

Parameter Estimation of the Temporal Point Process Model through the Bayesian Approach (Case study : Malaria disease data from Wahidin Hospital in Makassar City)

Darwis^{1*}, Sunusi N², Kresna A. J³

^{1, 2, 3}Mathematics Department, Faculty of Mathematics and Natural Sciences,
 Hasanuddin University, Makassar
 Email: awidarwis09@gmail.com*

Abstrak	Informasi Artikel
<p>Penelitian ini bertujuan mengestimasi parameter melalui pendekatan Bayesian dari model <i>temporal point process</i>. Parameter intensitas bersyarat model tersebut dipandang sebagai suatu <i>renewal process</i> yang selanjutnya digunakan melalui pendekatan <i>Squared Error Loss Function (SELF)</i>. Parameter intensitas bersyarat model <i>temporal point process</i> diestimasi menggunakan metode maximum likelihood estimation melalui persamaan <i>likelihood point process</i>. Selain itu, penelitian ini mengkaji metode estimasi maksimum likelihood dan metode Bayes untuk menganalisis fungsi resiko dari hasil penaksir parameter intensitas bersyarat. Pada aplikasi estimasi parameter ini, studi kasus yang digunakan adalah menganalisa data orang yang terkena penyakit malaria yang datanya berasal dari Rumah Sakit Wahidin Kota Makassar. Studi kasus tersebut menghasilkan nilai $-1 \leq \theta < 0,4$ yang merupakan nilai resiko penaksir MLE yang lebih tinggi dibandingkan dengan menggunakan Metode Bayes sedangkan nilai $0.5 \leq \theta \leq 1$ merupakan hasil nilai resiko dari penaksir MLE yang lebih kecil dibandingkan dengan menggunakan Metode Bayes.</p>	<p>Sejarah Artikel: Diajukan 20 Jan 2020 Diterima 1 Apr 2020</p> <hr/> <p>Kata Kunci : <i>Temporal Point Process</i>, Penduga Bayes, Sebaran Eksponensial</p>
Abstract	Keywords:
<p>This study aims to estimate parameters through the Bayesian approach from the temporal point process model. The conditional intensity parameter of the model is seen as an update process which is then used through the Squared Error Loss Function (SELF) approach. Generally the conditional intensity parameter for the temporal point process is estimated using the maximum likelihood estimation method that utilizes the likelihood point process equation but only limits the sample information extracted from a likelihood function without regard to prior information the sample. Another thing examined in this study is determining the Risk Function of the results of the conditional intensity parameter estimator using the maximum likelihood estimation (MLE) and the Bayes Method. As a parametric estimation application, an analysis of Malaria disease data was collected, the data obtained from Wahidin Hospital in</p>	<p>Keywords: Temporal Point Process, Bayesian Estimation, Exponential Distribution</p>

Makassar City. Based on the case study, the value is $-1 \leq \theta < 0.4$, MLE risk scale estimator is higher than risk scale made by estimator using Bayes while $0.5 \leq \theta \leq 1$, MLE risk scale estimator is smaller than estimator using Bayes method.	
--	--

1. Introduction

Some research shows that climate change causes an increase in malaria incidence. Malaria is an infectious disease that can be transmitted by mosquitoes called Anopheles. This disease is most common in tropical and subtropical regions where the Plasmodium parasite can thrive as well as the Anopheles mosquito vector. Based on data in the world, malaria kills one child every 30 seconds. Around 300-500 million people are infected, and around one million people die from this disease every year [1]. Based on these cases, it is difficult for the community to predict when this malaria will befall them. Because of the difficulty in predicting the emergence of malaria, therefore, the authors try to solve the cases that occur.

In handling the case, the authors use the temporal point process conditional intensity parameter, which will be estimated using the Bayesian approach as done [2]. However, in this study, the intensity of the temporal point process is seen as a renewal process.

