

Declining Cogging Torque Technique of an Integral Slot Number for Permanent Magnet Machines

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Abstract—The existence of cogging torque in electric equipment has been considered undesirable. This kind of friction in the air-gap impacts the alignment of flux and the stator slots, resulting in the observed outcome. Consequently, the imposition of restrictions on the rotation of the rotor is employed to generate electrical energy. This research endeavor primarily aims to mitigate the cogging torque of electrical machinery. The current utilization involves employing a total of three permanent magnet machines, often known as inset PMMs, which possess a slot count of 24 and a pole count of 8. The employed technique involves the integration of an optimal pole arc method in conjunction with the implementation of slots cut into the magnet's edge. The machine model under investigation has two fundamental variants, namely Models 1 and 2. These models are equipped with one-stage slotted (OSS) and two-stage slotted (TSS) edges on each magnet, in addition to pole arc optimization. The simulation was conducted using the Finite Element Method Magnetics (FEMM) 4.2 software together with LUA scripts, with a focus on rotor rotation ranges of 1°. Model 2 exhibited a decrease in cogging torque of 0.01 Nm, whereas Model 1 demonstrated a reduction of 0.015 Nm, and the basic model had a decrease of 0.02 Nm. When implementing a dual-layered cutting edge on a magnet and attempting to optimize its pole arc, it is imperative to consider that the cogging torque's peak magnitude becomes substantially diminished or entirely eliminated.

Keywords: *cogging torque, declining, finite element, integral slot number, inset pmm*

I. INTRODUCTION

Nowadays, machines utilizing permanent magnets known as permanent magnet machine (PMM) have found widespread application in various fields. The rationale lies in the advantages of the machine in comparison to alternative types. The machine offers several advantages, such as its robust form, simple and unassuming mechanical design, ease of maintenance, high torque density, and performance [1–4]. Nevertheless, an often-observed constraint in applying PMM is the fact that they consist of cogging torque (CT). The magnetically connected stator teeth and rotor core make contact with one another, which facilitates the establishment of the CT. The primary role of the CT in a PMM is to provide attractive forces that contribute to maintaining the proper alignment involving the permanent magnet (PM) with its stator [1].

The presence of a CT in industrial settings may give rise to unfavorable characteristics, such as vibrations and noise, hence limiting its usage to applications that

necessitate particular control. In scenarios characterized by low speeds, such as those commonly observed in wind turbine applications, the primary mover encounters challenges in rotating the permanent magnet rotor, hence impeding the generation of electrical power. Based on the research findings, it can be asserted that the occurrence of any CT in permanent magnets is a significant challenge encountered during the design phase.

In recent years, there has been a global increase in the implementation of CT reduction techniques. Several studies have concentrated on altering the design parameters of PMMs from the rotor perspective. These modifications include magnet pole shifting [5–9] and segmentation [1–3, 10–16], optimization of pole arc [3, 17–21], magnet skewing or step skewing [10, 13, 22–30], reduction of magnet strength [1, 19], variation and offset of pole arc length [1, 31, 32], and slotting the magnet edge [3, 33, 34]. Additional studies have been conducted to investigate methods for reducing CT from the stator perspective. These techniques include the implementation of dummy

slotting [35, 36], dummy teeth [37], stator skewing [13, 28, 29], asymmetric stator teeth [38, 39], optimization of slot opening width [40, 41], and modification in stator shoe height [42].

One approach that has demonstrated significant efficacy in reducing the maximum CT values for an integral number of slots is the implementation of a two-stage slotting (TSS) technique at the magnet's edge, along with the augmentation of the magnet's pole arc. The implementation of the slotted edge magnet process, in conjunction with a suitable pole arc, represents a further efficacious method for reducing CT. The act of inserting each magnet edge into slots in the PMM is used to effectively decrease both the flux density and overall flux dispersion through an air gap. In a theoretical framework, one possible technique to reduce the magnetic strength of a magnet is to optimize the pole arcs positioned on the outside surface of the magnet. The combined application of multiple CT reduction strategies can substantially decrease the maximum CT in PMM by approximately 99.5% compared to its original structure.

