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ORIGINAL RESEARCH

Study of mathematical models of type 2 diabetes mellitus and its complications

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Abstract. The rapid increase in type 2 diabetes mellitus (DM) cases in Indonesia, driven by hereditary factors and unhealthy lifestyles, poses significant health challenges. This study develops a compartmental model to analyze the progression of type 2 DM and the onset of complications, classifying individuals into susceptible, patients without complications, patients with complications, and those disabled due to complications. The model examines the influence of two key factors on the recovery from complications: habitual factors, including medication adherence, physical activity, dietary habits, smoking history, and environmental factors, such as stress levels, environmental support, patient trust, and compliance. The results indicate that habitual factors have a more substantial impact on mitigating complications compared to environmental factors, suggesting that lifestyle interventions are crucial in improving patient outcomes. The model also shows that an increase in behavioral interactions leading to disease progression results in instability, emphasizing the need for early and consistent behavioral interventions. This research offers valuable insights for healthcare providers and policymakers. By identifying the most influential factors in managing complications, the model can guide the development of targeted interventions that prioritize habitual changes, such as medication adherence and physical activity. Public health strategies can be tailored to emphasize the importance of these habitual factors, potentially reducing the burden of type 2 DM complications. Overall, this study highlights the critical role of personalized, behavior-focused interventions in the management and prevention of complications in patients with type 2 DM, offering a practical framework for improving patient care.

Keywords: Diabetes Mellitus, Type 2 DM, Complications, Equilibrium Points, Basic Reproductive Numbers

INTRODUCTION

Disease is typically brought on by germs, viruses, climatic or meteorological changes, or expulsion from the environment. Yet, similar to DM, the illness can develop due to genetics and lifestyle choices. An increase in blood sugar levels is a symptom of one chronic illness, diabetes mellitus [1]. Type 1 diabetes mellitus (type 1 DM), type 2 diabetes mellitus (type 2 DM), gestational diabetes mellitus, and a unique type of diabetes mellitus that develops as a result of other factors such diabetes monogyny syndrome are the four classifications into which diabetes is generally divided. Nevertheless, diabetes can be divided into three types: type 1 DM, type 2 DM, and gestational diabetes mellitus, depending on the rise in blood sugar levels in the body [2, 3]. Type 1 and type 2 DM are forms that are extremely prevalent in society. Type 1 diabetes is brought on by the pancreas' inability to generate insulin, which raises blood sugar levels in the body; type 2 diabetes is brought on by the loss of insulin's ability to carry glucose from the bloodstream to body cells [4].

According to projections made by the International Diabetes Federation in 2017, there would be an additional 425 million diabetics globally by the year 2045. Type 2 diabetes accounts for 90% of all cases of diabetes worldwide. Among of ten nations, Indonesia is the seventh with the lowest blood sugar management rates and a predicted increase in sufferers in 2045 [5]. DM presents a growing global health challenge, particularly due to its potential to cause severe and life-threatening complications if left untreated. If DM, especially type 2 DM, is not adequately managed or cured, it can lead to a range of complications, including cardiovascular diseases, kidney failure, nerve damage, and vision loss. These complications can significantly reduce life expectancy, increase the risk of disability, and impair the quality of life for patients. The burden of untreated DM also extends beyond individual health, leading to increased healthcare costs, loss of productivity, and strain on healthcare systems worldwide.

Diabetes sufferers will experience difficulties if blood sugar levels are higher. A type 2 DM patient if fasting blood sugar ≥ 126 mg/dl or random blood glucose ≥ 200 mg/dl will present some symptoms associated with diabetes. However, if fasting blood sugar is out of control, which is ≥ 240 mg/dl, the condition of patients suffering from diabetes is out of control so it can cause complications [6, 7]. Diabetes complications can be categorized as macro vascular complications and micro vascular complications. Someone who is in prediabetes or who suffers from type 2 DM has a risk of experiencing microvascular or macrovascular complications, and even mortality from these complications [8–10]. Macrovascular complications such as cardiovascular are closely related to insulin resistance that occurs in patients with type 2 diabetes, so this type of complication is more common in patients with type 2 DM [8]. For those with type 2 DM, micro vascular problems are the primary causes of kidney failure, foot ulceration and amputation, and possibly blindness. Macrovascular and microvascular issues can occur in people without diabetes, and prediabetes may also present with similar difficulties [9].

Those with complications of type 2 DM must put in time and effort to recover from their illness. Patients in the complication group have three possible conditions: complication-related mortality, disability, or recovery. Individuals with type 2 DM issues should choose to start the healing process right away, rather than waiting until things worsen. There are steps that can be taken, starting with working out, taking medication, and maintaining a healthy diet and way of life [6]. The environment in the complications group can build a positive and supportive aura so that a sense of enthusiasm is obtained and can reduce stress for complications sufferers. Type 2 DM patients who experience complications can be cured by consuming Western medicines and Chinese medicines. Both are considered comparable conditions, of course also strengthened by using recommendations by doctors and supporting given by family [10, 11]. Of these various aspects, there are two groups that can help recovery, namely habit factors such as food; and environmental factors such as support received from the sufferer's environment, both from family and medical personnel.