Therefore, this research will be written in a thesis entitled "Study temporal point process as a process of renewal (Renewal Process), Case Study Analyzing the time of emergence of malaria from data sourced from Wahidin Hospital in Makassar in 2011-2013".

2. Literature review

2.1 Temporal Point Process

A temporal point process is a model of point processes in one dimension (time) that is useful for sequences of random times if a case occurs. For example, the time when an earthquake occurs can be modeled as a temporal point process [3].

2.2 Renewal Process

The process of calculating the time between cases that are free and identical in a particular distribution is called the renewal process. The Renewal Process $(N(t))$ shows the number of cases during the interval $[0, t]$. Generally, X_n case is interpreted as the time between $n-1$ and n cases, which are commonly referred to as intercase time and S_n as the time until n -cases occur.

$$S_n = \sum_{i=1}^n X_i, \quad n \geq 1 \quad \text{and} \quad S_0 = 0 \quad (1)$$

2.3 Conditional Intensity Function

Conditional intensity function $\lambda(t|\mathcal{H}_t)$ is defined as $P(\text{one case in}(t, t + \Delta)|\mathcal{H}_t) = \lambda(t|\mathcal{H}_t)\Delta + o(\Delta)$, or

$$P(N(t, t + \Delta) = 1|\mathcal{H}_t) = \lambda(t|\mathcal{H}_t)\Delta + o(\Delta) \quad (2)$$

2.4 Risk Functions

Definition:

It is said that the L function is a loss function. The expected (average) value of the loss function is called the risk function. If $g(y; \theta)$, $\theta \in \Omega$ is the opportunity density function of y . Risk functions $R(\theta, \delta)$ is as follows:

$$R(\theta, \delta) = E\{\mathcal{L}[\theta, \delta(y)]\} = \int_0^\infty \mathcal{L}[\theta, \delta(y)]g(y; \theta)dy \quad (3)$$

3. Research Methodology

In this research, the study of malaria time was taken. The data that will be used for this case study is malaria disease data sourced from Wahidin Sudirohusodo Hospital Makassar in the period 2011-2013. The variables used in this study are: the response variable in the study is malaria data that originated from Wahidin Sudirohusodo Hospital Makassar in the period 2011-2013.

1. Response variable :
 The response variable in the study was malaria disease data sourced from the Wahidin Sudirohusodo Hospital in Makassar.
2. Predictor variable :
 Time of Occurrence (Date, Month, and Year) and Time between events.

The implementation of this research begins with a theoretical review which is then applied to a real phenomenon, namely the emergence of a disease in a particular area. The steps taken are as follows:

1. State the conditional intensity of the temporal point process model as a renewal process.
2. Formulate the likelihood temporal point process function which is seen as a renewal process.
3. Determines the cumulative distribution function for the time distribution between events.
4. Define the prior sample distribution on the gamma distribution.
5. Formulate a posterior distribution.
6. Estimating conditional intensity parameters using the Bayes method.
7. Calculates the conditional intensity for the time between events having an exponential distribution.
8. The results of the conditional intensity parameter estimation obtained through the Bayesian approach are compared with the results of the parameter estimation using the MLE approach through the risk function

4. Results and Discussion

4.1 Renewal Process of Conditional Intensity Functions

As a renewal process, Conditional Intensity Function, the relationship between time between cases will be demonstrated in determining the conditional intensity function.

$$P(\text{nothing applied on } (t_{i-1}, t_i]) = \exp\left(-\int_{t_{i-1}}^{t_i} \lambda(s|\mathcal{H}_s) ds\right) \quad (4)$$

