Complex Eigenvalue Analysis Of A Motorcycle Disc Brake Squeal

Zahrul Fuadi and M. Nizar Machmud
Department of Mechanical Engineering, Syiah Kuala University
Jl. Tgk. Syech Abdurrauf No. 7 Darussalam – Banda Aceh 23111, INDONESIA
Phone/Fax.: +62-651-7428069
e-mail: zahrul_fuadi2004@yahoo.com

Abstract

This paper discusses the phenomenon of brake squeal of a disc brake system used at motorcycle with a capacity of 125 cc. The occurrence of disc brake squeal is associated with the dynamic instability of the system with the influence coefficient of friction. The dynamic instability of the system is predicted using finite element complex eigenvalue analysis. The analysis predicted three unstable modes in the frequency range below 5 kHz. The unstable modes occurred at the frequency of 1250 Hz, 1920 Hz, and 2306 Hz. The occurrence of unstable mode is indicated by the presence of positive real part of the corresponding complex eigenvalue. Unstable mode is shown by the occurrence of coupled vibration between eigenvectors of disc and brake pads vibrating at similar frequency which consists of diametral mode of disc and bending modes of pads. The analysis also showed that the unstable condition could occur at a value of coefficient of friction as low as 0.2.

Keywords: Motorcycle disc brake squeal, complex eigenvalue analysis, dynamic instabilities

1. Introduction

Brake squeal have been receiving attention from automotive industry for a long time because it causes the problem of inconvenience to both the occupant and the people surrounding. This problem has been a great complain and cause a high maintenance cost [1, 2]. Although various techniques and approach are applied, the problem of brake squeal is still remains. One suggested method for eliminating brake squeal can usually work for a particular model of brake system but has little effect for the other model [2]. This means that the real mechanism of brake squeal is still not fully understood yet.

Brake squeal is defined as a high pitch frequency noise of the brake system at a frequency ranging from 1 kHz to 15 kHz. Of this frequency range, the one between 1 kHz to 5 kHz is considered having a high sound pressure level so that it could give the biggest annoyance [3].

One of the methods of analyzing the phenomenon of brake squeal is by using the complex eigenvalue analysis using the finite element method [2, 3-6] in which the occurrence of squeal is associated with the dynamic instability of the brake system. The method is highly flexible in which the modification can easily be made to the model to find out the effect of various parameters to the occurrence of brake squeal.

Most of previous analyses of brake squeal were conducted to the automotive brake system. However, this present analysis is conducted for a front disc type of a light weight motorcycle with engine capacity of 125 cc. The analysis is conducted using the finite element complex eigenvalue analysis targeting the frequency range from 1 kHz to 5 kHz.

The aim of this analysis is to predict the frequency of squeal at this particular disc brake system and to identify the vibration mode of the unstable condition. Identifying the modes that involves in squeal occurrence can help determining future refinement of the structure to avoid squeal occurrence.

However, the following assumptions are considered for this analysis.

1. The material is homogenously elastic,
2. The friction is uniformly distributed in the friction interfaces.

2. Friction Interface Model

The interface between brake pads and disc rotor is the source of system's instability leading to brake squeal. Therefore, modeling the interface is the most crucial part in the analysis. In this analysis the interface at the frictioning surface is modeled based on Ripin et.al [7]. In this model, the disc and pads are assumed to have some flexibility in that they can move relative to each other.

A simplified model of the disc and pads is given in Figure 1. Both the outside and the inside pads are connected to the disc with spring having stiffness of k_c. The total stiffness of contact can then be considered as K_c.

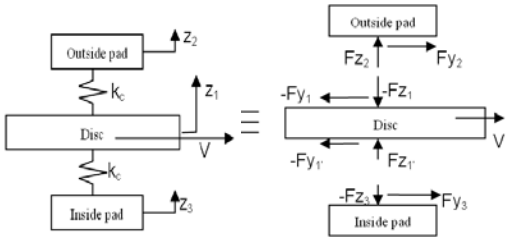

Figure 1. Simplified model of the disc and pads [7]

The general equation of motion of a vibrating system is:

$$[M]\ddot{x} + [C]\dot{x} + [K]x = F \qquad (1)$$

Where M, C, K is the matrices of mass, damping and stiffness, respectively and F is the friction force vector between the mating surfaces of the pads and the disc.