The use of mathematical modeling has helped in solving many health problems such as covid, tuberculosis, and possibly type 2 DM.Several mathematical models have been applied to tackle problems related to Type 2 DM, such as the glucose-insulin model, the SEIIT (Susceptible-Exposed-Infected-Insulin Therapy) model, and the Diabetes Complication (DC) model [12, 13]. A recent advancement in this area is the Susceptible Diabetes Complication (SDC) model, which extends the DC model. The SDC (Susceptible Diabetes Complication) model, a development of the DC model that aims to predict changes in the prevalence of diabetes in the United States with results sufficient to predict the prevalence of the disease [14], is another mathematical model in addition to the three previously mentioned models. Many methods are utilized to create mathematical models that aid in the solution of DM type 2 problems. In the SDC model, various parameters such as transmission rates, complication rates, and treatment factors are modeled using dynamic systems, integrating principles of differential.

The growing complexity of managing Type 2 diabetes mellitus (DM) and its associated complications necessitates advanced mathematical tools for understanding disease progression and optimizing treatment strategies. Complications arising from Type 2 DM, such as cardiovascular disease, kidney failure, and neuropathy, are influenced by multiple physiological and environmental factors. These factors interact in non-linear ways, making it challenging to predict outcomes and tailor interventions effectively. To address this challenge, mathematical modeling provides a framework to simulate the dynamic processes governing the onset and progression of complications in diabetic patients. By incorporating differential equations, optimization algorithms, and statistical methods, models can capture key interactions between glucose levels, insulin response, and complication risk over time. The objective of this study is to develop a comprehensive model that focuses on these Type 2 DM complications, aiming to improve predictions of patient outcomes and, ultimately, enhance the probability of recovery. This model will leverage real-world clinical data and employ numerical techniques to optimize treatment strategies, offering a more precise approach to managing complications and potentially improving long-term patient health.

METHODOLOGY

The mathematical model utilized to solve this issue comes from model development [14] and has four compartments: vulnerable, people without issues, people with complications, and individuals with disabilities as a result of complications. Regarding the compartment for patients with complications, a decision is made depending on the patient's preferred healing factor. Complications with habitual factors and difficulties with environmental variables were introduced as two new sections. Non-linear differential equations make up this model, which will be resolved by determining the equilibrium point and the fundamental reproduction number. In this study, an independent t-test will be employed for statistical analysis to examine the relationship between two linked factors in greater detail. The data were collected using a Likert scale, which assigns weights ranging from 0 to 1. The scale assumes that as individuals approach 0, they exhibit greater disobedience, while values closer to 1 indicate increasing obedience. This weighting system forms a core

Table 1. Weighted data of variables from each health-contributing factor

Factor	Variable	Assumed Weight	Compliance
Habitual	Consumption of drugs	1	obey
		0.3	Disobey
	Sport	0.9	obey
		0.2	Disobey
	Dietary habit	0.95	obey
		0.5	Disobey
	Smoking history	0.97	obey
		0.15	Disobey
Environmental	Environmental support	0.98	obey
		0.33	Disobey
	Stress	0.75	obey
		0.12	Disobey
	Patient trust	0.65	obey
		0.4	Disobey
	patient compliance	1	obey
		0.5	Disobey

assumption in the model, representing behavior on a continuous spectrum of compliance. By applying this assumption, the model aims to quantify and analyze the variations in obedience levels across the study population.

The Likert scale in Table 1 data provides insights into the levels of compliance or agreement with various habitual and environmental factors. For habitual factors, drug consumption shows a strong adherence with a score of 1 (Obey), indicating full compliance, while a score of 0.3 (Disobey) suggests a low level of adherence. Participation in sports is generally high with a score of 0.9 (Obey), but there is a notable drop to 0.2 (Disobey) for those with less engagement. Dietary habits receive a high compliance score of 0.95 (Obey), yet a score of 0.5 (Disobey) reflects moderate disagreement or lower adherence. Smoking history shows very high compliance with a score of 0.97 (Obey), compared to a significantly lower score of 0.15 (Disobey). In terms of environmental factors, environmental support is highly regarded with a score of 0.98 (Obey), whereas a score of 0.33 (Disobey) indicates less agreement. Stress management is moderately supported with a score of 0.75 (Obey), but there is a minimal agreement of 0.12 (Disobey). Patient trust in the system is moderately high at 0.65 (Obey) and shows moderate disagreement at 0.4 (Disobey). Patient compliance receives the highest compliance score of 1 (Obey), but a score of 0.5 (Disobey) reflects moderate adherence. Overall, these scores illustrate varying degrees of adherence and agreement with different factors influencing habitual and environmental contexts.