4.2 Conditional Intensity of Time Between Cases with an Exponential Distribution

As is known the opportunity density function for exponential random variables, stated as

$$f(\tau_i) = \theta e^{-\theta(\tau_i)} \quad , \tau_i \geq 0 \text{ dan } 0 < \theta < \infty$$

and the cumulative density function is,

$$F(\tau_i) = 1 - e^{-\theta(\tau_i)} \quad , \tau_i \geq 0$$

then the Conditional Intensity function:

$$\begin{aligned} \lambda(t_i|\mathcal{H}_{t_i}) &= \frac{f(\tau_i)}{1 - F(\tau_i)}, \\ &= \frac{\theta e^{-\theta(\tau_i)}}{1 - (1 - e^{-\theta(\tau_i)})}, \end{aligned}$$

$$= \theta . \tag{5}$$

4.3 Bayesian Estimation of Exponential Distribution

Bayesian estimates for θ with the SELF approach are:

$$\hat{\theta}_s = \frac{n + \alpha}{\sum_{i=1}^n x_i + \frac{1}{\beta}} \tag{6}$$

where n =many cases, $\sum_{i=1}^n x_i$ = Amount of time inter-cases

$$\alpha = 1 , \beta = \frac{1}{\bar{x}}$$

4.4 Temporal Point Process Application

The plot data is obtained at Figure 1.

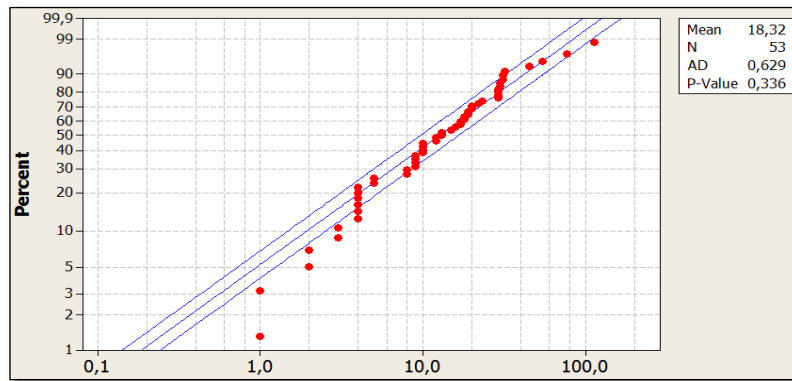


Figure 1 Probability The time plot inter-cases is exponentially distributed

Figure 1 shows the Probability of the test plot of the suitability of the exponential distribution with the time distribution between cases. The p-value and Anderson Darling (AD) can be seen in Table 1 as follows.

Table 1 Probability Value The time plot inter-cases has an Exponential distribution

Mean	N	AD	p-value
18.3	53	0.63	0.34

Table 1 It can be seen that the p-value from the Anderson Darling test above is higher than 0.34, which means it is greater than the alpha level of 0.05. So it is known that the data follows an exponential distribution.

4.5 Estimated Conditional Intensity Parameters

Table 2 Estimated Results of Conditional Intensity parameters for the time inter-cases with an Exponential distribution

Interval	Number of cases	Conditional Intensity
(0,1]	2	0.230531814
(1,2]	2	0.057632149
(2,3]	2	0.037459239
(3,4]	2	0.029378792
⋮	⋮	⋮

Interval	Number of cases	Conditional Intensity
(25,26]	3	0.004241373
(26,27]	2	0.003039283
(27,28]	1	0.001830873

4.6 MLE method

The parameter estimation results using the MLE method are obtained as follows: [4]

$$\lambda(t_i|\mathcal{H}_{t_i}) = \frac{n}{T}, \text{ where } T = \sum x_i$$

Furthermore, the risk level of the parameter estimator using the MLE method will be determined as follows:

For example: $\sum x_i = y$

Noted that $L(\theta, \delta) = (\delta - \theta)^2$ as δ is an estimator for θ , then:

$$[(\delta - \theta)^2] = E\left[\left(\theta - \left(\frac{n}{y}\right)\right)^2\right] \tag{7}$$

Based on equations (7),i.e.:

$$R(\theta, \delta) = E\{\mathcal{L}[\theta, \delta(y)]\} = \int_0^\infty \mathcal{L}[\theta, \delta(y)]g(y; \theta)dy$$