The objective of this study was to reduce CT. In the framework of this research, novel methodologies for reducing CT were introduced. The method consists of the integration of the TSS method and pole-arc optimization technique. This technology has the capability to be employed in conjunction with extended magnetic pole arcs and offset magnetic ends, resulting in a reduction of CT. The manufacturing of magnets initially consisted of two steps. The maintenance of a consistent separation around the rotor and its stator results in the strengthening of the boundary region of the air gap through the inclusion of slots positioned along the outer edge of the magnet. The CT phenomenon can be effectively reduced by incorporating slots along the outer edge of the magnet. This design modification results in a decrease in both the residual magnetic flux and the amount of energy kept within the air gap. It is important to note that the stator structure remains unchanged throughout this modification.

II. METHODE AND DESIGN

This section contains two main discussions: the theoretical framework employed to ascertain CT through the utilization of various references, and the design portion incorporated within the model simulation.

A. The Permanent Magnet Machine Study

A calculation of CT is conducted under the condition that the winding of the PMM's stator remains free of any current flow. Additionally, the effects of magnetic saturation and armature response are considered to be insignificant. Moreover, it is assumed a magnet's magnetization has a radial direction. Several mathematical equations have been utilized, one of which involves the computation of CT. To determine the CT (T_c), it is imperative to utilize the following equation [3]:

$$T_c = -\frac{1}{2} \phi_g^2 \frac{dR_g}{d\theta} \quad (1)$$

where ϕ_g^2 denote the air-magnetic gap's flux. The resistance of the air gap through which the current flows can be denoted as R_g , while the rotational angle of the machinery can be represented by θ . The regular square configuration of the magnetic fluxes suggests that the impact from the fluxes flowing in relation to air space on a CT is negligible (1). The CT exhibits variability in tandem with the fluctuations in its inclination towards space. The overall magnetic flux going through the gap is influenced by the flux densities (B) in the normal array and the cross-sectional area (A), as stated in reference [2]:

$$\phi_g = \int B \cdot dA. \quad (2)$$

Equation (3) provides additional insight into the concept of the Fourier transform as a CT phenomenon:

$$T_c = \sum_{k=1}^{\infty} T_{mk} \sin(mk\theta) \quad (3)$$

where m represents the numerical measurement of a least common multiple (LCM) involving the quantity of slots () and the quantity of poles (N_p). A parameter called k tends to be a single integer, whereas T_{mk} denotes a Fourier coefficient [3].

The PMM being studied is an inset type with a configuration consisting of eight poles and twenty-four slots. The CT phenomenon occurs when the magnetic poles are aligned in phase with or parallel to the slots of the stator [3]. The impact of each magnet on CT can be stated as below:

$$T_c = N_p \sum_{k=1}^{\infty} T_{pN_s k} \sin(N_s k\theta) \quad (4)$$

where $T_{pN_s k}$ represents the CT factor associated with each magnet. The investigation of an air-gap reluctance (1) can be conducted using this expression:

$$R_g = \frac{l_g}{\mu_0 A_g} \quad (5)$$

where l_g is used to indicate the distance of the air gap. Furthermore, the parameter A_g is employed to denote the dimension of the cross-section within an air gap, whereas the value μ_0 signifies the characteristics of the magnet of an air gap.

A formula such as (6) according to the cited reference [43] can be employed to create predictions for all of the flux over the gap in air (B_g), assuming that the magnet is of unit magnitude that can be written as follows:

$$B_g = \frac{B_r l_m}{(r_r + l_g) \ln \frac{r_r + l_g}{r_r - l_m}} \quad (6)$$

where B_r is frequently employed to indicate the remanence of magnetic material. Similarly, l_m is used to represent the

Table 1. Experimented with inset PMM setups

Parameters	Original model	Preliminary Model (Model 1)	Proposed Model (Model 2)
Diameter of the stator, d_{stator} (m)	0.121275	0.121275	0.121275
Lengths of the air gap, l_g (m)	0.00241699	0.00241699	0.002000
Teeth angle of the rotor, α_{rh} (deg.)	4.03189°	4.03189°	4.03189°
opening width of the slot, w_{so} (m)	0.00250949	0.00250949	0.00250949
Diameter of the shaft, d_{shaft} (m)	0.0423	0.0423	0.0423
Length of the magnet, l_m (m)	0.0368891	0.0368891	0.0368891
Height of the magnet, h_m (m)	0.00603865	0.00603865	0.00603865
Magnetic radius, r_m (m)	0.0582204	0.0582204	0.0382204
Rotor core radius, r_r (m)	0.052186	0.052186	0.052186
Inner stator radius, r_s (m)	0.0606374	0.0606374	0.0606374
Outer stator radius, r_o (m)	0.095	0.095	0.095
Magnet pole cross area, A_m (m ²)	0.000236899	0.000224353	0.000221628
Air gap cross section area, A_a (m ²)	0.000845496	0.000931228	0.000960732

magnetic distance, r_r , symbolizes the rotor radii.