Mathematical Model

The model was developed by adding several compartments related to complications, namely the subpopulation with disabilities after complications (T), sufferers of complications supported by habitual factors (H), and sufferers of complications supported by environmental factors (E). The total population in this model is denoted as N with subpopulations in this model, namely: subpopulation susceptible to diabetes mellitus (S), subpopulation with diabetes without complications (D), subpopulation with diabetes and complications (C), subpopulation with disabilities due to complications (T), subpopulation with complications supported by habitual factors (H), and subpopulation with complications supported by environmental factors (E). The total population can be written as follows:

$$\mathrm{N} = \mathrm{S} + \mathrm{D} + \mathrm{C} + \mathrm{T} + \mathrm{H} + \mathrm{E}.$$

Symbols of variable and parameters used in the research model and parameters values used in the research model are tabulated in Table 2 and 3, respectively. In this model, several key assumptions are made to describe the progression and potential outcomes for individuals at risk of or diagnosed with type 2 diabetes mellitus (DM). These assumptions include:

- Every individual who is born will be included in the susceptible group for diabetes mellitus (S).
- Individuals who have descendants who have had type 2 DM will be included in the uncomplicated diabetes group (D).
- Individuals who are in susceptible groups are at risk of developing type 2 DM if the behavior or lifestyle that is applied daily does not comply with health recommendations. Individuals who have type 2

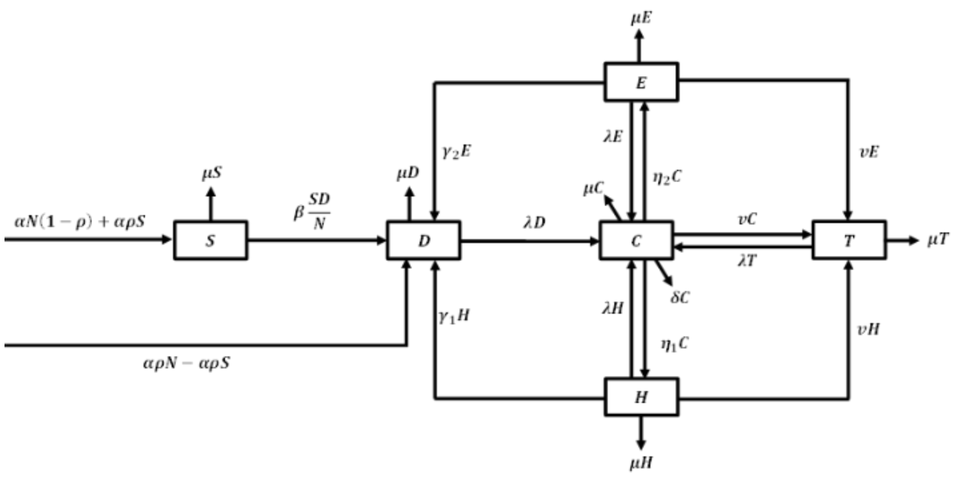

Figure 1. Transfer diagram of type 2 DM.

Table 2. Symbols of variable and parameters used in the research model

Symbol/ Parameter	Description
N	The number of individuals in a population
S	The number of individuals who are susceptible to diabetes mellitus
D	Number of individuals with diabetes without complications
C	Number of individuals with diabetes with complications
T	Number of individuals with disabilities due to complications
H	The number of individuals with complications supported by habitual factors
E	The number of individuals with complications supported by environmental factors
α	The natural birth rate of the individual population
μ	The natural death rate of the individual population
ρ	The proportion of individuals who have genetic factors with diabetes mellitus
β	The proportion of the number of individuals who have behavioral factors become people with diabetes mellitus
λ	Rate of occurrence of complications
v	Rate of disability due to complications
γ_1	The cure rate of complications becomes without complications by using habitual factors
γ_2	The cure rate for complications becomes uncomplicated using environmental factors
η_1	Individual rates of complications are supported by habitual factors
η_2	Individual levels of complications are supported by environmental factors

Table 3. Parameters values used in the research model

Parameter	Value	Sources/ References
α	0.00116817	[15]
μ	0.001167379	[15]
ρ	0.077	[14]
β	(0.48; 0.0009)	[16, 17]
λ	0.125	[18]
v	0.05	[13]
γ_1	0.37141	[14]
γ_2	0.08	[13]
η_1	(0.19; 0.04; 0.015)	Assumption
η_2	(0.08; 0.18; 0.019)	Assumption

DM without complications (D) are at risk for complications.