Where:

$$\mathcal{L}[\theta, \delta(y)] = \left[\theta - \left(\frac{n}{y}\right)\right]^2.$$

Because of the sample $x_1, x_2 \dots x_n$ having an exponential distribution then the corresponding peer prior distribution is the Gamma distribution so that:

$$\begin{aligned} R(\theta, \delta) &= E\left[\theta - \left(\frac{n}{y}\right)\right]^2 = \int_0^\infty \left[\theta - \left(\frac{n}{y}\right)\right]^2 \cdot \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy \\ &= \int_0^\infty \left[\theta - \left(\frac{n}{y}\right)\right]^2 \cdot \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy \\ &= \int_0^\infty \left[\theta^2 - 2n\theta \frac{1}{y} + n^2 \frac{1}{y^2}\right] \cdot \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy \\ &= \int_0^\infty \theta^2 \cdot \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy - \int_0^\infty 2n\theta \frac{1}{y} \cdot \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy + \int_0^\infty n^2 \frac{1}{y^2} \cdot \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy \\ &= \theta^2 \int_0^\infty \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy - 2n\theta \int_0^\infty \frac{1}{y} \cdot \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy + n^2 \int_0^\infty \frac{1}{y^2} \frac{\theta^n}{(n-1)!} y^{n-1} e^{-\theta y} dy \\ &= \theta^2 \cdot 1 - \frac{2n\theta^{n+1}}{(n-1)!} \int_0^\infty y^{(n-1)-1} e^{-\theta y} dy + \frac{n^2\theta^n}{(n-1)!} \int_0^\infty y^{(n-1)-2} e^{-\theta y} dy \\ &= \theta^2 - \frac{2n\theta^{n+1}}{(n-1)!} \cdot \int_0^\infty y^{(n-1)-1} e^{-\theta y} dy + \frac{n^2\theta^n}{(n-1)!} \cdot \int_0^\infty y^{(n-1)-2} e^{-\theta y} dy, \end{aligned} \tag{8}$$

where :

$$\int_0^\infty y^{(n-1)-1} e^{-\theta y} dy = \int_0^\infty y^{(n-2)} e^{-\theta y} dy \tag{9}$$

and

$$\int_0^\infty y^{(n-1)-2} e^{-\theta y} dy = \int_0^\infty y^{(n-3)} e^{-\theta y} dy. \tag{10}$$

Equation (10) could be solved as follows :

Let: $\theta y = u$ thus $du = \theta dy$.

Therefore $y = \frac{u}{\theta}$, thus $y^{n-2} = \left(\frac{u}{\theta}\right)^{n-2}$,

then obtained $dy = \frac{du}{\theta}$.

Equation (10) can be written as:

$$\int_0^{\infty} y^{(n-2)} e^{-\theta y} dy = \int_0^{\infty} \left(\frac{u}{\theta}\right)^{n-2} e^{-u} \cdot \frac{du}{\theta}$$

The results are obtained as follows:

$$\begin{aligned} \int_0^{\infty} y^{(n-2)} e^{-\theta y} dy &= \int_0^{\infty} \frac{u^{n-2}}{\theta^{n-2}} e^{-u} \frac{1}{\theta} du \\ &= \frac{1}{\theta} \cdot \frac{1}{\theta^{n-2}} \int_0^{\infty} u^{n-2} e^{-u} du, \\ &= \frac{1}{\theta} \cdot \frac{1}{\theta^n} \int_0^{\infty} u^{n-2} e^{-u} du, \\ &= \frac{1}{\theta} \cdot \frac{\theta^2}{\theta^n} \int_0^{\infty} u^{n-2} e^{-u} du, \\ &= \frac{\theta}{\theta^n} \int_0^{\infty} u^{n-2} e^{-u} du, \\ &= \frac{\theta}{\theta^n} \cdot (n-2)! \\ &= \frac{(n-2)!}{\theta^{n-1}}. \end{aligned} \tag{11}$$