The force vector *F* can be linearized as

$$F = K_f\, x \qquad (2)$$

where K_f is the friction force matrices.

Using Figure 1, for outside pad:

$$\begin{bmatrix} Fy_1 \\ Fz_1 \\ Fy_2 \\ Fz_2 \end{bmatrix} = \begin{bmatrix} 0 & -\mu k_c & 0 & \mu k_c \\ 0 & -k_c & 0 & k_c \\ 0 & \mu k_c & 0 & -\mu k_c \\ 0 & k_c & 0 & -k_c \end{bmatrix} \begin{Bmatrix} y_1 \\ z_1 \\ y_2 \\ z_2 \end{Bmatrix} = [K_f]\{q\} \qquad 3)$$

and for inside pad

$$\begin{bmatrix} Fy_{1'} \\ Fz_{1'} \\ Fy_{3'} \\ Fz_{3'} \end{bmatrix} = \begin{bmatrix} 0 & \mu k_c & 0 & -\mu k_c \\ 0 & -k_c & 0 & k_c \\ 0 & -\mu k_c & 0 & \mu k_c \\ 0 & k_c & 0 & -k_c \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{Bmatrix} = [K_f]\{q\} \qquad 4)$$

Substituting Equation (3) and (4) into (1), a homogenous equation of motion is obtained, i.e.

$$[M]\ddot{x} + [C]\dot{x} + [K - K_f]x = 0 \qquad (5)$$

Equation 5 is an equation of motion of a free vibrating system with friction force included into its stiffness matrices. Ignoring damping, the solution of Equation (5) can be in complex form of:

$$S_i = \sigma_i \pm \omega_i j \qquad (6)$$

where S_i is the i-th complex eigenvalue, j = $\sqrt{-1}$, ω_i is the natural frequency of complex mode i, and σ_i is the real part of the i-th eigenvalue.

The occurrence of system's instability, i.e. occurrence of squeal, can be determined by analyzing the real part of a corresponding mode's eigenvalue. If the real part of an eigenvalue is positive, it indicates that the corresponding mode has growing amplitude of vibration leading to system's instability [1,4,6,8].

3. Free Vibration Analysis of the Finite Element Model

The finite element model of a motorcycle disc brake and pad is given in Figure 2. The interface of the disc and pads is the main source of the squeal noise so that it is quite reasonable to model only the disc and a pair of pads for this analysis.

The free vibration analysis of both the disc and pads is conducted to know the natural frequencies and mode shapes of the components. The information about this mode shape is important to determine in which mode the system is when dynamic instability occurs. This information is also important for improving the dynamic stability of the system by altering the harmonic of the component targeting certain natural mode shape. This method is also known as "tuning" and nowadays become one of the successful methods in dealing with brake squeal.

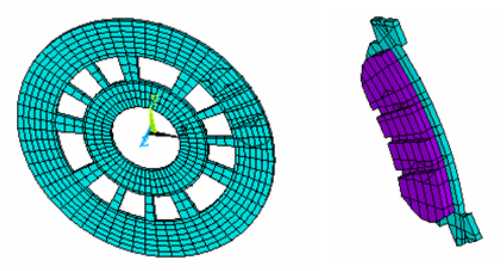

Figure 2. Finite element model of the disc and pad

In previous studies about disc brake squeal, is it revealed that diametral modes of disc and bending mode of pads are important modes in the occurrence of disc brake squeal [2, 7]. In the present analysis, free vibration analysis is conducted particularly to identify the frequency of the diametral mode of the disc and the bending mode of the pads.