- If individuals who are in the type 2 DM group without complications (D) experience complications, they will be included in the type 2 DM group with complications (C).
- Individuals who experience complications have the possibility of recovery if they choose two factors for their healing process, namely habitual factors or environmental factors.
- Individuals who have implemented these factors will then be included in the group of people with complications using habit factors (H) or the group of people with complications using environmental factors (E).
- Individuals who are in the group of sufferers from complications have a chance to recover, but if they experience complications again during healing, they will be in the type 2 DM group with complications (C).
- Individuals who experience complications have the condition to die or experience disability after complications.
- Individuals with post-complication disabilities will be in the post-complication disability group (T).
- Each group experienced natural death with the same mortality rate and type 2 DM could not be saved.

The mathematical model can be constructed as follows:

Based on Figure 1, a system of non-linear equations can be formed as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= \alpha N(1-\rho) + \alpha\rho S - \beta\frac{SD}{N} - \mu S,\\
\frac{dD}{dt} &= \alpha\rho N - \alpha\rho S + \beta\frac{SD}{N} + \gamma_1 H + \gamma_2 E - (\lambda+\mu)D,\\
\frac{dC}{dt} &= \lambda(D+T+H+E) - (\delta+\mu+\eta_1+\eta_2+v)C,\\
\frac{dT}{dt} &= v(C+H+E) - (\mu+\lambda)T,\\
\frac{dH}{dt} &= \eta_1 C - (\mu+\lambda+\gamma_1+v)H,\\
\frac{dE}{dt} &= \eta_2 C - (\mu+\lambda+\gamma_2+v)E.
\end{aligned} \tag{1}
$$

Normalization of state variables is done by assuming:

$$s = \frac{S}{N};\ i_d = \frac{D}{N};\ i_c = \frac{C}{N};\ i_t = \frac{T}{N};\ r_h = \frac{H}{N};\ r_e = \frac{E}{N}.$$

The state variables used must then be divided by the total population:

$$\frac{ds}{dt} = \alpha(1\text{-}\rho) + \alpha\rho s - \beta s i_d - \mu s,$$
$$\frac{di_d}{dt} = \alpha\rho - \alpha\rho s + \beta s i_d + \gamma_1 r_h + \gamma_2 r_e - (\lambda+\mu) i_d,$$
$$\frac{di_c}{dt} = (i_d+i_t+r_h+r_e)\lambda - (\delta+\mu+\eta_1+\eta_2+v) i_c, \qquad (2)$$
$$\frac{di_t}{dt} = (i_c+r_h+r_e)v - (\mu+\lambda) i_t,$$
$$\frac{dr_h}{dt} = \eta_1 i_c - (\mu+\lambda+\gamma_1+v) r_h,$$
$$\frac{dr_e}{dt} = \eta_2 i_c - (\mu+\lambda+\gamma_2+v) r_e,$$

where the initial conditions for Equations 2 are $s = 2.9\%$; $i_d = 3\%$; $i_c = 65{,}4\%$; $i_t = 11\%$; $r_h = 5.2\%$; and $r_e = 12.5\%$.

The use of two values for beta and three assumed values for the last two parameters stems from the need to explore multiple scenarios and their potential impact on the model. By incorporating different values for beta, we aim to account for variability in certain factors, such as the rate of change in the complications or responsiveness to treatment. Similarly, assigning three assumed values for the last two parameters allows for a more comprehensive analysis of how these variables influence the model's outcomes. These varying parameter values are not based on empirical data but are hypothetical cases constructed to simulate different potential real-world conditions. This approach enables the model to assess a broader range of possibilities and provides insights into how these assumptions affect the dynamics of the system being studied.

RESULTS AND DISCUSSION

Equilibrium Point

The equilibrium point of a non-linear system is crucial for stability analysis, which is performed by linearizing the system around this point [19]. To find the equilibrium points, the non-linear equations are linearized, resulting in a Jacobian matrix. The process begins by setting each equation to be time-invariant. The complication-free equilibrium point is a situation where there are no complications in the region, so the results are as $E_0(s^*, i_d^*, 0, 0, 0, 0)$, where $s^* = \frac{\alpha(1\text{-}\rho)}{\mu\text{-}\alpha\rho+\beta i_d^*}$ and $i_d^* = \frac{\alpha\rho s^*\text{-}\alpha\rho}{\beta s^*\text{-}\mu\text{-}\lambda}$. It is related to the complication equilibrium points, namely the situation where there are complications in that conditions. The complication equilibrium point obtained is $E_1(s^*, i_d^*, i_c^*, i_t^*, r_h^*, r_e^*)$, where $s^* = \frac{\alpha(1\text{-}\rho)}{\mu\text{-}\alpha\rho+\beta i_d^*}$, $i_d^* = \frac{\alpha\rho s^*\text{-}\alpha\rho\text{-}\gamma_1 r_h^*\text{-}\gamma_2 r_e^*}{\beta s^*\text{-}\lambda\text{-}\mu}$, $i_c^* = \frac{(i_d^*+i_t^*+r_h^*+r_e^*)\lambda}{\delta+\mu+\eta_1+\eta_2+v}$, $i_t^* = \frac{(i_c^*+r_h^*+r_e^*)v}{\mu+\lambda}$, $r_h^* = \frac{\eta_1 i_c^*}{\mu+\lambda+\gamma_1+v}$ and $r_e^* = \frac{\eta_2 i_c^*}{\mu+\lambda+\gamma_2+v}$.