An experimental study made use of the settings outlined in Table 1. The magnetic remanence () value of NdFeB permanent magnets is estimated to be around 1.2 Tesla. Equation (1) facilitates the estimation of the CT for each individual piece of experimental equipment. The impact of rotor slots on the CT can be evaluated by examining the geometric details of the magnet pole cross-sections as described in (1), (2), and (3). This research aims to analyze and compare the impact of magnet notching on CT performance across different machine types.

B. Design of Permanent Magnet Machine

The procedures of this investigation were implemented in accordance with the predetermined design. Three PMM models are used in this simulation, including the original (conventional) model, the preliminary (Model 1), and the proposed (Model 2) versions of the design. The Finite Element Method Magnetics (FEMM) software is utilized to acquire magnetic flux distribution and CT measurements for every model. The specifications of each model are outlined in Table 2.

The initial model refers to the original version in which no modifications or enhancements have been made to the

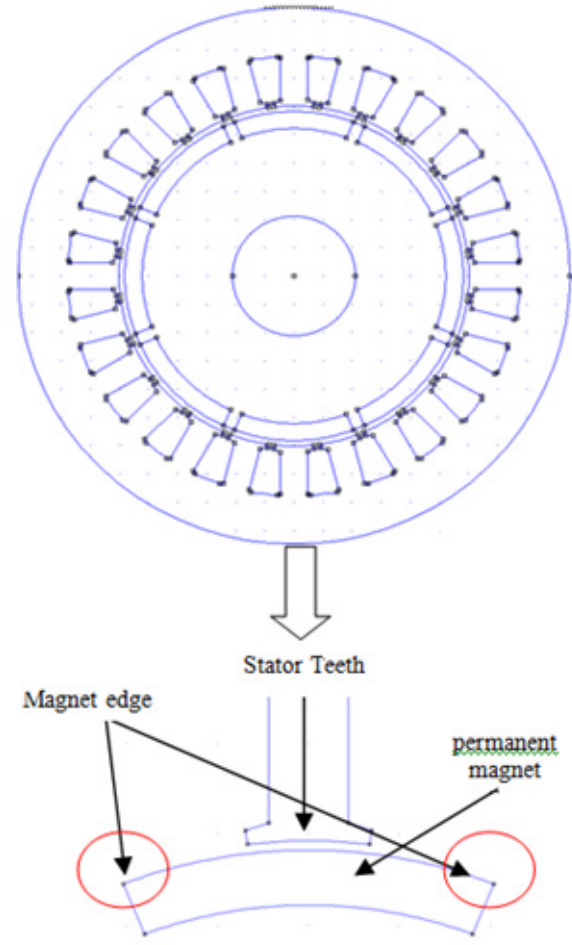


Figure 1. One cycle ECG illustration

permanent magnet design. Model 1 can be considered a preliminary version of the original model, wherein modifications were made to the magnet's edge shape with the inclusion of a one-step slotting process. Model 2 is a proposed design from the original model that, using a technique known as two-step slotting, goes through additional slotting twice at both ends of the PM.

The magnets from the early PMM designs examined in this research are shown in Figure 1. Starting out, neither the magnetic rotor nor the rotor's core were changed (see Figure 1). To reduce the machine's CT, slots are present on both magnet edges (Figure 2 (a) and (b)).

Figure 2 provided herein represents a further advancement or elaboration of the content depicted in Figure 1. The inclusion of slots at both ends of each magnetic edge in the illustrated figure serves the function of decreasing the cogging torque. The approaches utilized in the aforementioned procedure consist of the one-step slotting method (Model 1) and the two-step slotting method (Model 2), correspondingly. The permanent magnet configuration is the main difference between the proposed device and the current model. Model 1 has square (1 step) cuts around the edges of the permanent magnet, but Model 2 has two levels of cuts (2 steps of slotting) around the edges of the magnet.