The free vibration analysis is conducted in the frequency range below 5 kHz. In that frequency range, it is observed that 5 diametral modes of the disc and 4 bending modes of the pad exist. Figure 3 shows diametral modes of the disc and Figure 4 shows bending modes of the pad.

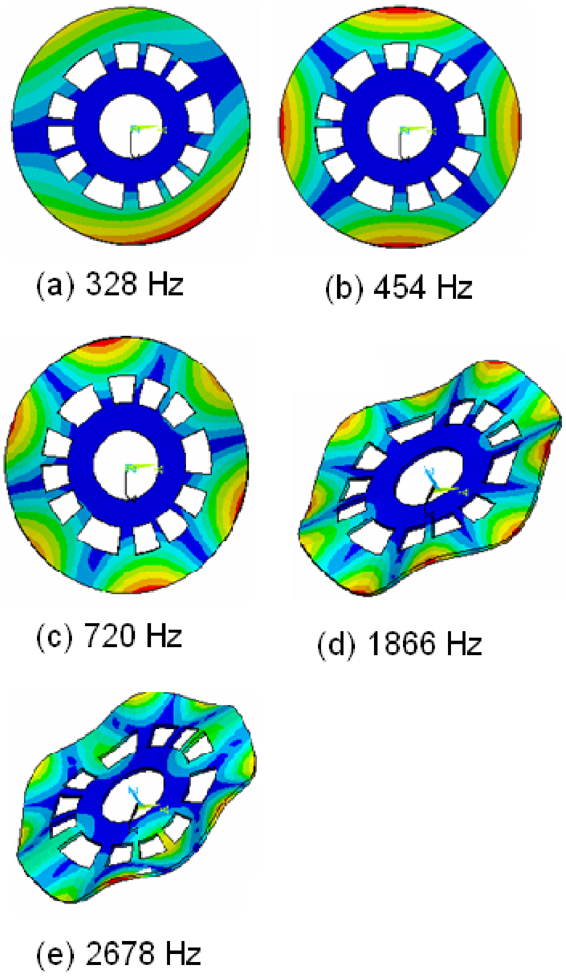

Figure 3. Diametral modes of the disc

The obtained mode shapes in the free vibration analysis can then be compared with the obtained complex mode shape in the complex eigenvalue analysis.

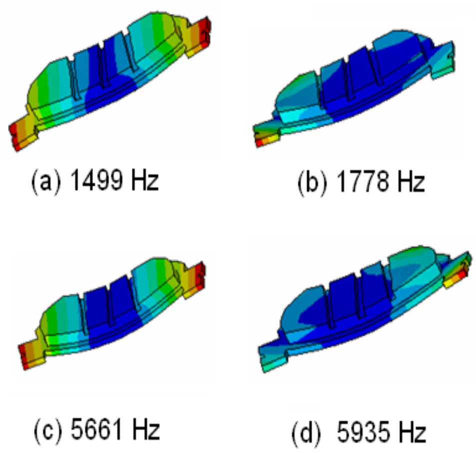

Figure 4. Bending modes of the pad

4. Complex Eigenvalue Analysis of the Coupled Disc-Pads System

The coupled model of the disc-pads is given in Figure 5. There are two contacts modeled in the analysis; one between the disc-pads and one between the pads and the support. The contact stiffness as in Equation 3 and 4 is modeled for the disc-pads contact. For the latter one, the contact is modeled with an array of springs each with assumed stiffness of 2 MN/m.

The complex eigenvalue analysis is conducted within the value of contact stiffness K_c from 0 MN/m to 250 MN/m with the increase of 50 MN/m. This is somewhat equal to the brake pressure of 0 to 5 bar [8]. The initial value of coefficient of friction is taken to be 0.4. The complex eigenvalue analysis is conducted for the frequency range below 5 kHz.