However, in this research, the authors focus on two equilibrium points for several reasons. First, these two points may represent the most relevant or significant states for the specific context of the study, such as a stable disease state and a recovery state. Second, the analysis might be simplified to focus on the equilibrium points that are biologically or clinically meaningful, while other potential equilibria may be less relevant or unstable, making them less significant for the practical application of the model.

Basic Reproduction Number

The basic reproduction number (R_0) is a number obtained from secondary cases of infectious conditions from an area so that over time it can suppress time so that infection cases can decrease [20]. If $R_0 < 1$ is obtained, the disease will decrease over time, but if $R_0 > 1$ then disease can develop around the population and can cause epidemics [21]. The basic reproductive number (R_0) has an important role because it can help to control the number of cases of complications in the region or population so that complications can be reduced. The subpopulations involved were diabetes without complications (D), diabetes with complications (C), complications subpopulation supported by habitual factors (H), and complications supported by environmental factors (E), where the equations these can be written as follows:

$$\frac{dD}{dt} = \alpha\rho N - \alpha\rho S + \beta\frac{SD}{N} + \gamma_1 H + \gamma_2 E - (\lambda+\mu)D,$$
$$\frac{dC}{dt} = \lambda(D+T+H+E) - (\delta+\mu+\eta_1+\eta_2+v)C, \qquad (3)$$
$$\frac{dH}{dt} = \eta_1 C - (\mu+\lambda+\gamma_1+v)H,$$
$$\frac{dE}{dt} = \eta_2 C - (\mu+\lambda+\gamma_2+v)E.$$

From these equations, a matrix $G = FV^{-1}$ can be formed, where F represents the matrix of transmission rates and V is the matrix of the rate of disease recovery. The basic reproduction number (R_0) is then obtained as the spectral radius (the dominant eigenvalue) of the matrix G. Then, it is obtained the basic reproduction number as follows:

$$R_0 = \frac{\eta_1\lambda(\gamma_2+\lambda+\mu+v)(\delta+\eta_1+\eta_2+\mu+v)}{(\gamma_1+\lambda+\mu+v)(\delta+\eta_1+\eta_2+\mu+v)} \qquad (4)$$

From choosing different values for the two parameters β to look for R_0, it can be showed that as individuals increasingly choose factors that contribute to the resolution of complications, the overall incidence of complications in a given area decrease. This implies that enhanced adherence to effective treatment strategies or health-promoting behaviors directly correlates with a reduction in complications, ultimately leading to a lower R_0 and improved health outcomes within the population.

Stability Analysis

Stability analysis involves examining the eigenvalues of the system of equations to determine the nature of the equilibrium points. By calculating the eigenvalues, we can assess the stability of these points; if all eigenvalues have negative real parts, the equilibrium point is considered stable, indicating that perturbations will decay over time. Conversely, if any eigenvalue has a positive real part, the equilibrium point is unstable, suggesting that small disturbances could lead to significant changes in the system's behavior. The model used is a form of a system of non-linear equations, so a linearization step must be carried out first before finding the eigenvalues of the system of equations. The linearization process by assuming the equation first will then partially derive the variables in the related functions by forming a Jacobian matrix. The eigenvalues obtained are as in Table 4 and Table 5.

Table 4. Eigen Values in Complication-Free States

No	Eigen value	
	β = 0.48	***β = 0.0009***
1	$\lambda_{e1} = 0.369476217421$	$\lambda_{e1} = -0.556597224184$
2	$\lambda_{e2} = -0.635299449130$	$\lambda_{e2} = -0.543533967750$
3	$\lambda_{e3} = -0.001292859697$	$\lambda_{e3} = -0.000254276136$
4	$\lambda_{e4} = -0.438599709666$	$\lambda_{e4} = -0.032014743987$
5	$\lambda_{e5} = -0.088506451209$	$\lambda_{e5} = -0.125879510988$
6	$\lambda_{e6} = -0.240726059221$	$\lambda_{e6} = -0.256084398442$
	Not stable	Stable