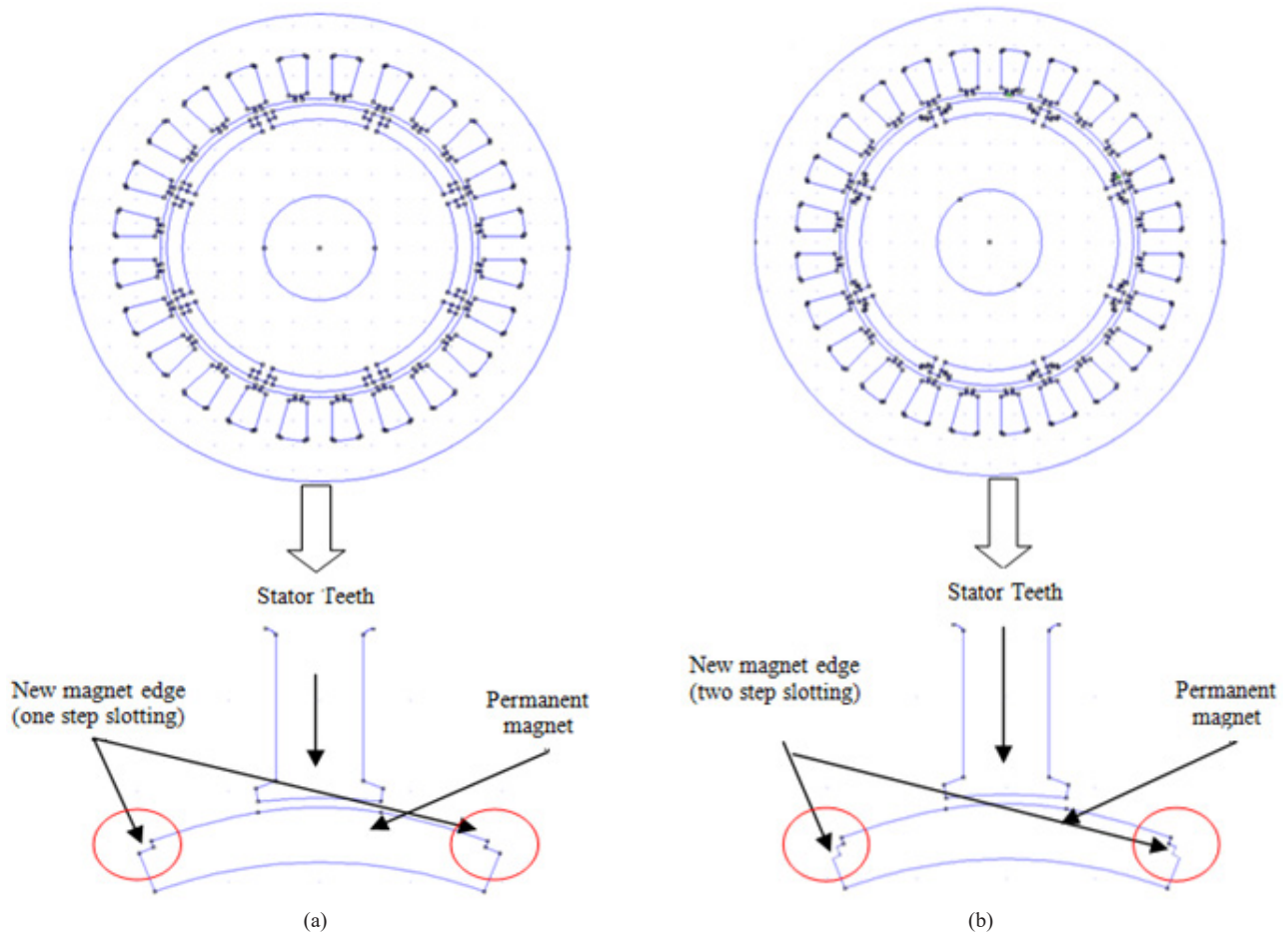


Figure 2. Design of inset PMMs: (a) preliminary (Model 1) and (b) proposed (Model 2)