The same thing is done for Equation (11) so that it is obtained as follows:

$$\int_0^{\infty} \frac{1}{y^2} y^{(n-3)} e^{-\theta y} dy = \frac{(n-3)!}{\theta^{n-2}}. \tag{12}$$

Based on Equation (11) and (12) obtained that:

$$\begin{aligned} R(\theta, \delta) &= E \left[\theta - \left(\frac{n}{y}\right) \right]^2 = \theta^2 - \frac{2n\theta^{n+1}}{(n-1)!} \cdot \int_0^{\infty} y^{(n-1)-1} e^{-\theta y} dy + \frac{n^2\theta^n}{(n-1)!} \cdot \int_0^{\infty} y^{(n-1)-2} e^{-\theta y} dy \\ &= \theta^2 - \frac{2n\theta^{n+1}}{(n-1)!} \cdot \frac{(n-2)!}{\theta^{n-1}} + \frac{n^2\theta^n}{(n-1)!} \cdot \frac{(n-3)!}{\theta^{n-2}} = \theta^2 - \frac{2n\theta^2}{(n-1)} + \frac{n^2}{(n-1)(n-2)} \theta^2, \\ &= \frac{(n-1)(n-2)\theta^2 - (n-2)2n\theta^2 + n^2\theta^2}{(n-1)(n-2)}, \\ &= \frac{(n^2 - 3n + 2)\theta^2 - (2n^2 - 4n)\theta^2 + n^2\theta^2}{(n-1)(n-2)}, \\ &= \frac{n^2\theta^2 - 3n\theta^2 + 2\theta^2 - 2n^2\theta^2 + 4n\theta^2 + n^2\theta^2}{(n-1)(n-2)}, \\ &= \left(\frac{(n+2)}{(n-1)(n-2)} \right) \theta^2. \end{aligned} \tag{13}$$

4.7 Bayes methods

Noticed that :

$$\hat{\theta}_s = \frac{n + \alpha}{\sum_{i=1}^n x_i + \frac{1}{\beta}}$$

The Equation can be written as follows:

$$\begin{aligned} \frac{n + \alpha}{\sum_{i=1}^n x_i + \frac{1}{\beta}} &= \frac{n}{y + \frac{1}{\beta}} + \frac{\alpha}{y + \frac{1}{\beta}} \\ &= \frac{n}{y} \left(\frac{y}{y + \frac{1}{\beta}} \right) + \alpha \beta \left(\frac{1}{(y + \frac{1}{\beta}) \beta} \right). \end{aligned}$$

Let

$$\begin{aligned} A &= \frac{y}{y + \frac{1}{\beta}} \text{ and } B = \frac{1}{(y + \frac{1}{\beta}) \beta} = \frac{1}{y\beta + 1} \\ A &= \frac{y\beta}{y\beta + 1}, \quad B = 1 - A \end{aligned}$$

Thus

$$\begin{aligned} \frac{n + \alpha}{\sum_{i=1}^n x_i + \frac{1}{\beta}} &= \frac{n}{y + \frac{1}{\beta}} + \frac{\alpha}{y + \frac{1}{\beta}} \\ &= \frac{n}{y} \left(\frac{y}{y + \frac{1}{\beta}} \right) + \alpha \beta \left(\frac{1}{(y + \frac{1}{\beta}) \beta} \right) \\ &= \frac{n}{y} \cdot A + \alpha \beta \cdot B. \end{aligned}$$

Since $E(\theta) = \hat{\theta}$ and $\hat{\theta} = \frac{n + \alpha}{\sum_{i=1}^n x_i + \frac{1}{\beta}}$,

then

$$= \frac{n + \alpha}{\sum_{i=1}^n x_i + \frac{1}{\beta}} = \frac{n}{y} \cdot A + \alpha \beta \cdot B .$$