Table 5. Eigen Values in Complication States

No	Eigen value at $\eta_1 = 0.19$ and $\eta_2 = 0.08$	
	β = 0.48	***β = 0.0009***
1	$\lambda_{e1} = 0.3694811316382$	$\lambda_{e1} = -0.556597225478$
2	$\lambda_{e2} = -0.635299659298$	$\lambda_{e2} = -0.543533966388$
3	$\lambda_{e3} = -0.001292846799$	$\lambda_{e3} = -0.000254275612$
4	$\lambda_{e4} = -0.438599431613$	$\lambda_{e4} = -0.032014743807$
5	$\lambda_{e5} = -0.088506390462$	$\lambda_{e5} = -0.125879510849$
6	$\lambda_{e6} = -0.240726007384$	$\lambda_{e6} = -0.256084398403$
	Not stable	Stable
	Eigen value at $\eta_1 = 0.04$ and $\eta_2 = 0.18$	
	β = 0.48	***β = 0.0009***
7	$\lambda_{e1} = 0.362296348928$	$\lambda_{e1} = -0.000257546170$
8	$\lambda_{e2} = -0.001340472245$	$\lambda_{e2} = -0.035481957928$
9	$\lambda_{e3} = -0{,}071838699711$	$\lambda_{e3} = -0.125757182127$
10	$\lambda_{e4} = -0.224475046082$	$\lambda_{e4} = -0.255936539259$
11	$\lambda_{e5} = -0.476026218445$	$\lambda_{e5} = -0.499193875972$
12	$\lambda_{e6} = -0.573560433150$	$\lambda_{e6} = -0.547737019517$
	Not stable	Stable
	Eigen value at $\eta_1 = 0.015$ and $\eta_2 = 0.019$	
	β = 0.48	***β = 0.0009***
13	$\lambda_{e1} = 0.357283576313$	$\lambda_{e1} = -0.000260656097$
14	$\lambda_{e2} = -0.550460946365$	$\lambda_{e2} = -0.547589116598$
15	$\lambda_{e3} = -0.001439583534$	$\lambda_{e3} = -0.061081428686$
16	$\lambda_{e4} = -0.074469171329$	$\lambda_{e4} = -0.125539124739$
17	$\lambda_{e5} = -0.294736347086$	$\lambda_{e5} = -0.287937027012$
18	$\lambda_{e6} = -0.235124698271$	$\lambda_{e6} = -0.255956768257$
	Not stable	Stable

Complication-free states refer to conditions or scenarios within a model where individuals do not experience any adverse health outcomes or complications related to a specific disease, such as Type 2 diabetes. In the context of a mathematical or epidemiological model, these states represent an optimal health scenario in which patients maintain their health without progressing to more severe conditions that would typically result from the disease.

Table 4 shows two scenarios with distinct β values, revealing that one of these configurations results in a stable condition, as indicated by the presence of all negative eigenvalues. This stability suggests that perturbations in the system will decay over time, maintaining the equilibrium point. Table 5, on the other hand, explores a wider range of β values, leading to the identification of additional equilibrium points. This increase in variations demonstrates the model's complexity and its ability to capture various dynamics influenced by changes in β. Notably, stability was achieved for certain combinations of parameters η_1, η_2, and β, indicating specific conditions under which the system can maintain equilibrium despite the presence of complications.

Graphic Illustration

A graphic illustration provides a simulation using the value of the parameter β or the level of behavioral interaction. β that can cause diabetes mellitus is 0.48 and 0.0009. The results to be shown are in the form of percent (%) of the population as the y-axis and time (month) = t as the x-axis.

Figure 2(a) is a simulation of the parameter $\beta = 0.48$ with $\eta_1 = 0.19$ and $\eta_2 = 0.08$ indicating that the S subpopulation experienced a decrease in rate resulting in a decrease in population, but then experienced an increase in rate at $t = 26$ to bring a population change of 18%. This affects the D subpopulation which has increased by 18% when $t = 7$, but has decreased when $t \geq 8$ namely 0.2%. Subpopulation C will experience a declining rate, resulting in a 15% decrease in its population. This decline occurs because individuals with complications are choosing one of two factors that can aid in their recovery. Conversely, subpopulation H will see an increase of 32%, with a subsequent decrease to 1% of its population as individuals either recover and transition back to subpopulation C or move to subpopulation T. Subpopulation T, on the other hand, will increase by 0.4% initially but will then experience a decline of 9% to 0.2% of the population at $t = 9$. Additionally, subpopulation E will witness a 2% increase in its population, followed by a slight decrease of 0.1%.