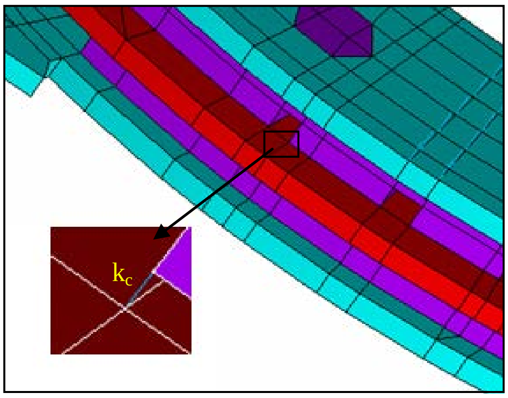

Figure 5. Coupled model of disc-pads

The analysis is initially conducted for the value of contact stiffness K_c of 0 MN/m. This value of contact stiffness means that there is no pressure given at the brake. As expected, there is no unstable mode existed at this value. The value of contact stiffness is then raised to 50 MN/m. At this value, there are three unstable modes exist indicated by the presence of three eigenvalues that have positive real part. The first unstable mode occurred at frequency of 1250 Hz with the value of real part 50 rad/s (50 + 1250 Hz). This unstable mode is paired with a stable mode of the same frequency (-58 – 1250 Hz) as also observed at other published result [2, 7].

The second unstable mode occurred at the frequency of 1920 Hz (94 + 1920 Hz) and its pair at (-74 – 1920 Hz). The third unstable mode occurred at the frequency of 2306 Hz (38 + 2306 Hz) and its pair at (-29 – 2306 Hz). The complex eigenvalue at contact stiffness of 50 MN/m is given in Figure 6 and the unstable mode is called as Mode A, Mode B, and Mode C beginning from the one that has the lowest frequency.

The mode shape of Mode A, Mode B, and Mode C is given in Figure 7. Mode A consists of disc vibrating in its 5th diametral mode indicated by the presence of 10 nodes and antinodes. In this mode, the pads are in the bending mode. Mode B consists of the same disc diametral mode with Mode A but with different bending mode of the pads. Mode C consists of disc vibrating at its 6th diametral mode and bending mode of the pads.

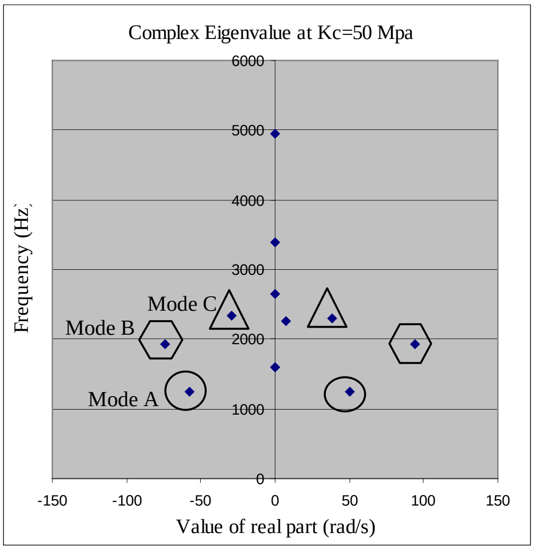

Figure 6. Complex eigenvalue at $K_c = 50$ MN/m

As the comparison, the 5th and the 6th diametral mode of the disc is vibrating at the frequency of 1866 Hz and 2678 Hz, respectively, in the free-free vibration. The coupling with the pads with friction force has shifted the frequency of these mode shapes. The 5th diametral mode is vibrating at the frequencies of 1250 Hz and 1920 Hz and the 6th diametral mode is vibrating at the frequency of 2306 Hz.

The analysis is then continued for the value contact stiffness until 250 MN/m with the increment of 50 MN/m. Analysis of the resulted complex eigenvalue is aiming at finding the unstable modes that have mode shape similar to the previous three unstable modes. The resulted complex eigenvalues are recorded and plotted as in Figure 8. Figure 8 then shows the effect of contact stiffness for the occurrence of dynamic instability of the analyzed brake model.