Furthermore

$$R(\theta, \delta) = E \left[\left[\theta - \left(\frac{n + \alpha}{y + \frac{1}{\beta}} \right) \right]^2 \right] = E \left[\left[\theta - \frac{n}{y} A + (B) \alpha \beta \right]^2 \right],$$

or

$$\begin{aligned} &E \left[\left[\theta - \left(A \frac{n}{y} + (1 - A) \alpha \beta \right) \right]^2 \right] \\ &= E \left[\theta^2 - 2\theta \left(A \frac{n}{y} \right) - 2\theta(1 - A) \alpha \beta - 2A(1 - A) \frac{n}{y} \alpha \beta + ((1 - A) \alpha \beta)^2 + A^2 \left(\frac{n}{y} \right)^2 \right], \\ &= E(\theta^2) - E \left(2A\theta \frac{n}{y} \right) - E(2\theta(1 - A) \alpha \beta) - E \left(2A(1 - A) \frac{n}{y} \alpha \beta \right) + E(((1 - A) \alpha \beta)^2) + E(A^2 \left(\frac{n}{y} \right)^2) \\ &= \theta^2 - 2A\theta n E \left(\frac{1}{y} \right) - 2\theta(1 - A) \alpha \beta - 2A(1 - A) \alpha \beta n E \left(\frac{1}{y} \right) + ((1 - A) \alpha \beta)^2 + A^2 n^2 E \left(\frac{1}{y^2} \right), \\ &= \theta^2 - 2A\theta \frac{n}{n-1} \theta - 2\theta(1 - A) \alpha \beta - 2A(1 - A) \alpha \beta \frac{n}{n-1} \theta + (1 - A) \alpha \beta)^2 + A^2 n^2 \frac{\theta^2}{(n-1)(n-2)}, \\ &= \left(1 - 2A \frac{n}{n-1} + \frac{A^2 n^2}{(n-1)(n-2)} \right) \theta^2 - \left(2(1 - A) \alpha \beta + 2A(1 - A) \alpha \beta \frac{n}{n-1} \right) \theta + (1 - A) \alpha \beta, \end{aligned}$$

$$= \left(\frac{n^2(1 - 2A + A^2) - (3 - 4A)n + 2}{(n - 1)(n - 2)} \right) \theta^2 - \left(\frac{(2A\alpha\beta - 2A^2\alpha\beta)n + 2(2 - 2A)\alpha\beta}{(n - 1)} \right) \theta + (1 - A)(\alpha\beta)^2 \quad (14)$$

4.8 The differences in the risk functions of the MLE and Bayes methods

Based on the case study, the risk function is obtained as follows :

For Bayes method,

$$R(\theta, \hat{\theta}) = 0.02065\theta^2 + 0.00217\theta - 0.00102$$

For,

$$MLER(\theta, \hat{\theta}) = 0.02073\theta^2$$

Table 3 The score of risk estimator scale for the of MLE method and Bayes for the score $-1 \leq \theta \leq 1$

Score θ	Risk Scale		Score θ	Risk Scale	
	MLE	BAYES		MLE	BAYES
-1	0.020739	0.01746	0	0	-0.00102
-0.9	0.016799	0.01375	0.1	0.000207	-0.00060
-0.8	0.013273	0.01046	0.2	0.00083	0.00024
-0.7	0.010162	0.00758	0.3	0.001867	0.00149
-0.6	0.007466	0.00511	0.4	0.003318	0.00315
-0.5	0.005185	0.00306	0.5	0.005185	0.00523
-0.4	0.003318	0.00142	0.6	0.007466	0.00772
-0.3	0.001867	0.00019	0.7	0.010162	0.01062
-0.2	0.00083	-0.00063	0.8	0.013273	0.01393
-0.1	0.000207	-0.00103	0.9	0.016799	0.01766
0	0	-0.00102	1	0.020739	0.02180

Table 3 indicates that: $-1 \leq \theta < 0.4$, MLE risk scale estimator is higher than risk scale made by estimator using Bayes while $0.5 \leq \theta \leq 1$, MLE risk scale estimator is smaller than estimator using Bayes method.