For the simulation $\beta = 0.48$, $\eta_1 = 0.04$ and $\eta_2 = 0.18$ (see Figure 2(b)), the S subpopulation decreases when $t = 3$ by 3% of the population, but it also increases when $t \geq 39$ by 17% of the population. For subpopulation D, the rate will change until $t = 8$, close to 10% of the population. This increase was influenced by a decrease in subpopulation S, but subpopulation D will again experience a decrease when $t = 10$ onwards as much as 1% of the population. Subpopulation C will decrease to $t = 8$ as much as 13% of the population, but will again experience a decrease at a constant rate of 0.06% of the population. H and E subpopulations will experience an increase because individuals in subpopulation C choose one of the two factors to help their healing, but the decline in the rate in subpopulation H is faster, namely when $t = 7$ as much as 1% of the population compared to subpopulation E at $t = 11$ as many as 0.3% of the population. For the T subpopulation it will start with a significant increase until $t = 0.6$ approaching 2.8% of the population and will decrease by 0.89% of the population when $t = 11$. The simulation when $\beta = 0.48$, $\eta_1 = 0.015$ and $\eta_2 = 0.019$ is shown by Figure 2(c). The figure shows that the S subpopulation has decreased in rate resulting in a population decline of 11.7%, but will increase again by 22.3%. Subpopulation D will experience an increase in rate when $t = 9$ is close to 4% of the population, followed by a decrease in rate when $t \geq 10$ by 0.7% of the population. Subpopulation C will experience an increase

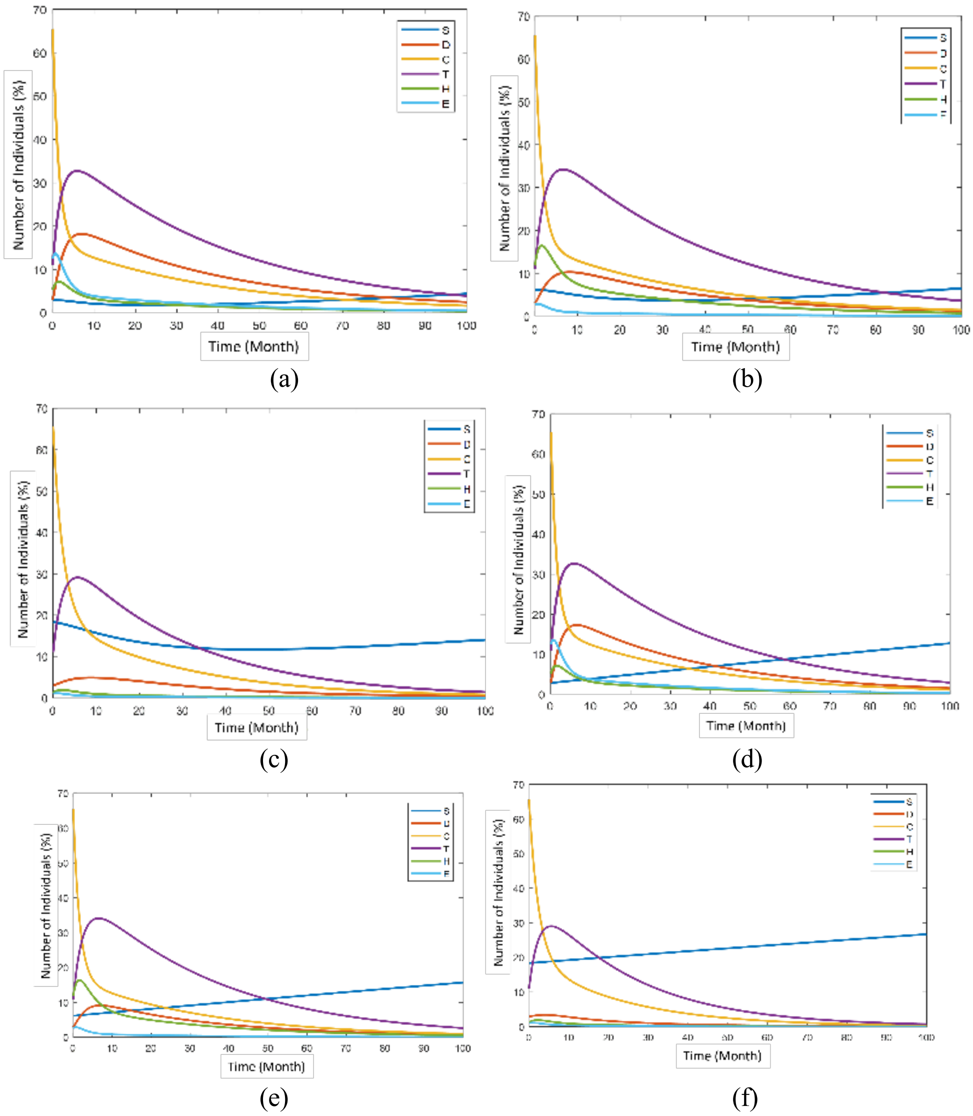

Figure 2. Graphic illustrations related to complications of type 2 DM

of close to 14% of the population, but will again experience a decrease of close to 0.06% of the population. Subpopulations H and E will experience changes in their rate of increase followed by a decrease when t = 5 and t = 2 approach 29% of the population and 1.8% of the population and will experience a further decrease of up to end of simulations. For the T subpopulation, the rate will increase but with a short span of 1 month and will decrease again until t = 12, approaching 0.3% of the population.