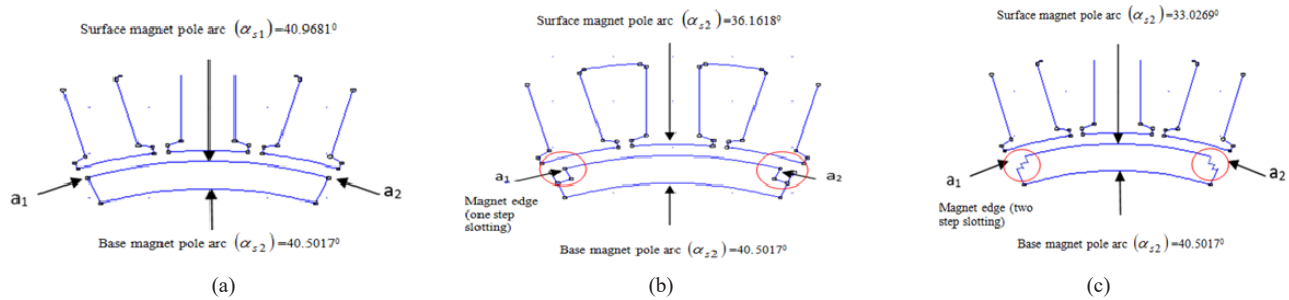


Figure 3. Specifics on the design of experimental inset PMM: (a) original model, (b) preliminary (Model 1), and (c) proposed (Model 2)

Figure 3 illustrates the comprehensive configuration of the experimental setup. The values denoted as α_{s1} and α_{s2} in Figure 3(a) correspond to the leading and trailing edges of the PM NdFeB that comprise the experimental machine. The value of the initial magnet pole arc remains unchanged. As depicted in Figure 3(b) and 3(c), the pole arc at the surface diminishes in both the first and second models due to the presence of slots on the magnetic outer side.

Given the findings derived from a finite element simulation, a surface magnet pole arc is observed to be diminished to a value of 36.16180 for the first model, however, for the second model, it is reduced to 33.02690. It is essential to acknowledge the magnetic pole arc improvement of 33.02690 in relation to the proposed

strategy. By implementing a slotting technique along the magnet edge, it is possible to decrease the amount of additional material required. This reduction in material allows for a decrease in both the magnetic pole pitch and the cross-sectional area. The magnet pole arc lengths for the majority of experimental machinery were measured to be 0.0416287 meters for the basic design, 0.0367514 meters for Model 1, and 0.0335585 meters for Model 2. The length of the leading and trailing edges along the path of the magnet rotor decreases as both the arc length and the magnet pitch decrease. Given that a magnetic edge contains slots, the width of the sectional throughout an air gap increase. The rotor's surfaces are also changed by the fact that the magnet has slots on its outside ends. The research findings indicate that there is an absence of stator-

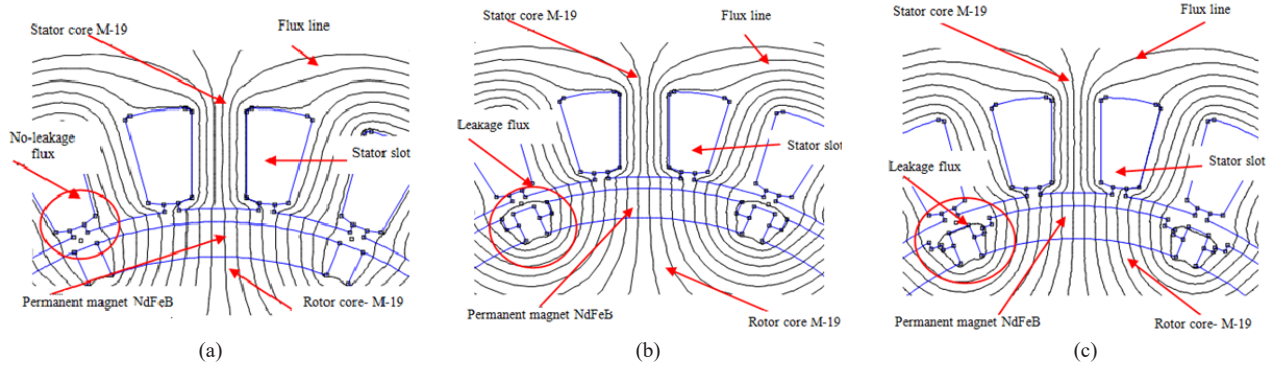


Figure 4. Flux dispersion studies from the research inset PMMs: (a) original model, (b) preliminary (Model 1), and (c) proposed (Model 2)

based dummy slots in all of the machines analyzed.

