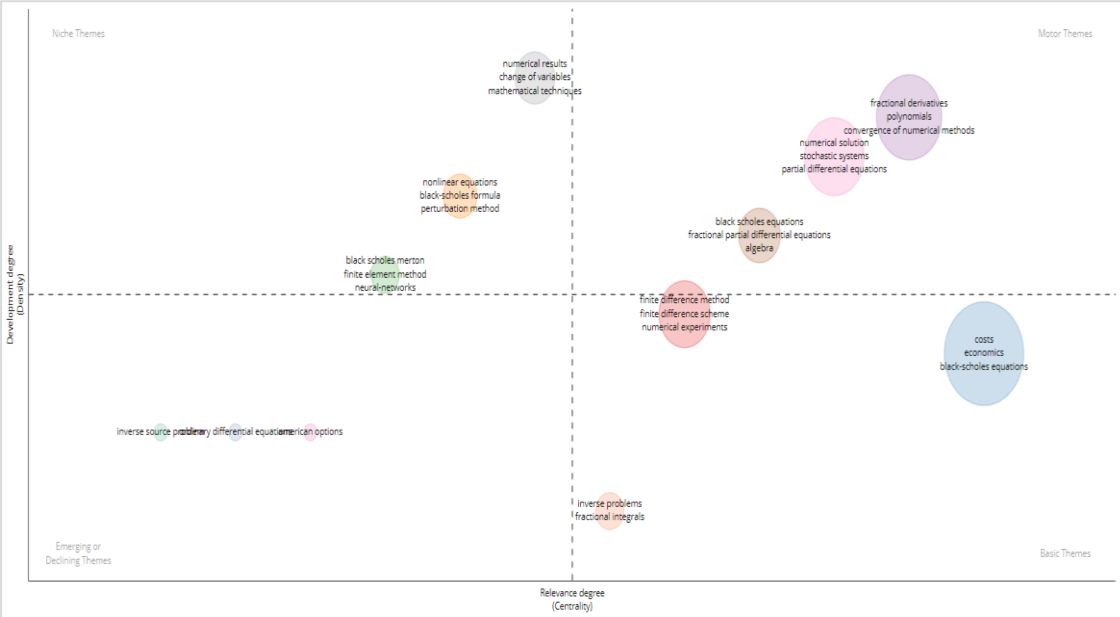
Overall, the position and size of the bubble in the diagram show that the Black-Scholes model has become an integral part of economic and financial research over the past decade, confirming its strong link to the economy. It also signifies the continued growth in research that is based on or utilizes this model, reflecting the wide adaptability and relevance of the Black-Scholes model in the modern world of finance.

4.4 2021-2023: Towards Methodological Novelty and Product Complexity

In recent periods, there has been an increased interest in 'numerical results' and 'fractional derivatives', indicating a move towards more sophisticated methodologies and the search for solutions to newer and more complex financial instruments. The reliance on "finite difference methods" and "numerical experiments" highlights the developments in computational techniques required to address the needs of today's markets. Topics such as 'error analysis' and 'perturbation techniques' also appear frequently, indicating more detailed research in the search for model accuracy and its application in more dynamic and uncertain market scenarios.

To explore the relationship between the 'Relevance Degree' and 'Co-occurrence Score' of research themes related to the Black-Scholes model during the period towards methodological novelty and product complexity (2021-2023), we can analyze figure 10 below.

Figure 10. Evolution of Black-Scholes Related Research Themes for the 2021-2023 Period



Themes related to 'Fractional Derivatives', 'Polynomials', and 'Convergence of Numerical Methods' are located in the top-right quadrant and labeled as motor themes. Meanwhile, 'Black Scholes Equations' is next to 'Costs' and 'Economics', which are placed in the middle of the bottom-right quadrant as Basic Themes. This indicates that, while still relevant, this topic may no longer be the main focal point in current research, but it is still an important basic theme.

The theme 'Fractional Differential Equations', which is closer to the center, indicates that this is an evolving area with broader linkages. In the context of Black-Scholes, it shows the exploration of using fractional methods in the context of option pricing and finance. In this period, the research focus has shifted towards the development and application of fractional differential equations, as indicated by the dominant position of the 'Fractional Derivatives' theme in the Motor Themes quadrant. This suggests that there is a strong push in research to integrate fractional methods with the Black-Scholes model, which may be related to efforts to model more complex market behavior or achieve a deeper understanding of financial asset price dynamics.

The 'Black Scholes Equations' bubble located between the Basic Themes indicates that, although this theme may no longer be an actively developing focal point, it is still a strong foundation for current research. It also signifies that the Black-Scholes model, despite being several decades old, remains relevant and useful in modern financial modeling. Overall, the position of these themes in the diagram confirms that the Black-Scholes model not only continues to be used but also inspires new approaches in finance research, particularly in the context of fractional differential equations that can offer new insights and more sophisticated methods to analyze and solve complex problems in finance.

From the analysis of the entire period (1978–2023), it can be concluded that the Black-Scholes model has undergone significant evolution over four decades, from its initial acceptance in finance to its continued integration and exploration in various aspects of economics and mathematics. This evolution reflects not only the importance of the model in financial theory and practice but also its adaptability and relevance in the face of new challenges and research questions.

5. Conclusion

Bibliometric analysis reveals that the Black-Scholes Model has evolved significantly in trends, contributors, and research themes since its first introduction four decades ago. The analysis metrics, which include annual publication and citation trends as well as the geographical distribution of research, show a continuous increase in its academic and practical relevance. The model, launched in 1978, has adapted its application according to changing market conditions. The research is dominated by contributors from the United States and the United Kingdom, with significant global contributions. Authors such as Kim J. and Kananthai A., as well as institutions such as Korea University, have made important contributions.

Period 1978–2000: Early research focus was limited to the theoretical underpinnings of the Black-Scholes Model, with a low frequency of occurrence and moderate relevance in the academic literature. The model emerged as a new tool in the finance literature but has yet to become a major reference in economics and finance research. Period 2001–2010: There was a significant increase in the relevance of 'Black Scholes Equations', 'Finite Difference Method', and 'Partial Differential Equations' in economics. These developments marked the Black-Scholes Model as a major focus in the study of risk and socio-economic impact. Period 2011–2020: The Black-Scholes Model continues to be integrated into economic research, with intensive exploration of its applications. Despite the decline in WII and II values, the high frequency of occurrence signals a great interest in exploring new applications of the model, reflecting its adaptability and broad relevance in the modern world of finance. Period 2021-2023: The research focus shifts to exploring applications of the model in the context of fractional differential equations, marking the adaptation of the model to address new and complex market challenges and more specific research into applications of the Black-Scholes Model in these new contexts, reflecting a shift in research focus and innovation of the model's applications in applied mathematics.

The evolution of the Black-Scholes Model has had a far-reaching impact on modern finance, becoming an essential element in economic and financial research. The model drives new approaches in financial research, aiding financial innovation, education, policy, and cross-disciplinary applications. Financial institutions use variations of the model to better hedge strategies and reduce portfolio risk. In computer science, the model is integrated with machine learning to predict market trends and investor behavior. Its application in fractional differential equations also opens up research opportunities in applied mathematics. Financial market authorities use the model in global systemic risk analysis.

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