As it is shown in Figure 8, two eigenvalues that are initially separated become closer together as the value of contact stiffness increases until both of the eigenvalues are united together. At the same time, the real part of the united eigenvalues becomes positive. In the practical condition, this can be seen as follows: at low brake pressure, the rotor and the pads might have some vibration but occurs at their own particular natural frequencies. But as the brake pressure increased, the contact stiffness at the interfaces become so high that all the components now can vibrate together at one particular frequency. Because the energy fed to the system is larger that the energy converted to heat and wear, the rest of the energy will cause amplitude of vibration become larger and larger resulting in high frequency vibration called squeal.

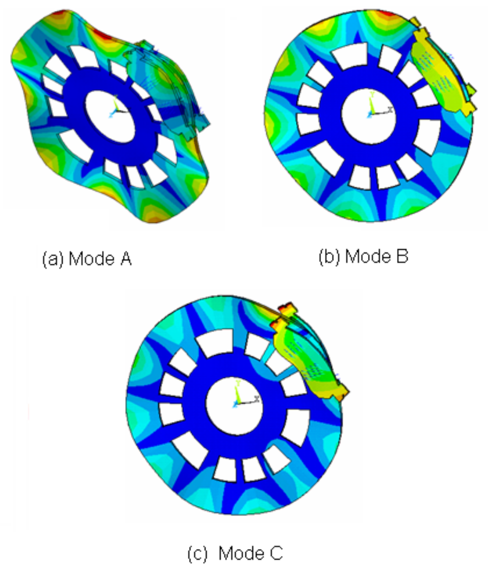

Figure 7. Unstable mode shapes

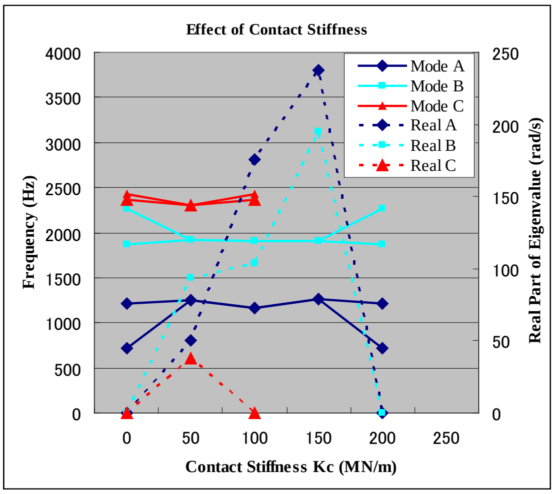

Figure 8. Effect of Contact Stiffness

Because each of the components has many vibration modes according to the frequency, several unstable modes could be ignited at the same time. This is observed in this analysis that at the same contact stiffness, there are three different unstable modes occurred. These unstable modes continued to exist for higher value of contact stiffness until become stable again at certain high value.

The coupling of eigenvalues when unstable modes occur and the breaking out of these eigenvalues when the contact stiffness increased to certain high value was also reported by other researchers [7-9].

Effect of coefficient of friction

The initial value of coefficient of friction used in the analysis is 0.4. In further analysis various value of coefficient of friction is applied to find out its effect to the dynamic stability of the system. In this analysis, the value of contact stiffness is kept at a value of 100 MN/m.