III. RESULT AND DISCUSSION

A. Simulation Result of Magnetic Flux Distribution

The general features of the inset-PMM have been thoroughly examined using the FEMM version 4.2 of the finite element analysis software, integrated with LUA script version 4.0 [17, 19, 43, 44]. The integration of both components facilitates the simulation of the particular inset-PMM. The ability of LUA to execute parallel computations is an added advantage. The Auto-CAD software initiates the generation of a simulated structure at the onset of each simulation, subsequently allowing for its integration into the FEMM software. The models utilized in this study are characterized by their two-dimensional rectangular shape. This work presents simulation outcomes for both stator and rotor components of an inset PMM. The simulations take into account a collection of slots located on the outer edges of the magnet. The primary aim of this research project was to contrast and evaluate the dispersion pattern of flux surrounding an air gap with CT in experimental machines.

Figure 4(a) illustrates a basic magnet without any implemented optimization strategies. As a consequence of this phenomenon, there is a significant presence of magnetic flux with extremely high magnitudes that permeates the surrounding air gap of these stator slots. Hence, the intensity of interaction between the magnetic surfaces and the slots increases, resulting in higher resistance against the prime mover's rotation of a rotor. This resistance arises from the opposing force generated by the attractive force between the magnetic surfaces with high magnetic properties.

Figure 4(b) depicts an altered version of Figure 4(a), wherein a slot has been introduced at the magnetic edges. Consequently, there is a significant decline in the magnetic. The phenomenon of flux reduction concentration at the magnet's edge leads to a decline in the level of contact friction experienced between the magnetic surfaces and each individual slot. Furthermore, this approach demonstrates a significant degree of effectiveness in reducing the flow of a magnetic field that traverses an air

gap and then enters the slots.

The data presented in Figure 4(c) demonstrates that employing a two-step slotting configuration at the magnet's ends results in a further reduction in magnetic flux density. This outcome is superior to that achieved with a one-step slotting structure. This behavior can be explained by the comparatively diminished surface area of the magnet when comparing both a magnet with a one-step slotting (OSS) design and a magnet without slots. Moreover, an additional element that contributes to this phenomenon is the existence of two-step slots positioned at the outer edges of the magnets. The occurrence of these slots leads to an augmented separation between the edges of the magnets and the stator core, resulting in a significant reduction in the level of contact between the magnets and the stator slots. Nevertheless, under specified conditions and at specific points on the magnet's surface, it becomes feasible to identify a resemblance in the decrease of magnetic flux density between OSS and TSS designs.

For the machine under discussion, Figure 4 shows the pattern of flux dispersion, including the fluxes directed out of the magnetic rotors toward the stator's surface. This arrangement of flux lines of magnets flowing out of a magnetic rotor, in accordance with the concept of least reluctance, is made possible by the inclusion of magnetic slots. This principle might be considered as an inclination for the flow to selectively navigate through its surrounding space, following the most advantageous pathways available at any given time. In the context of magnet slotting, it is worth noting that these actions necessitate a rather moderate magneto-motive force. The occurrence of the event leads to the emergence of flux leakage on the rotor's surface, resulting in a reduction in the magnitude of normal fluxes that are oriented towards the gap in air.

A simulation results demonstrate that the normal magnetic flux of each Model 2 is 0.602157 Tesla, which is lower than both the initial model's flux of 0.65508 Tesla and the flux of Model 1, which is 0.612056 Tesla. Using a mathematical formula in (6), the data in Figure 5 is used to figure out how the flux spreads in a normal direction at different rotor angles for each of the machines that were tested.

A flux waveform over an air gap has been distorted, as shown in Figure 5. The phenomenon is predicted to begin

at a particular location taking into account the width of the stator slot. Putting a magnet slot on the outside edges of these modification models, especially Models 2 and 1, has no effect on the frequency of the fluctuations in this flux waveform at the machines observed. The lack of impact on the magnetic pull balancing within the air gap in Model 2 can be attributed to the presence of the TSS in its magnets.