The simulation shown in Figure 2(d) to Figure 2(f) is a simulation when $\beta = 0.0009$. Figure 2(d) is a simulation when $\eta_1 = 0.19$ and $\eta_2 = 0.08$ where for the S subpopulation it can be seen that there is a change in rate at a certain time. The change in rate increase that occurred in the S subpopulation was significant. However, as time goes by the slower the rate increases. Subpopulation D initially experienced an increase in rate to close to 6.7% of the population at t = 6 which then decreased until end of simulation to reach stability. Subpopulation C begins with a decrease in rate of 14% until t = 6, then experiences an increase in time until the end of simulations which is caused by individuals choosing one of the factors that help their recovery. The H and E subpopulations experienced a significant increase of 32.6% and 7% of the population respectively, which will further decrease towards stability. The T subpopulation also experienced a significant increase when t = 0.6 of 13.6% of the population and then experienced a decrease in rate which resulted in a population decrease of 3.7%.

Figure 2(e) using the parameters $\eta_1 = 0.04$ and $\eta_2 = 0.18$ shows that the S subpopulation experienced a significant change in rate up to close to 9% of the population at t = 7. It affects other subpopulations such as subpopulation D which has experienced a decrease in rate due to individuals experiencing complications so that the decline will further decrease towards stability until the end of simulation. Subpopulation C will experience a decrease in the rate of increase over time towards stability up to 13% of the population. The H and E subpopulations will experience a significant increase as a result of the selection made by people with complications, but over time they will also experience a decrease until they reach stability until the end of simulation. The T subpopulation experienced a change in rate at a certain time starting with an increase to close to 2.8% at t = 2 and then experienced a decrease back towards stability starting at t = 11 of 0.8%.

The simulation shown by Figure 2(f) using $\eta_1 = 0.015$ and $\eta_2 = 0.019$. The S subpopulation shows that the changes that occur are very significant, experiencing an increase and will experience a slower rate over time. Subpopulation D also experienced a significant increase in rate to close to 3% of the population at t = 4, but again experienced a decrease in rate towards stability until the end of simulation. Subpopulation C shows that there is a decreasing rate of change until t = 10 for 13% of the population, but it will increasingly be towards stability until the end of simulation. Subpopulations H and E will experience a significant increase in rate, each approaching 28% and 1.7% of the population, which will then also experience a decrease towards stability until the end of simulation. For the t subpopulation, it shows that a significant change in rate only occurs in a short time interval, namely when t = 0 to t = 1.7, then it will also experience a decrease in rate towards stability until the end of simulation.

The decrease in the individual that occurs in subpopulation S is due to behavioral interactions that contribute to the development of Type 2 diabetes mellitus (DM) among at-risk individuals. As a result, many susceptible individuals are diagnosed with Type 2 DM, leading to an increase in subpopulation D. Individuals who have been affected by type 2 DM will have the opportunity to experience complications if their sugar levels exceed the predetermined limits, so that subpopulation C will experience an increase in the population. Furthermore, the individual is given the option to choose one of the factors that can help in the process of healing complications. If they choose habitual factors, they will transition to subpopulation H; if they choose environmental factors, they will move to subpopulation E. As a result, subpopulation C will experience a decrease in its rate, while subpopulations H and E will see an increase. Additionally, subpopulation D may experience a rise in its return rate or overall population, as individuals who have successfully recovered from complications transition back into this subpopulation. Individuals who experience disabilities as a result of complications will belong to subpopulation T, leading to an increase in its rate. Consequently, this will result in a rise in the overall number of individuals within this subpopulation.

CONCLUSION

Patients with type 2 DM who experience complications can choose one of the factors that can help in the healing process, namely habitual factors and environmental factors. The model was analyzed at the equilibrium point and its stability against complications. Increased behavioral interactions contribute to a progressively unstable condition, which can adversely affect the number of individuals suffering from Type 2 diabetes mellitus (DM) in a given area. This instability may also lead to a rise in the number of Type 2 DM sufferers who experience complications, exacerbating the overall health crisis in the population. As for the model that has been completed, habitual factors dominate to help cure complications in type 2 DM sufferers compared to environmental factors by complying with all conditions and orders given by medical experts. Based on this, the number of complications in an area can decrease. Meanwhile, after being reviewed using statistical tests, mathematical models that have been built and applied by related individuals and when applied with good principles of rules and recommendations, will provide a chance of recovery of up to 74%. However, sample size and diversity may affect the generalizability of the findings.

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