It can be seen in the graph below :

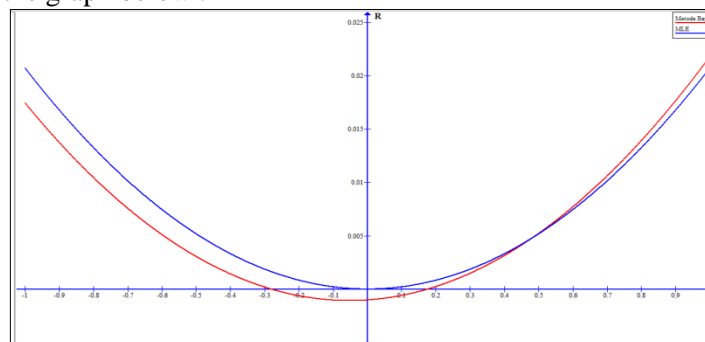


Figure 2 Graph of the relationship between parameter score θ with risk scale using MLE and Bayes methods

Note:

$$Q = \theta \text{ and } R = \text{Risk Scale}$$

Figure 2 shows that score $-1 \leq \theta \leq 1$, MLE estimator (blue line) presents a higher risk of error than the risk made by estimator using the Bayes method (red line).

5. Conclusion

Based on the description from the previous chapters, it can be concluded as follows: The estimated conditional intensity parameters of the temporal point process model through the Bayesian approach are carried out as follows:

First, stating the conditional intensity time between cases, then constructing the likelihood temporal point process:

$$L = \left(\prod_{i=1}^N f(t_i | \mathcal{H}_{t_i}) \right) P(N_{(t_N, T]} = 0),$$

$$= \left(\prod_{i=1}^N \lambda(t_i | \mathcal{H}_{t_i}) \right) \exp \left\{ - \int_0^T \lambda(s | \mathcal{H}_s) dt \right\},$$

Furthermore, the estimation of conditional intensity parameters using the Bayesian approach is done through the SELF approach. Estimation results obtained as follows:

$$\lambda(t_i | \mathcal{H}_{t_i}) = \frac{n + \alpha}{\sum_{i=1}^n x_i + \frac{1}{\beta}}.$$

1. Risk function of each estimator:

Bayes estimator :

$$R(\theta, \hat{\theta}) = \left(\frac{n^2(1 - 2A + A^2) - (3 - 4A)n + 2}{(n - 1)(n - 2)} \right) \theta^2 - \left(\frac{(2A\alpha\beta - 2A^2\alpha\beta)n + 2(2 - 2A)\alpha\beta}{(n - 1)} \right) \theta + (1 - A)(\alpha\beta)^2.$$

MLE estimator :

$$R(\theta, \hat{\theta}) = \left(\frac{(n + 2)}{(n - 1)(n - 2)} \right) \theta^2.$$

2. From the data recorded at Wahidin Sudirohusodo Hospital illustration is obtained:

MLE estimator score presents a higher error than the risk that made by estimator using the Bayes method.

6. Recommendation

In this study, the study of the emergence of malaria for the foreseeable future can not be predicted because research is still limited to determining conditional intensity. It is recommended in subsequent studies to make parametric methods for the conditional intensity that has been obtained.

References

- [1] Sumampouw O.J, dkk (2019), "the trend of dengue hemorrhagic fever in the city of Manado in 2018-2019. *KESMAS*.8(6).
- [2] Ogata.(1999), "Seismicity analysis through point process modeling :A review. *Pure Appl. Geophys.* 155,471-507.
- [3] Daley, D.J. and Vere-Jones, D. (2003) "Introduction to the Theory of Point Processes". Vol 1: Elementary Theory and Methods(2nd edition). Springer, New York.
- [4] Sunusi, N, dkk , (2010), " Study of Eartquake Forecast through Hazard Rate Analysis, *Int.J.App. Math and Stat*, vol 1(J.10) : p. 96-103