The presence of eccentricity in a PMM machine results in an imbalance, leading to an increased force of attraction across the air gap. Furthermore, the rate of fluctuation in flux density is higher in an eccentric PMM machine compared to a healthy one [45]. In order to examine the concentration of magnetic fluxes across each of the stator and rotor cores, finite element modeling was implemented in the research.

Among the several simulated models, it is observed that Model 1 and 2 exhibit the most prominent peak values of magnetic field strength, measuring 1.36446 and 1.36286 Tesla, respectively. For a better understanding, Figure 1 shows the stator teeth with the highest flux density. Based on the findings of an experimental investigation, it has been determined that the implementation of slotted magnetic edges does not yield a noticeable effect on the flux density that traverses the rotor and stator cores. The stator core flux of Model 2 exhibited a significantly higher magnitude compared to that of Model 1. The rotor core of Model 2 exhibits the highest flux density among all machines, measuring at 0.888384 Tesla. This is followed by model 1, which has a flux density of 0.866344 Tesla, and the basic model, with a flux density of 0.870284 Tesla. The presence of slots on each magnetic edge may have an impact on

the increase of flux throughout the rotor core, considering the previous limitation of the rotor core's flux density to a range of 1.5 to 1.6 Tesla. Exceeding a flux density value of 1.6 Tesla within the rotor could negatively influence the machine's performance. Consequently, modifications were implemented to the configuration of the core.

Figure 6(a) illustrates the results obtained from the PMMs without optimization, indicating that the total magnetic flux within the rotor core measures 0.872028 Tesla. The normal magnetic flux density is measured at 0.784701 Tesla, whereas the tangential magnetic flux density is recorded at 0.380364 Tesla. Once the magnet ends are slotted (as depicted in Figure 6(b)), the resultant magnetic flux distribution within the rotor core is measured to be 0.866344 Tesla. The magnetic flux is distributed with a magnitude of 0.801836 Tesla in the normal direction and 0.328042 Tesla in the tangential direction.

The inclusion of a two-step slot at the end of the magnet edge provides a more uniform dispersion of the overall flux to the rotor core in comparison to a PMM without slots on the magnetic edge (Figure 6(c)). Specifically, the total magnetic flux measures 0.888384 Tesla in the former case, whereas it amounts to 0.768943 Tesla and 0.444917 Tesla in the latter case, following a normal tangential magnetic flux distribution.

Figure 6 illustrates that the positioning of the slot at the rotor's magnet edges generates minimal or negligible impact on the development of fluxes in the design of the machines. Based on the available evidence, it is reasonable to infer that the implementation of Model 2 is an achievable option. It should be noted that the primary components (M-19) comprising the rotor and stator, as employed in this study, have demonstrated optimal performance within the magnetic field range of 1.5 to 1.6 Tesla.

The computational technique of finite element analysis was employed to examine the CT at a rotation angle of 180° and with the rotor undergoing 10 stages of rotation. For the purpose of clarity, Figure 7 represents a mechanical angle of only 60° . The CT machine was found to employ distinct pulses for every 60° mechanical degrees of rotation, distinguishing it from other machines. The initial model exhibits a peak CT value of roughly 0.02 nm. This model was considered suitable due to its air-gap reluctance being the highest among the models that were examined. The peak CT of Model 1 is lower compared to the Basic Model, measuring at 0.015 Nm. The decrease