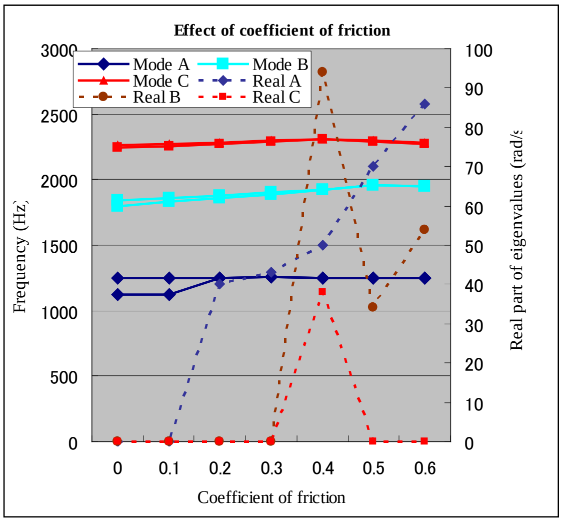

Figure 9. Effect of coefficient of friction

Figure 9 shows the result of the analysis. At the coefficient of friction of 0.1, no unstable mode exists although the contact stiffness is as high as 100 MN/m. The coupling of eigenvalues of Mode A starts at the value of coefficient of friction 0.2. But at this point, the other two modes still do not exist. Mode B and Mode C start occurring at the value of coefficient of 0.4. While Mode C disappears as soon as the coefficient of friction increased to 0.5, the other two modes continued occurring up to the coefficient of friction 0.6. In this particular case of analysis, it can be said that the critical coefficient of friction for the occurrence of system's dynamic instability is 0.2.

4. Conclusion

The present analysis has shown the possible propensity of disc brake squeal at a particular motorcycle disc brake system. In this case, squeal is associated to the occurrence of system's dynamic instability indicated by the presence of unstable mode in the complex eigenvalue analysis of a finite element model.

The analysis predicted that the main instability of the system can occur at the frequency of 1250 Hz, 1920 Hz, and 2306 Hz. The vibration mode involved in this instable condition is the diametral mode of the disc and the bending mode of the pads.

The dynamic instability can occur at the brake contact stiffness of 100 MN/m or at the brake pressure as low as 2 bar if the coefficient of friction at the contact surface is 0.2. The analysis also suggested that several dynamic unstable conditions can exist at the same time.

Although the result of this analysis has not been verified experimentally, most of the information obtained from the analysis is qualitatively comparable with the previous published researches in the same field.

One suggestion for avoiding the occurrence of squeal that can be made as the result of this analysis is to increase the stiffness of either the pads or the disc to shift the diametral or the bending mode frequency of this component. In this way, it is possible to separate (decoupled) the eigenvalue so that the system can be maintain in stable condition.

However, further analysis is still needed especially to find out the influence of parameters such as stiffness, damping, and dimension of the disc to the dynamic instability of the system. If such analysis can be done, it could be possible to suggest a more to the point ideas for preventing the occurrence squeal at the particular disc brake model, particularly in early design stage.

Acknowledgements

The authors would like to thank the Syiah Kuala University for providing the fund for this research through the program of Hibah SP4 2005.

Literatures

[1] Papinniemi, A., Joseph C., Lai, S., Zhao, J. & Loader, L. (2002). Brake squeal: a literature review, *Applied Acoustic*. 63(4), 391-400.

[2] Liles, G.D. "Analysis of disc brake squeal using finite element method", SAE paper 891150. 01-0142

[3] Hulten, J. (1995). Some drum brake squeal mechanisms. Chalmer University of technology, Sweden.

[4] Dihua, G. and Dongying, J., (1998). A study oc disc brake squeal using the finite element method. SAE paper #890547

[5] Dunlap, K.B., Riehle, M.A., Longhouse, R.E.(1999). An investigative overview of automotive disc brake noise. SAE paper 1999

[6] Kung, S.W., Dunlap, K.B. & Ballinger, R.S. (2000). Complex eigenvalue analysis for reducing low frequency brake squeal. *SAE* paper 00-01-0444.

[7] Ripin, Z.B.M., Barton, D.C. & Crolla, D.A. (1995). Analysis of disc brake squeal using the finite element method, *Institution of Mechanical Engineering, Autotech 95* paper C498/12/074.

[8] Ghesquire, H. & Castel, L. (1992). Brake squeal noise and prediction. Proc. I. Mech.E, Conf. *Autotech 1992*, paper C389/257.

[9] Sherif, H.A., Blouet, J., Cretegny, J.F., Gras, R., & Vialard, S. (1989). Experimental investigation of self-excited squeal. *SAE Technical Paper Series*, paper 872451