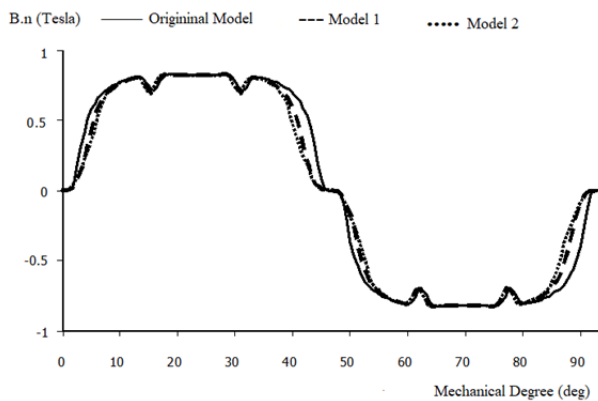


Figure 5. Normal flux simulation findings for each examined PMM model

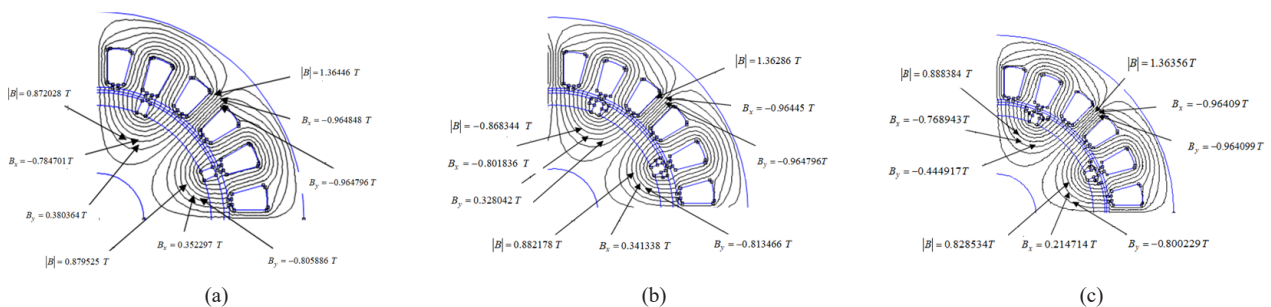


Figure 6. Specifics on the design of experimental inset PMM: (a) original model, (b) preliminary (Model 1), and (c) proposed (Model 2)

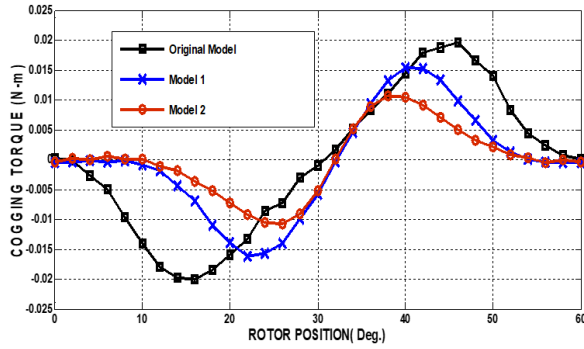


Figure 7. Analysis of CT at various inclination angles

in operational efficiency can be attributed to the interplay between the rotor and its magnet, along with the reluctance induced by an air gap. The highest peak torque (CT) recorded for Model 2 was 0.01 Nm, which was the smallest value compared to the other three experimental models.

The proposed Model 2, including a configuration of eight poles and twenty-four slots, effectively reduces cogging torque. Based on the data provided by Figure 7, it is readily apparent that Model 2 has a significantly higher level of productivity in comparison to the other models. This is evident from its lowest CT values for the initial twelve mechanical degrees of rotation (0° – 22°) as well as the last twelve mechanical degrees of rotation (38° – 60°). The proposed Model 2 exhibits its highest CT when the rotor is fully turned around 26° from its starting mechanical position. In comparison, the original model and Model 1 have similar values of approximately 15° and 22° , respectively. This suggests that Model 2 requires less mechanical energy compared to the other two models in order to attain a comparable initial rotational velocity when starting from a state of rest or low speed. The possibility of a varied distribution of CT arises due to the softer features of Model 2.

IV. CONCLUSION

The present research work was focused on the comprehensive implications of the width of slot openings on a specific region of magnetic poles. According to the outcomes of the models, it can be shown that a decrease in the magnet pole's cross-sectional area is the main cause of a drop in CT and normal flux in an air gap of inset PMMs. The most important advancement in Model 2 is the ability to change a magnet's pole arc and spacing without affecting the rotor radii or stator structure. The proposed machine exhibits mechanical integrity, although the inclusion of magnets with rotor teeth diminishes centrifugal force. A positive association has been seen involving the widening of an air gap within the machine and the reduction in reluctance. The flows of critical flux onto the surfaces of the rotor's teeth result in a significant CT reduction in the magnetic circuit within the air space. The proposed inset PMM Model 2 has the potential to be employed in energy-constrained applications, specifically in low-speed wind turbine generators.

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